Study on Discrete Mathematics Sets

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Abstract

A set is a well-defined collection of distinct objects. The objects that make up a set (also known as the elements *or members of a set) can be anything: numbers, people, letters of the alphabet, other sets, and so on. When they are considered collectively they form a single set of size three, written. Sets are one of the most fundamental concepts in mathematics. A set is a gathering together into a whole of definite, distinct objects of our perception or of our thought which are called* elements *of the set. Sets are conventionally denoted with* capital letters. *Sets A and B are equal if and only if they have precisely the same elements. The most basic properties are that a set has elements, and that two sets are equalled (one and the same) if and only if every element of one set is an element of the other. This abstract specify the different uses of union intersection and different.*

Keywords:

Sets theory ; Union; Intersection; Elements; Functions; Subset; Complements

Introduction

In mathematics, a **set** is a collection of distinct objects, considered as an object in its own right. For example, the numbers 2, 4, and 6 are distinct objects when considered separately, but when they are considered collectively they form a single set of size three, written {2,4,6}. Sets are one of the most fundamental concepts in mathematics. Developed at the end of the 19th century, set theory is now a ubiquitous part of mathematics, and can be used as a foundation from which nearly all of mathematics can be derived. In mathematics education, elementary topics such as Venn diagrams are taught at a young age, while more advanced concepts are taught as part of a university degree. The German word *Merge*, rendered as "set" in English, was coined by Bernard Bolzano in his work The Paradoxes of the Infinite.

Definition

A set is a well-defined collection of distinct objects. The objects that make up a set (also known as the elements or members of a set) can be anything: numbers, people, letters of the alphabet, other sets, and so on. Georg Cantor, the founder of set theory, gave the following definition of a set at the beginning. [1]

Describing Sets

There are two ways of describing, or specifying the members of, a set. One way is by intensional definition, using a rule or semantic description:

- *A* is the set whose members are the first four positive integers.
- *B* is the set of colors of the French flag.

The second way is by extension – that is, listing each member of the set. An extensional definition is denoted by enclosing the list of members in curly brackets:

$C = \{4, 2, 1, 3\}$
$D = \{\text{blue, white, red}\}$.

There are two important points to note about sets. First, a set can have two or more members which are identical, for example, {11, 6, 6}. However, we say that two sets which differ only in that one has duplicate members are in fact exactly identical (see Axiom of extensionality). Hence, the set {11, 6, 6} is exactly identical to the set {11, 6}. The second important point is that the order in which the elements of a set are listed is irrelevant (unlike for a sequence or tuple). We can illustrate these two important points with an example:

$$\{6, 11\} = \{11, 6\} = \{11, 6, 6, 11\} .$$

For sets with many elements, the enumeration of members can be abbreviated. For instance, the set of the first thousand positive integers may be specified extensionally as:

$$\{1, 2, 3, ..., 1000\},$$

where the ellipsis ("...") indicates that the list continues in the obvious way. Ellipses may also be used where sets have infinitely many members. Thus the set of positive even numbers can be written as {2, 4, 6, 8, ... }.

The notation with braces may also be used in an intensional specification of a set. In this usage, the braces have the meaning "the set of all ...". So, E = {playing card suits} is the set whose four members are ♠, ♦, ♥, and ♣. A more general form of this is set-builder notation, through which, for instance, the set F of the twenty smallest integers that are four less than perfect squares can be denoted:

$$F = \{n^2 - 4 : n \text{ is an integer; and } 0 \leq n \leq 19\}.$$

In this notation, the colon (":") means "such that", and the description can be interpreted as "F is the set of all numbers of the form $n^2 - 4$, such that n is a whole number in the range from 0 to 19 inclusive." Sometimes the vertical bar ("|") is used instead of the colon.

One often has the choice of specifying a set intensionally or extensionally. In the examples above, for instance, $A = C$ and $B = D$. [2]

Special sets

There are some sets that hold great mathematical importance and are referred to with such regularity that they have acquired special names and notational conventions to identify them. One of these is the empty set, denoted {} or ∅. Another is the unit set {x}, which contains exactly one element, namely x.[2] Many of these sets are represented using black board bold or bold typeface. Special sets of numbers include:

- **P** or ℙ, denoting the set of all primes: **P** = {2, 3, 5, 7, 11, 13, 17, ...}.
- **N** or ℕ, denoting the set of all natural numbers: **N** = {1, 2, 3, . . .} (sometimes defined containing 0).
- **Z** or ℤ, denoting the set of all integers (whether positive, negative or zero): **Z** = {..., −2, −1, 0, 1, 2, ...}.
- **Q** or ℚ, denoting the set of all rational numbers (that is, the set of all proper and improper fractions): $\mathbf{Q} = \{a/b : a, b \in \mathbf{Z}, b \neq 0\}$. For example, $1/4 \in \mathbf{Q}$ and $11/6 \in \mathbf{Q}$. All integers are in this set since every integer a can be expressed as the fraction $a/1$ ($\mathbf{Z} \subsetneq \mathbf{Q}$).
- **R** or ℝ, denoting the set of all real numbers. This set includes all rational numbers, together with all irrational numbers (that is, numbers that cannot be rewritten as fractions, such as $\sqrt{2}$, as well as transcendental numbers such as π, e and numbers that cannot be defined).

- **C** or ℂ, denoting the set of all complex numbers: **C** = $\{a + bi : a, b \in \mathbf{R}\}$. For example, $1 + 2i \in \mathbf{C}$.
- **H** or ℍ, denoting the set of all quaternions: **H** = $\{a + bi + cj + dk : a, b, c, d \in \mathbf{R}\}$. For example, $1 + i + 2j - k \in \mathbf{H}$.

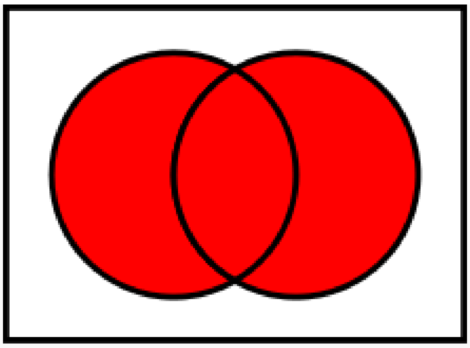
Positive and negative sets are denoted by a superscript - or +, for example: $\mathbb{Q}^+$ represents the set of positive rational numbers. [3]

Each of the above sets of numbers has an infinite number of elements, and each can be considered to be a proper subset of the sets listed below it. The primes are used less frequently than the others outside of number theory and related fields.

Basic operations

There are several fundamental operations for constructing new sets from given sets.

Unions



The **union** of *A* and *B*, denoted $A \cup B$

Main article: Union (set theory)

Two sets can be "added" together. The *union* of *A* and *B*, denoted by $A \cup B$, is the set of all things that are members of either *A* or *B*.

Examples:

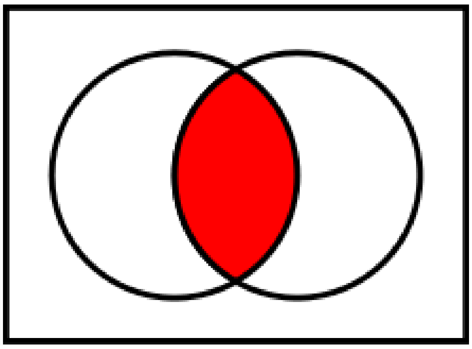
- $\{1, 2\} \cup \{1, 2\} = \{1, 2\}$.
- $\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$.
- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

Some basic properties of unions:

- $A \cup B = B \cup A$.
- $A \cup (B \cup C) = (A \cup B) \cup C$.
- $A \subseteq (A \cup B)$.
- $A \cup A = A$.
- $A \cup \emptyset = A$.
- $A \subseteq B$ if and only if $A \cup B = B$.

Intersections

A new set can also be constructed by determining which members two sets have "in common". The *intersection* of *A* and *B*, denoted by $A \cap B$, is the set of all things that are members of both *A* and *B*. If $A \cap B = \emptyset$, then *A* and *B* are said to be *disjoint*.



The **intersection** of *A* and *B*, denoted $A \cap B$.

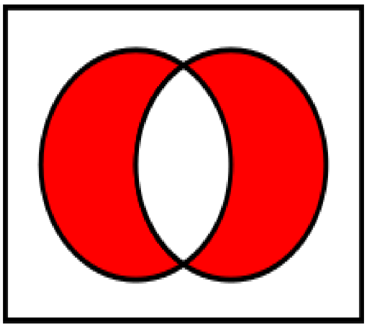
Examples:

- $\{1, 2\} \cap \{1, 2\} = \{1, 2\}$.
- $\{1, 2\} \cap \{2, 3\} = \{2\}$.

Some basic properties of intersections:

- $A \cap B = B \cap A$.
- $A \cap (B \cap C) = (A \cap B) \cap C$.
- $A \cap B \subseteq A$.
- $A \cap A = A$.
- $A \cap \emptyset = \emptyset$.
- $A \subseteq B$ if and only if $A \cap B = A$.

Complement of *A* in *U*



The symmetric difference of *A* and *B*

Two sets can also be "subtracted". The *relative complement* of *B* in *A* (also called the *set-theoretic difference* of *A* and *B*), denoted by $A \setminus B$ (or $A - B$), is the set of all elements that are members of *A* but not members of *B*. Note that it is valid to "subtract" members of a set that are not in the set, such as removing the element *green* from the set {1, 2, 3}; doing so has no effect.

In certain settings all sets under discussion are considered to be subsets of a given universal set *U*. In such cases, $U \setminus A$ is called the *absolute complement* or simply *complements* of *A*, and is denoted by A'. [4]

Examples:

- $\{1, 2\} \setminus \{1, 2\} = \emptyset$.
- $\{1, 2, 3, 4\} \setminus \{1, 3\} = \{2, 4\}$.
- If *U* is the set of integers, *E* is the set of even integers, and *O* is the set of odd integers, then $U \setminus E = E' = O$.

Some basic properties of complements:

- $A \setminus B \neq B \setminus A$ for $A \neq B$.
- $A \cup A' = U$.
- $A \cap A' = \emptyset$.
- $(A')' = A$.
- $A \setminus A = \emptyset$.
- $U' = \emptyset$ and $\emptyset' = U$.
- $A \setminus B = A \cap B'$.

An extension of the complement is the symmetric difference, defined for sets *A*, *B* as

$$A \,\Delta\, B = (A \setminus B) \cup (B \setminus A).$$

For example, the symmetric difference of {7,8,9,10} and {9,10,11,12} is the set {7,8,11,12}.

Complements

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Applications

Set theory is seen as the foundation from which virtually all of mathematics can be derived. For example, structures in abstract algebra, such as groups, fields and rings, are sets closed under one or more operations.

One of the main applications of naive set theory is constructing relations. A relation from a domain A to a codomain B is a subset of the Cartesian product $A \times B$. Given this concept, we are quick to see that the set F of all ordered pairs (x, x^2), where x is real, is quite familiar. It has a domain set **R** and a codomain set that is also **R**, because the set of all squares is subset of the set of all reals. If placed in functional notation, this relation becomes $f(x) = x^2$. The reason these two are equivalent is for any given value, y that the function is defined for, its corresponding ordered pair, (y, y^2) is a member of the set F.

Reference

1.Dauben, Joseph W., *Georg Cantor: His Mathematics and Philosophy of the Infinite*, Boston: Harvard University Press (1979) ISBN 978-0-691-02447-9.

2.Halmos, Paul R., *Naive Set Theory*, Princeton, N.J.: Van Nostrand (1960) ISBN 0-387-90092-6.

3.Stoll, Robert R., *Set Theory and Logic*, Mineola, N.Y.: Dover Publications (1979) ISBN 0-486-63829-4.

4.Velleman, Daniel, *How To Prove It: A Structured Approach*, Cambridge University Press (2006) ISBN 978-0-521-67599-4