Set and Set Theories

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Introduction

A ***set*** *is a collection of objects that have something in common or follow a rule. The objects in the set are called its elements. Set notation uses curly braces, with elements separated by commas.*

A general set is denoted within curly braces.

A = {1, 2, 3, 4}. Here A is a set and inside the set are the elements of the set.

Only the capital letters are used to denote a set and lowercase letters are used to denote subsets.

Operations on Set

The symbol ∪ is employed to denote the union of two sets. Thus, the set $A \cup B$—read "*A* union *B*" or "the union of *A* and *B*"—is defined as the set that consists of all elements belonging to either set *A* or set *B* (or both). For example, suppose that Committee *A*, consisting of the 5 members Jones, Blanshard, Nelson, Smith, and Hixon, meets with Committee *B*, consisting of the 5 members Blanshard, Morton, Hixon, Young, and Peters. Clearly, the union of Committees *A* and *B* must then consist of 8 members rather than 10—namely, Jones, Blanshard, Nelson, Smith, Morton, Hixon, Young, and Peters.

This is the example of a Union.

The intersection operation is denoted by the symbol ∩. The set $A \cap B$—read "*A* intersection *B*" or "the intersection of *A* and *B*"—is defined as the set composed of all elements that belong to both *A* and *B*. Thus, the intersection of the two committees in the foregoing example is the set consisting of Blanshard and Hixon.

If *E* denotes the set of all positive even numbers and *O* denotes the set of all positive odd numbers, then their union yields the entire set of positive integers, and their intersection is the empty set. Any two sets whose intersection is the empty set are said to be disjoint.

When the admissible elements are restricted to some fixed class of objects *U*, *U* is called the universal set (or universe). Then for any subset *A* of *U*, the complement of *A* (symbolized by A' or $U - A$) is defined as the set of all elements in the universe *U* that are not in *A*. For example, if the universe consists of the 26 letters of the alphabet, the complement of the set of vowels is the set of consonants.

In analytic geometry, the points on a Cartesian grid are ordered pairs (x, y) of numbers. In general, $(x, y) \neq (y, x)$; ordered pairs are defined so that $(a, b) = (c, d)$ if and only if both $a = c$ and $b = d$. In contrast, the set $\{x, y\}$ is identical to the set $\{y, x\}$ because they have exactly the same members.

The Cartesian product of two sets *A* and *B*, denoted by $A \times B$, is defined as the set consisting of allordered pairs (a, b) for which $a \in A$ and $b \in B$. For example, if $A = \{x, y\}$ and $B = \{3, 6, 9\}$, then $A \times B = \{(x, 3), (x, 6), (x, 9), (y, 3), (y, 6), (y, 9)\}$.

This is called Intersection.

Relation In Set Theory

In mathematics, a relation is an association between, or property of, various objects. Relations can be represented by sets of ordered pairs (a, b) where a bears a relation to b. Sets of ordered pairs are commonly used to represent relations depicted on charts and graphs, on which, for example, calendar years may be paired with automobile production figures, weeks with stock market averages, and days with average temperatures.

A function f can be regarded as a relation between each object x in its domain and the value $f(x)$. A function f is a relation with a special property, however: each x is related by f to one and only one y. That is, two ordered pairs (x, y) and (x, z) in f imply that $y = z$.

A one-to-one correspondence between sets A and B is similarly a pairing of each object in A with one and only one object in B, with the dual property that each object in B has been thereby paired with one and only one object in A. For example, if $A = \{x, z, w\}$ and $B = \{4, 3, 9\}$, a one-to-one correspondence can be obtained by pairing x with 4, z with 3, and w with 9. This pairing can be represented by the set $\{(x, 4), (z, 3), (w, 9)\}$ of ordered pairs.

Many relations display identifiable properties. For example, in the relation “is the same colour as,” each object bears the relation to itself as well as to some other objects. Such relations are said to be reflexive. The ordering relation “less than or equal to” (symbolized by $\leq$) is reflexive, but “less than” (symbolized by $<$) is not. The relation “is parallel to” (symbolized by $\parallel$) has the property that, if an object bears the relation to a second object, then the second also bears that relation to the first. Relations with this property are said to be symmetric. (Note that the ordering relation is not symmetric.) These examples also have the property that whenever one object bears the relation to a second, which further bears the relation to a third, then the first bears that relation to the third—e.g., if $a < b$ and $b < c$, then $a < c$. Such relations are said to be transitive.

Relations that have all three of these properties—reflexivity, symmetry, and transitivity—are calledequivalence relations. In an equivalence relation, all elements related to a particular element, say a, are also related to each other, and they form what is called the equivalence class of a. For example, the equivalence class of a line for the relation “is parallel to” consists of the set of all lines parallel to it.

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