

Theory of Polarisation of Light

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Abstract:

In this Paper we have studied of Polarisation of Light, and explain Quarter wave plate half wave plate, Specific Rotation of Optical Substance

Introduction

Polarization (also polarisation) is a property applying to transverse waves that specifies the geometrical orientation of the oscillations.^{[1][2][3][4][5]} In a transverse wave, the direction of the oscillation is transverse to the direction of motion of the wave, so the oscillations can have different directions perpendicular to the wave direction.^[4] A simple example of a polarized transverse wave is vibrations traveling along a taut string (*see image*); for example, in a musical instrument like a guitar string. Depending on how the string is plucked, the vibrations can be in a vertical direction, horizontal direction, or at any angle perpendicular to the string. In contrast, in longitudinal waves, such as sound waves in a liquid or gas, the displacement of the particles in the oscillation is always in the direction of propagation, so these waves do not exhibit polarization. Transverse waves that exhibit polarization include electromagnetic waves such as light and radio waves, gravitational waves,^[6] and transverse sound waves (shear waves) in solids. In some types of transverse waves, the wave displacement is limited to a single direction, so these also do not exhibit polarization; for example, in surface waves in liquids (gravity waves), the wave displacement of the particles is always in a vertical plane.

An electromagnetic wave such as light consists of a coupled oscillating electric field and magnetic field which are always

perpendicular; by convention, the "polarization" of electromagnetic waves refers to the direction of the electric field. In linear polarization, the fields oscillate in a single direction. In circular or elliptical polarization, the fields rotate at a constant rate in a plane as the wave travels. The rotation can have two possible directions; if the fields rotate in a right hand sense with respect to the direction of wave travel, it is called right circular polarization, or, if the fields rotate in a left hand sense, it is called left circular polarization.

Light or other electromagnetic radiation from many sources, such as the sun, flames, and incandescent lamps, consists of short wave trains with an equal mixture of polarizations; this is called unpolarized light. Polarized light can be produced by passing unpolarized light through a polarizing filter, which allows waves of only one polarization to pass through. The most common optical materials (such as glass) are isotropic and do not affect the polarization of light passing through them; however, some materials—those that exhibit birefringence, dichroism, or optical activity—can change the polarization of light. Some of these are used to make polarizing filters. Light is also partially polarized when it reflects from a surface.

According to quantum mechanics, electromagnetic waves can also be viewed as streams of particles called photons. When viewed in this way, the polarization of an electromagnetic wave is determined by a

quantum mechanical property of photons called their spin. A photon has one of two possible spins: it can either spin in a right hand sense or a left hand sense about its direction of travel. Circularly polarized electromagnetic waves are composed of photons with only one type of spin, either right- or left-hand. Linearly polarized waves consist of equal numbers of right and left hand spinning photons, with their phase synchronized so they superpose to give oscillation in a plane.

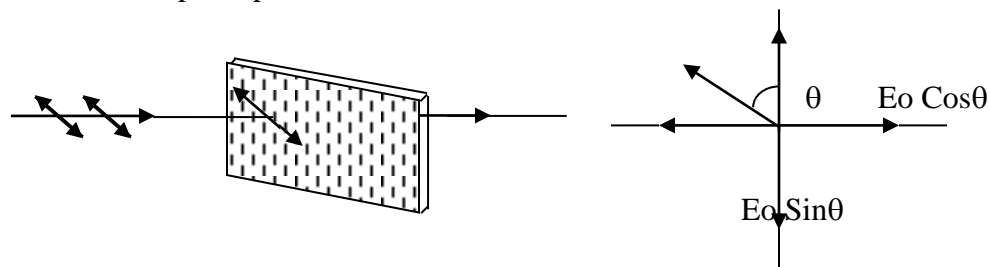
Polarization is an important parameter in areas of science dealing with transverse waves, such as optics, seismology, radio, and microwaves. Especially impacted are technologies such as lasers, wireless and optical fiber telecommunications, and radar.

Production of Circularly & Elliptically Polarized Light:

When two plane-polarized waves of same frequency whose electric vectors are perpendicular to each other and have a fixed phase difference, are superimposed, then the

resultant wave is elliptically polarized light. Under certain conditions this elliptically polarized light is converted into plane polarized or circular polarized light.

If a plate is cut from the calcite crystal with its optic axis parallel to its faces as shown in fig. If a plane polarized lights is incident normally on the face of plate making an angle θ with its optic axis, then on passing through the plate its amplitude splits into two components $E_o \cos\theta$ along the optic axis & $E_o \sin\theta$ perpendicular to the optic axis. The component $E_o \cos\theta$ having vibrations parallel to the optic axis forms the E – ray (extraordinary ray) and the component $E_o \sin\theta$ having vibrations perpendicular to the optic axis forms the O – ray (Ordinary ray). Both rays travels in the same direction but with different speeds. In calcite crystal E – ray has velocity greater than that of O – ray i.e. $V_E > V_O$. As a result a phase difference is introduced between the waves. This phase difference depends on the thickness of the plate.



If the thickness of the plate is d and the refractive indices of the calcite crystal for E – ray & O – rays are μ_E & μ_O respectively. Then the optical path of these rays will be $\mu_E d$ & $\mu_O d$ respectively.

So the phase difference between E – ray & O – ray will be

$$\delta = 2\pi/\lambda(\text{path difference})$$

$$\delta = 2\pi/\lambda (\mu_{O}d - \mu_{E}d)$$

$$\delta = 2\pi/\lambda (\mu_{O} - \mu_{E}) d$$

λ = wavelength of polarized light

If the incident wave is represented by $E = E_0 \sin \omega t$, then

$$\begin{aligned} \text{For E - ray} & \quad E_x = E_0 \cos\theta \sin(\omega t + \delta) \\ \text{For O - ray} & \quad E_y = E_0 \sin\theta \sin \omega t \end{aligned}$$

For convenience, let $E_x = x$, $E_y = y$, $E_0 \cos\theta = a$ & $E_0 \sin\theta = b$, so the above equations will become

$$\begin{aligned} (1) \quad \text{For E - ray} & \quad \text{—————} \quad x = a \sin(\omega t + \delta) \\ (2) \quad \text{For O - ray} & \quad \text{—————} \quad y = b \sin \omega t \end{aligned}$$

The nature of wave can be obtained by eliminating t from equations (1) & (2),
 $x/a = \sin \omega t \cos\delta + \cos \omega t \sin\delta$

From equation (2),

$$\sin \omega t = y/b \quad \& \quad \cos \omega t = \sqrt{(1 - y^2/b^2)}$$

$$\therefore x/a = (y/b) \cos\delta + \sin\delta \sqrt{(1 - y^2/b^2)}$$

$$\text{or } x/a - (y/b) \cos\delta = \sin\delta \sqrt{(1 - y^2/b^2)}$$

squaring it,

$$x^2/a^2 + y^2/b^2 \cos^2 \delta - (2xy/ab) \cos\delta = \sin^2 \delta (1 - y^2/b^2)$$

$$\text{or } x^2/a^2 + y^2/b^2 - (2xy/ab) \cos\delta = \sin^2 \delta \quad \text{—————(3)}$$

This equation in general is an equation of ellipse. Thus the light emerging from the calcite plate is elliptically polarized light but under certain conditions it is converted into plane or circularly polarized light.

Special cases:

(i) **Plane polarized light** : If the thickness d of the plate be such that

$$(a) \quad \delta = 0, 2\pi, 4\pi \dots \dots \dots = 2n\pi, \text{ where } n = 0,1,2,3 \dots \text{ then}$$

$$\sin\delta = 0 \quad \text{and} \quad \cos\delta = 1$$

Thus from equation (3),

$$x^2/a^2 + y^2/b^2 - (2xy/ab) = 0$$

$$\text{or } [x/a - y/b]^2 = 0$$

$$\text{or } \pm [x/a - y/b] = 0 \quad \text{—————} \quad (4)$$

This represents a pair of straight lines through the origin having a positive

slope (b/a)

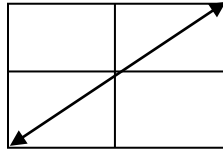
(b) If $\delta = \pi, 3\pi, 5\pi \dots \dots \dots = (2n + 1)\pi$, where $n = 0,1,2,3 \dots$ then

$$\sin \delta = 0 \quad \text{and} \quad \cos\delta = - 1$$

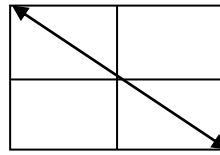
$$x^2/a^2 + y^2/b^2 + (2xy/ab) = 0$$

$$\pm [x/a + y/b] = 0 \quad \text{—————} \quad (5)$$

This equation also represents a pair of straight lines through the origin having a negative slope ($-b/a$)



$$\delta = 0, 2\pi, 4\pi \dots$$



$$\delta = \pi, 3\pi, 5\pi \dots$$

(ii) Elliptically and circularly polarized light symmetric about optic axis :

(a) $\delta = \pi/2, 5\pi/2, 9\pi/2 \dots = (2n + 1/2) \pi$, then

$$\sin\delta = 1 \quad \text{and} \quad \cos\delta = 0$$

From equation (3),

$$x^2/a^2 + y^2/b^2 = 1 \quad \text{-----} \quad (6)$$

This is an equation of ellipse. So the emergent light will be elliptically polarized light.

Now if the angle between the plane of vibration of plane polarized light and the optic axis of calcite crystal is 45° then,

$$a = E_o \cos \theta = E_o / \sqrt{2} \quad \& \quad E_o \sin \theta = E_o / \sqrt{2}$$

i.e. in this case $a = b$.

So equation (6) will become

$$x^2 + y^2 = a^2 \quad \text{-----} \quad (7)$$

This is an equation of circle. So in this case the emergent light will be circularly polarized light.

To determine the direction of rotation of the electric vector in elliptically or circularly polarized substitute $\delta = (2n + 1/2) \pi$ in equation (1) & (2),

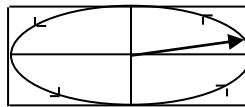
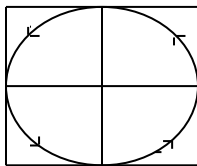
$$x = a \cos \omega t$$

$$y = b \sin \omega t$$

Let us now locate the tip of the rotating vector at $t = 0$ & $t = \Delta t$

$$\text{At } t = 0 \quad x = a \quad \& \quad y = 0$$

$$\text{At } t = \Delta t \quad x \approx a \quad \& \quad y \approx b\omega\Delta t = \text{positive}$$



$$t = \Delta t$$

$$t = 0$$

$$a = b$$

$$\delta = \pi/2, 5\pi/2, 9\pi/2 \dots$$

$$a \neq b$$

$$\delta = \pi/2, 5\pi/2, 9\pi/2 \dots$$

Thus the tip of the vector rotates anti-clockwise and the emergent light, in this case will be an left handed elliptically or circularly polarized light.

(b) $\delta = 3\pi/2, 7\pi/2, 11\pi/2 \dots = (2n + 3/2) \pi$

$$\sin\delta = 1 \quad \text{and} \quad \cos\delta = 0$$

From equation (3),

$$x^2/a^2 + y^2/b^2 = 1 \quad \text{—————} \quad (8)$$

Again the emergent light will be elliptically polarized light provided that $a \neq b$ or $\theta \neq 45^\circ$ & if $\theta = 45^\circ$ the emergent light will be circularly polarized light.

To determine the direction of rotation of the electric vector substitute

$$\delta = (2n + 3/2) \pi \text{ in equation (1) \& (2),}$$

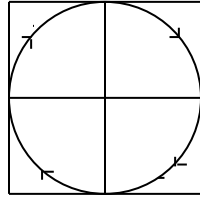
$$x = -a \cos \omega t$$

$$y = b \sin \omega t$$

Let us now again locate the tip of the rotating vector $t = 0$ & $t = \Delta t$,

$$\text{At } t = 0 \quad x = -a \quad \& \quad y = 0$$

$$\text{At } t = \Delta t \quad x \approx -a \quad \& \quad y \approx \omega \Delta t = \text{positive}$$

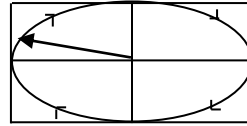


$$a = b$$

$$\delta = 3\pi/2, 7\pi/2, 11\pi/2 \dots$$

$$t = \Delta t$$

$$t = 0$$



$$a \neq b$$

$$\delta = 3\pi/2, 7\pi/2, 11\pi/2 \dots$$

Thus the tip of the vector rotates clockwise and the emergent light, in this case will be a right handed elliptically or circularly polarized light

Phase retardation plates: A doubly refracting crystal plate of uniform thickness whose refracting surfaces are parallel to its optic axis and produces a definite phase difference between the ordinary and the extraordinary ray, is called phase retardation plate.

In calcite crystal the velocity of E – ray v_E is greater than that of

O – ray's v_O . So the difference in time taken by these waves to cross the plate

$$\Delta t = d/v_O + d/v_E$$

Where d is the thickness of the plate.

\therefore Path difference between the E – ray and the O – ray

$$\Delta x = d (\mu_O - \mu_E)$$

where μ_O & μ_E are the refractive indices of calcite plate for O – ray & E –ray respectively. Hence the phase difference between the E – ray and the O – ray

$$\delta = (2\pi/\lambda) \Delta x = (2\pi/\lambda) d (\mu_O - \mu_E) \quad \dots \dots \dots (1)$$

where λ is the wavelength of the incident light.

Quarter Wave Plate: A doubly refracting crystal plate having a thickness such as to create a path difference of $\lambda/4$ or a phase difference of $\pi/2$ between the O – ray & E – ray, is called quarter wave plate.

If d is thickness of such a plate, then for negative crystals

$$\lambda/4 = d (\mu_O - \mu_E)$$

and for positive crystals

$$\lambda/4 = d (\mu_E - \mu_O)$$

Thus the thickness of quarter wave plate made from negative crystal (calcite) is given by

$$d = \lambda / \{4(\mu_O - \mu_E)\}$$

and the thickness of the quarter wave plate made from positive crystal (quartz) is given by

$$d = \lambda / \{4(\mu_E - \mu_O)\}$$

If the ordinary and extraordinary ray have a path difference of $(n + 1/4) \lambda$ even then the quarter wave plate would introduce path difference of $\lambda/4$ between O – ray & E – ray. For negative crystals

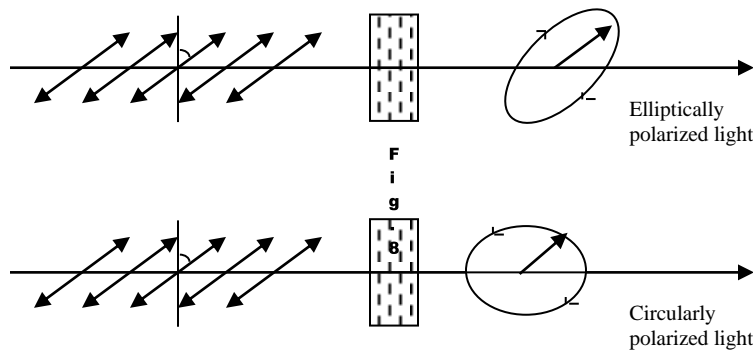
$$(n + 1/4) \lambda = d (\mu_O - \mu_E)$$

and for positive crystals,

$$(n + 1/4) \lambda = d (\mu_E - \mu_O)$$

where $n = 0, 1, 2, \dots$

whenever $n = 0$, the thickness of plate is minimum.



A quarter wave plate is used to produce and detect elliptically and circularly polarized lights

Wave Plate: A doubly refracting crystal plate having a thickness such as to create a path difference of $\lambda/2$ or a phase difference of π between the O – ray & E – ray, is called half wave plate.

If the thickness of the plate is d , then for negative crystals

$$\lambda/2 = d (\mu_O - \mu_E)$$

or

$$d = \lambda / \{2(\mu_O - \mu_E)\}$$

for positive crystals

$$\lambda/2 = d (\mu_E - \mu_O)$$

or

$$d = \lambda / \{2(\mu_E - \mu_O)\}$$

Whenever path difference between the ordinary and extraordinary waves is $(n + 1/2)\lambda$ the half wave plate will still introduce a path difference of $\lambda/2$ between O – ray & E – ray.

For negative crystal,

$$(n + 1/2)\lambda = d (\mu_O - \mu_E)$$

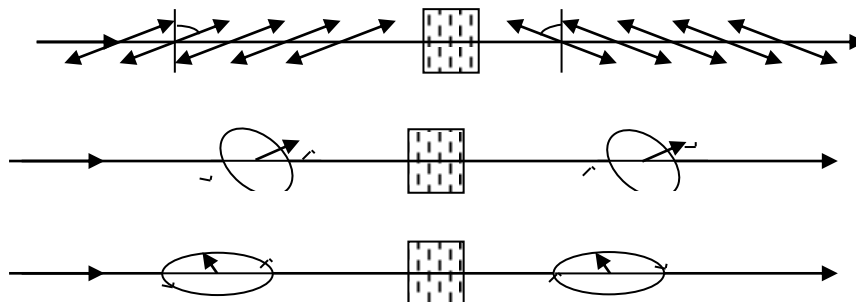
For positive crystal,

$$(n + 1/2) \lambda = d (\mu_E - \mu_O)$$

where $n = 0, 1, 2, \dots$

Half wave plate is used for changing the direction of plane of vibration of the plane polarized light. When a plane polarized light is incident normally on the half wave plate, the emergent ray is also plane polarized but the plane of vibration of emergent ray rotates through an angle 2θ

from the plane of vibration of incident ray, where θ is the angle between the plane of vibration of incident light and the optic axis of the half wave plate. HW plate can change left handed elliptically or circularly polarized light into respective right handed elliptically or circularly polarized light.



Specific Rotation: The term “Specific Rotation” is used to bring rotation of all optically active substances in a comparable form.

For Solids: At a constant temperature for plane polarized light of definite wavelength the angle of rotation (θ) of the plane of polarization is directly proportional to the length traveled by the polarized light in the solid, i.e.,

$$\theta \propto l$$

or $\theta = S l$

or $S = \theta/l$

Where S is specific rotation of solid. It is measured in terms of degrees/mm.

Thus at a constant temperature for plane polarized light of definite wavelength, the specific rotation of a solid is equal to the angle of rotation produced by 1mm length of solid.

For Liquids: The rotation of plane of polarization by a liquid is directly proportional to the length (l) traversed by the polarized light in liquid of density (d). Hence

$$\theta \propto ld$$

$$\theta = Sld$$

$$S = \theta/ld$$

Length of tube containing liquid is measured in decimeters.

Thus the specific rotation of liquid is equivalent to the angle of rotation of plane of polarization produced by a liquid of length 1 decimeter and density 1 gm/cm^3 . It is measured in degree $(\text{dm})^{-1} (\text{gm/cm}^3)^{-1}$.

For solutions: At constant temperature for plane-polarized light of constant wavelength, the angle of rotation (θ) of plane of polarization in optically active solutions is directly proportional to the distance (l) traversed in the solution of concentration (c) i.e.,

$$\theta \propto lc$$

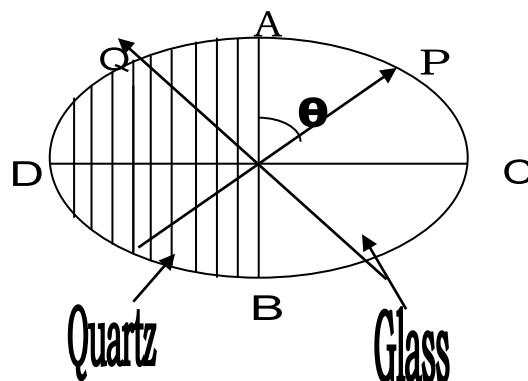
$$\theta = Slc$$

$$S = \theta/lc = \theta V/lm \quad [\because c = m/V \text{ gm/cm}^3]$$

Thus at a constant temperature for plane polarized light of definite wavelength specific rotation of solution is equal to the angle of rotation of plane of polarization produced by 1dm length of solution when the concentration of the solution is 1gm/cm^3 . Its unit is $\text{degree (dm)}^{-1}(\text{gm/cm}^3)^{-1}$.

Laurent's Half Shade Polarimeter: The experimental arrangement of this Polarimeter is shown in fig.16. In this polarimeter half shade device is used in between polarizer and polarimeter tube to increase its sensitivity.

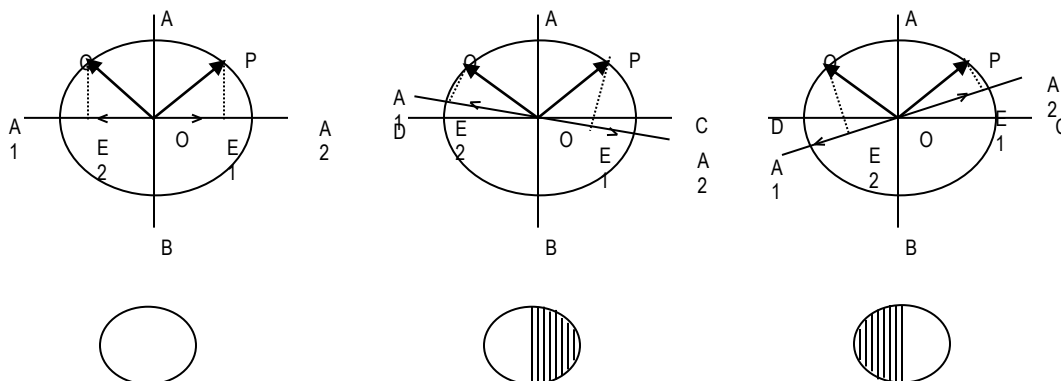
Half Shade Device: It consists of two semicircular plates of same radius cemented together along the diameter so as to form a complete circular plate.



One semicircular plate is half wave plate made from quartz in which optic axis is parallel to its phase and the other semicircular plate is a glass plate. Thickness of glass plate is so adjusted that it should absorb same amount of light as that by quartz plate. In this way the intensity of transmitted light from both semicircular plates remain same. Circular plate made by this device is called Laurent's half shade device. It is kept in between polarizer and tube in such a way that plane of vibrations of incident plane polarized light remain perpendicular to the face of Laurent's plate.

Working: Let the plane of vibration of the plane polarized light incident normally on half shade plate is along OP. Here OP makes an angle θ with diameter AOB. The vibrations emerge from the glass plate as such i.e. along the plane OP. Inside the quartz plate light is divided into two components, one ordinary component and the other extra – ordinary component. The two components travel along the same direction but with different speeds and a phase difference π will be introduced between them. Now the resultant of E – ray and O – ray will be OQ making angle θ with AOB. Thus the vibration of the beam emerging out of the quartz will be along OQ.

If the principal plane of the analyzer A is perpendicular to AOB, then both half portions of the field of view will be seen equally bright because the components OE_1 & OE_2 of vibrations of OP & OQ respectively along A_1OA_2 will be equal.

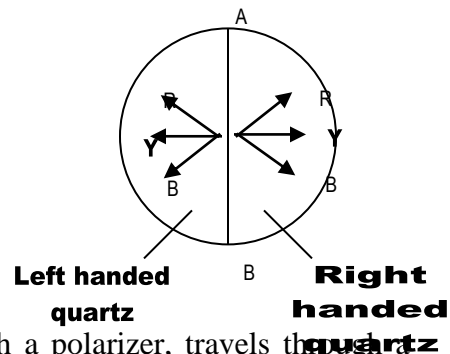


If the principal section A_1OA_2 of the analyzer A is slightly rotated in clockwise direction, then component OE_2 increases and OE_1 decreases. As a result the left half of the field of view appears brighter than the right half. On the other hand, if the principal section of analyzer A is slightly rotated in the anti – clockwise direction, then the component OE_2 becomes less than the component OE_1 and right half of the field of view now appears brighter than the left half.

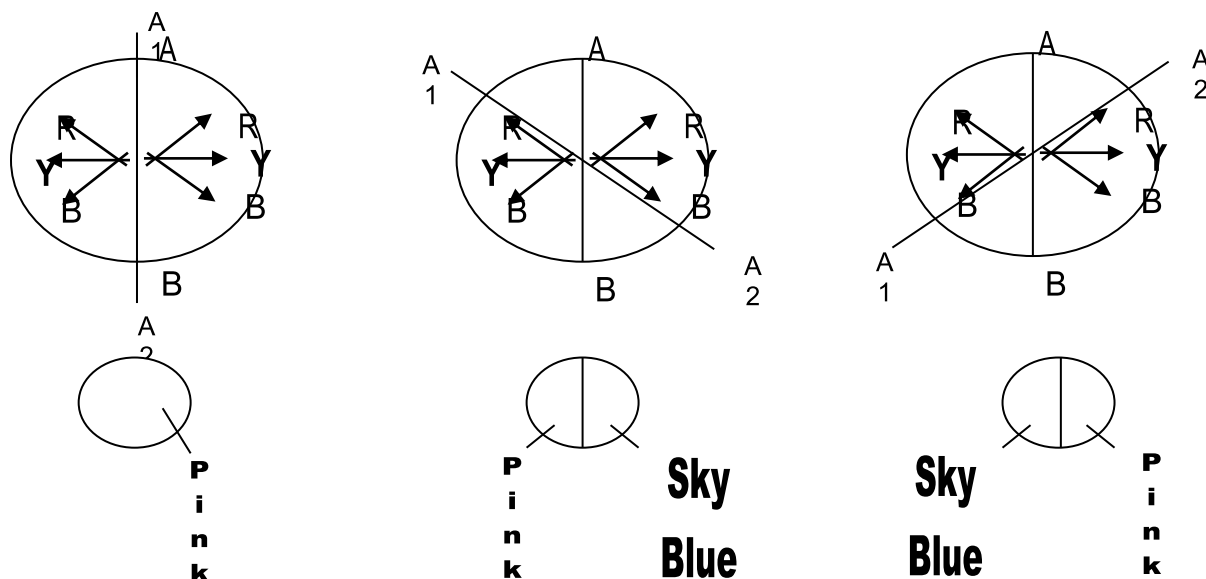
In order to determine the angle of rotation the analyzer A is rotated to such an extent that both halves may appear equally bright. Now the analyzer A is set in this position of equal brightness of entire field of view first by taking water in polarimeter tube and then by taking optically active solution in the polarimeter tube. Difference of two readings gives the angle of rotation. This angle is measured accurately by polarimeter. Main defect of this polarimeter is that it can be used only for that wavelength of light for which semicircular quartz plate acts as half wave plate. In general, this polarimeter is used with sodium light.

Biquartz Polarimeter: The experimental arrangement of this polarimeter is shown in fig.16 but white light source is used in this polarimeter. Biquartz device is used between polarizer and polarimeter tube.

Biquartz Device: It consists of two semicircular plates of quartz (one of left handed quartz and other of right handed quartz) each of thickness 3.75mm. Both are cut perpendicular to the optic axis and joined together along the diameter AOB as shown in fig. The thickness of each plate (3.75mm) rotates the plane of polarization for yellow light by 90° .



Working: When white light, rendered plane polarized with a polarizer, travels through biquartz normally, the phenomenon of rotatory dispersion occurs in both biquartz plates because the plane polarized light is traveling along the optic axis. The planes of vibration of different colours are rotated through different angles. The rotation of yellow colour is 90° .



If the principal plane of the analyzer is parallel to AOB, the yellow light will not be transmitted through the analyzer and appearance of two halves is similar. The two halves have a grayish violet tint, is called sensitive tint or tint of the passage.

If the principal plane of the analyzer is rotated slightly in anti – clockwise direction according to fig.20, then the analyzer will transmit vibrations parallel to A_1A_2 and stop the normal vibrations, i.e. it will make the left half of the field of view as pink and the right half as sky colour.

Now if the principal section of the analyzer A_1A_2 is rotated slightly in the clockwise direction according to fig.20, then it will make left half of the field of view as sky colour and right half as pink.

In order to determine the angle of rotation, analyzer A is set in the position of equal brightness of violet colour in whole field of view as shown in fig.20 first by taking pure water in the tube and then by taking optically active solution. These positions can be noted with great accuracy. If we rotate the analyzer slightly, tint of passage changes very rapidly. Due to this reason the accuracy of biquartz device is much higher. The difference of the two positions of the analyzer in the two conditions gives the angle of rotation of plane of polarization. If the concentration of solution is c gm/cm³ and length of path of light in solution is l in dm, then specific rotation can be determined from $S = \theta/lc$ relation.

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