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Design of a controller for reduced order model of an SMIB power system using pole placement technique

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Abstract- *In order to design a large interconnected system and to gurantee the system stability at a minimum cost it is very complex.From control theory point of view, power system is a higher order multivariable process which operates constantly in a changing environment. Hence, it is necessary to perfom the order reduction, for such a complex system. In this paper order reduction techniques i.e. Modal Analysis Approach methods have been used to reduce the stable higher order system to a stable lower order model.*

Index terms- SMIB power system,order reduction,modal analysis approach

I. INTRODUCTION

Planning and operation of modern power systems have become complex due to large number of interconnections involved in the system.Power system stability investigates the system behaviour when it is subjected to disturbances.Generally the stability of a linear system is entirely independent on the type of the input,magnitude of the input and the state of the system.But the stability of a non-linear system depends on the type of the input,magnitude of the input and state of the system.The stability of a nonlinear system is classified as follows.

1.Local stability(or)stability in small

2.Finite stability

3.Global stability(or)stability in large

In this paper the small signal stability performance of a single machine connected to large system through transmission lines is considered. Generally, during normal operation small signal stability refers to the system dynamics. In a most common way it is defined as the ability of the power system to maintain synchronism when subjected to small disturbances. The outline of this section includes four sections. Problem Statement will be discussed in Section 2. Model reduction techniques are discussed in Section 3. Design of a controller via Pole Placement technique is described in Section 4. The SMIB Power System has been represented in state space for which the reduction procedure is applied in Section 5. In the last section, i.e. Section 6 conclusion is stated.

II. PROBLEM STATEMENT

Most of the model reduction methods focus on linear time invariant continuous time and discrete time systems. Here in this paper Linear Continuous time systems are described by the following equations

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) + Du(t) \quad (2)$$

Where $x(t)$ is vector of state variables

$u(t)$ is vector of inputs

$y(t)$ is vector of outputs

A is known as state or plant matrix of order $n\times n$

B is known as control or input matrix of order $n\times m$

C is known as output matrix of order $p\times n$

D is known as Feedfoward matrix of order $p\times m$

Here n is the order of the system

m is no. of input variables

p is no. of output variables

The task of model reduction is to find a reduced order Linear time invariant systems of order 'r' ($r\leq n$) which is described by the equations

$$\dot{x_r}(t) = A_r x_r(t) + B_r u(t) \quad (3)$$

$$y_r(t) = C_r x(t) + D_r u(t) \quad (4)$$

III. DIFFERENT MODEL REDUCTION METHODOLOGIES

Most of the model reduction methods focus on linear systems, which in many cases provide

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accurate descriptions of the physical systems. There are two methods which are used in this paper to reduce the order of the system.

1. Modal Analysis Approach
2. Aggregation Method

Modal Analysis Approach

Modal reduction methods based on modal analysis approach identify and preserve the important properties of original system.It is one of the oldest model order reduction technique.In this proposed method of reducing linear system to a lower order, the system has the same dominant eigenvalues and eigenvectors of the original system.The principal of this method is to neglect the eigen values of the original system which are farthest from the origin and retain only dominant eigenvalues.

The algorithm of the proposed method is given below:

Step1:Find the eigenvalues of the original system, which is represented below

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Step2:Find the eigen vectors using the eigen values of the original system

Step3:Compute the Modal matrix of the original system as shown below

$$M = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \quad (5)$$

Where M_1 is of the order $(r\times r)$

M_4 is of the order (n-r)×(n-r)

Here n is the order of the system

r is the order of the reduced system

Step4:Partion the matrix 'A' into submatrices as shown below

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \quad (6)$$

Where A_1 is of the order $r \times r$

A_4 is of order of (n-r)×(n-r)

Step5:The reduced order state matrix 'A' which is represented as $'A_r'$ can be calculated using

$$A_r = A_1 + A_2 M_3 M_1^{-1} \quad (7)$$

Step6:Partion the matrix 'B' into sub-matrices as shown below

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad (8)$$

Where B_1 is of order (r×1)

B_2 is of order (n-r)×1

Step7:The reduced order input matrix 'B' which is represented as $'B_r'$ can be calculated using

$$B_r = B_1 \quad (9)$$

Step8:Partion the matrix 'C' into sub-matrices as shown below

$$C = [C_1 \quad C_2] \quad (10)$$

Step9:The reduced order output matrix 'C' which is represented as $'C_r'$ can be calculated using

$$C_r = C_1 \quad (11)$$

Step10:The reduced order state-space realization (A_r, B_r, C_r, D_r) is given below

$$\dot{x_r}(t) = A_r x_r(t) + B_r u(t)$$

$$y_r(t) = C_r x(t) + D_r u(t)$$

Here n is the order of the system

r is the order of the reduced system

2. DESIGN OF A CONTROLLER VIA POLE PLACEMENT TECHNIQUE

The performance of the reduced model is improved by designing a controller using pole placement technique.By using Pole placement method the closed loop poles are placed at desired location on s-plane.

The sufficient conditions that has to be satisfied before designing controller.

1. The system should be completely state controllable.
2. The state variables are measurable and available for feedback.
3. Control input is unconstrained.

During design of controller the choice of closed loop poles should satisfy the following

1. Not only the dominant poles, but all the poles are forced to lie at desired locations.
2. The poles which we choose should be close to the open loop poles.

[Figure. 1]

The state model of a system without using state feedback is given by Eqn.1

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From the above block diagram

q = system input when state variable feedback is employed

σ = Feedback signals obtained from state variables

u = Scalar control vector

The feedback signal 'σ' is obtained from state variables and it is given by σ =Kx,

Where K is the feedback gain matrix of order 1×r and it is given as $K = [K_1 \quad K_2 \quad \cdots \quad K_r]$

If the system employing state variable feedback then,

$$u=q-\sigma \text{ (or) } q-Kx \qquad (12)$$

The state equations of the compensated model is obtained by substituting Eqn. (16) in Eqn. (1) as

$$\dot{x}(t) = (A - BK)x(t) + Bu(t) \qquad (13)$$

$$y(t) = (C - DK)x(t) + Du(t)$$

Steps to Design the controller using Pole Placement technique

Step A: Check controllability of the system by using

$$Q_c = [B \quad AB \quad A^2B \quad \cdots \quad A^{r-1}B]$$

Step B: Consider the desired poles $\mu_1\ \mu_2 \cdots \mu_r$ at which the system is to be placed

Step C: By using Step B, we obtain the desired characteristic polynomial as

$$(s-\mu_1)\ldots(s-\mu_r) = s^2 + a_1 s^{r-1} + a_2 s^{r-2} + \cdots + a_r$$

Step D: By using state variable feedback, the characteristic polynomial of the system is given by

$$|(sI - A + BK)| = s^r + b_1 s^{r-1} + b_2 s^{r-2} + \cdots + b_r$$

Where $K = [K_1 \quad K_2 \quad \cdots \quad K_r]$

Step E: Equate Steps C and D to know the values of 'K'

Step F: The state equations using feedback is obtained from Eqn. (13)

IV CASE STUDY

Analyze the small signal stability of a thermal generating station consisting of four 555MVA, 24KV at 60Hz is represented below

[Figure .2]

The network reactance is taken in p.u and the resistances are to be neglected. Here the transmission circuit 2 is out of service, P= 0.9, Q=0.3(Over excited), $E_t = 1.0\angle 36^0$, $E_B = 0.995\angle 0^0$. The p.u fundamental parameters of the equivalent generator are $L_{adu} = 1.65$, $L_{aqu} = 1.60$, $R_{fd} = 0.0006$, $R_a = 0.003$ $L_{fd} = 0.153$.

[Figure. 3]

The state space form of Figure. 3 is represented as

$$\begin{bmatrix} \Delta\dot{\omega}_r \\ \Delta\dot{\delta} \\ \Delta\dot{\varphi}_{fd} \\ \Delta\dot{v}_1 \\ \Delta\dot{v}_2 \\ \Delta\dot{v}_s \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & a_{36} \\ 0 & a_{42} & a_{43} & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & a_{55} & 0 \\ a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \\ \Delta\varphi_{fd} \\ \Delta v_1 \\ \Delta v_2 \\ \Delta v_s \end{bmatrix}$$

$$\text{Or } \dot{x} = Ax$$

[Table 1]

[Table 2]

Modal Analysis Approach

1. The matrices from the above equations are

A

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -52.08 & -23.54 & 9.415 & -23.54 & -99.2 & 27.88 \\ -541.2 & 0.6953 & -2.216 & -36.23 & 24.15 & 664.2 & -362.3 \\ -1136 & 1.46 & -0.664 & -76.08 & -12.08 & 1395 & -760.8 \\ -541.2 & 0.6953 & 1.328 & -36.23 & -38.65 & 664.2 & -362.3 \\ 398 & 3.403 & 897.1 & -1346 & 897.1 & -53.83 & -26.91 \\ 298.5 & 2.552 & 672.9 & -1009 & 672.9 & -40.37 & -35.89 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{2.121} \\ \mathbf{0.6642} \\ \mathbf{-1.328} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{C} = [\mathbf{1} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}] \quad \boldsymbol{D} = [\mathbf{0}]$$

Modal Analysis Approach

Eigen values of the matrix "A" are -71.25 ± 636.27i, -0.37, -2.47, -48.85, -27.89 and -36.66

The Modal matrix consists the elements of Eigen vectors which is obtained from Eigen values as shown below

M =

$$\begin{bmatrix} -0.0001i & 0.0001i & 0.2989 & 0.2595 & -0.0204 & -0.0260 & -0.0112 \\ 0.0610 - 0.0155i & 0.0610 + 0.0155i & -0.1116 & -0.6401 & 0.9974 & 0.7244 & 0.4094 \\ -0.3067 + 0.0135i & -0.3067 - 0.0135i & -0.7930 & 0.5349 & 0.0042 & 0.0502 & 0.5588 \\ -0.6459 & -0.6459 & -0.4749 & 0.4295 & -0.0087 & 0.0228 & -0.0995 \\ -0.3084 - 0.0151i & -0.3084 + 0.0151i & -0.0403 & 0.0096 & -0.0137 & -0.0350 & -0.7080 \\ -0.0002 - 0.4990i & -0.0002 + 0.4990i & 0.2043 & 0.2282 & -0.0450 & -0.3447 & -0.0584 \\ 0.0091 - 0.3745i & 0.0091 + 0.3745i & -0.0233 & -0.0125 & -0.0496 & -0.5928 & -0.0737 \end{bmatrix}$$

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From Eqn.(5) and (6).

$$M_1 = \begin{bmatrix} -0.0001i & 0.0001i & 0.2989 \\ 0.0610-0.0155i & 0.0610+0.0155i & -0.1116 \\ -0.3067+0.0135i & -0.3067-0.0135i & -0.7930 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} -0.6459 & -0.6459 & -0.4749 \\ -0.3084-0.0151i & -0.3084+0.0151i & -0.0403 \\ -0.0002-0.4990i & -0.0002+0.4990i & 0.2043 \\ 0.0091-0.3745i & 0.0091+0.3745i & -0.0233 \end{bmatrix}$$

Similarly by using Eqn's (7),(9) &(11)the reduced order model is represented as

$$A_r = \begin{bmatrix} 0 & 1 & 0 \\ -2850 & -3111 & -637 \\ 13539 & 15152 & 2970 \end{bmatrix} \quad B_r = \begin{bmatrix} 0 \\ 0 \\ 2.121 \end{bmatrix}$$

$$C_r = [1 \quad 0 \quad 0] \qquad D_r = [0]$$

Under steady state conditions s→0, then

$$A_r = \begin{bmatrix} -140 & -408600 & -152500 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B_r = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_r = [-4.012 * 10\hat{}-011 \quad -3.122 * 10\hat{}-008 \quad -51676]$$

$$D_r = [0]$$

[Figure.4]

Design of a Controller

The reduced order model for the above system obtained from Modal analysis approach is

$$A_r = \begin{bmatrix} -140 & -408600 & -152500 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} B_r = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_r = [-4.012 * 10\hat{}-011 \quad -3.122 * 10\hat{}-008 \quad -51676]$$

$$D_r = [0]$$

From Steps A and B the system is state controllable and the desired poles are -0.484, -3.1564, -0.525

The desired characteristic polynomial from Step C is $(s^3 + 4.1654s^2 + 3.43881s + 0.80199)$

From Step D the characteristic polynomial is $(s^3 + (140 + K_1)s^2 + (408600 + K_2)s + (152500 + K_3))$

The gain values K_1 and K_2 are obtained from Step E as

$$K = [-135.8346 \quad -408596.56 \quad -152499.198]$$

The compensated state equations obtained from Eqn. 13 as

$$A_{rc} = \begin{bmatrix} -4.1654 & -3.44 & -0.802 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} B_{rc} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{rc} = [-4.012 * 10\hat{}-011 \quad -3.122 * 10\hat{}-008 \quad -51676]$$

$$D_{rc} = [0]$$

Under Steady State Conditions s→0, then

$$A_{rc} = \begin{bmatrix} -4.1654 & -3.44 & -0.802 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} B_{rc} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{rc} = [0 \quad 0 \quad -0.2720] \quad D_{rc} = [0]$$

[Figure. 5]

V. CONCLUSION

Increase in power system dimensions posses higher order transfer function.Behaviour of such systems makes it difficult to analyze.In the past many methods have been developed to approximate the large order system to lowest order.In this paper modal analysis approach method is used to reduce the system to its lowest order.The performance of the reduced order model has been improved by designing a controller using pole placement technique. The simulation results are presented in this paper

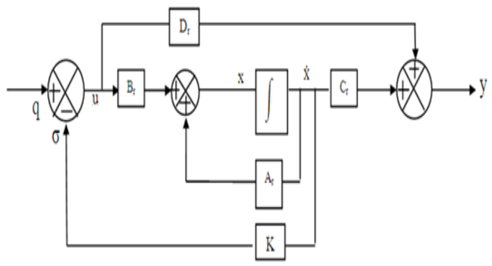

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Figure.1 Block Diagram of state variable feedbacksystem

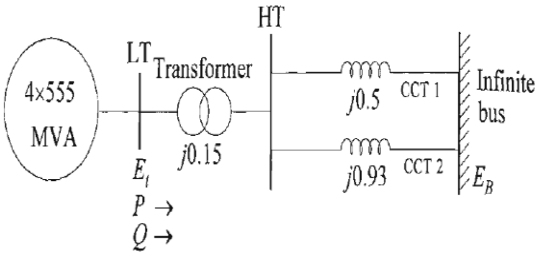

Figure.2 Schematic diagram of SMIB system

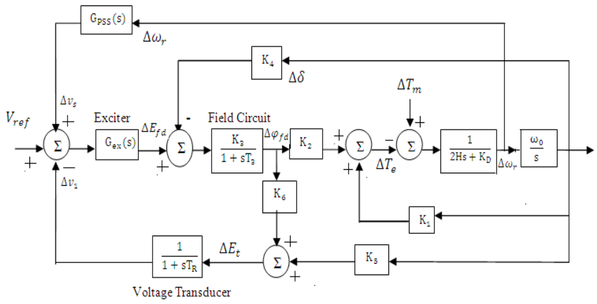

Figure.3 Block Diagram representation of Figure.2

Table 1: Elements of Matrix 'A'

$a_{11} = \frac{-K_D}{2H}$	$a_{12} = \frac{-K_1}{2H}$
$a_{13} = \frac{-K_2}{2H}$	$a_{21} = \omega_0 = 2\pi f_0$
$a_{32} = \frac{-\omega_0 R_{fd}}{L_{fd}} m_1 L'_{ads}$	$a_{33} = \frac{-\omega_0 R_{fd}}{L_{fd}} [1 - \frac{L'_{ads}}{L_{fd}} + m_2 L'_{ads}]$
$a_{34} = -b_{32} K_A = \frac{-\omega_0 R_{fd}}{L_{adu}} K_A$	$a_{36} = \frac{-\omega_0 R_{fd}}{L_{adu}} K_A$
$a_{42} = \frac{K_5}{T_R}$	$a_{43} = \frac{K_6}{T_R}$
$a_{44} = \frac{-1}{T_R}$	$a_{51} = K_{STAB}\, a_{11}$
$a_{52} = K_{STAB}\, a_{12}$	$a_{53} = K_{STAB}\, a_{13}$
$a_{55} = \frac{-1}{T_\omega}$	$a_{61} = \frac{T_1}{T_2} a_{51}$
$a_{62} = \frac{T_1}{T_2} a_{52}$	$a_{63} = \frac{T_1}{T_2} a_{53}$
$a_{65} = \frac{T_1}{T_2} a_{55} + \frac{-1}{T_2}$	$a_{66} = \frac{-1}{T_2}$

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Gain Constants
$K_1 = 0.7643$ $K_2 = 0.8649$ $K_3 = 0.3230$ $K_4 = 1.4187$ $K_5 = -0.1463$ $K_6 = 0.4168$ $K_{STAB} = 9.5$ $K_A = 200$
Time Constants
$T_1 = 0.154(s)$ $T_2 = 0.033(s)$ $T_3 = 2.365(s)$ $T_\omega = 1.4(s)$ $T_R = 0.02(s)$
H=3.5(MW.s/MVA), $K_D = 0$

Table 2: Gain and Time constants of the system

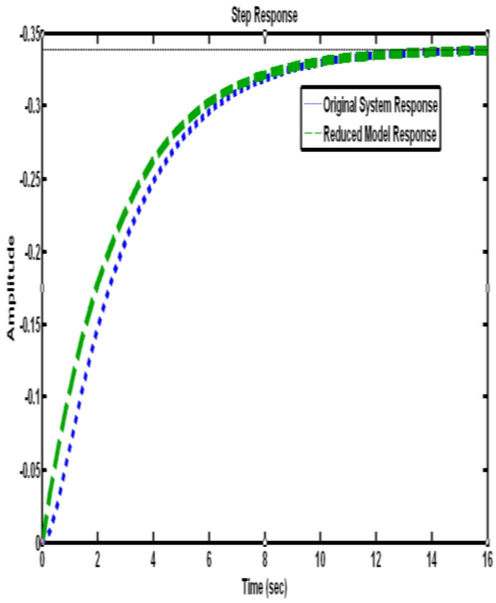

Figure 4: Step response of Original (6^{th}) order and reduced (2^{nd}) order system using Modal Analysis Approach method

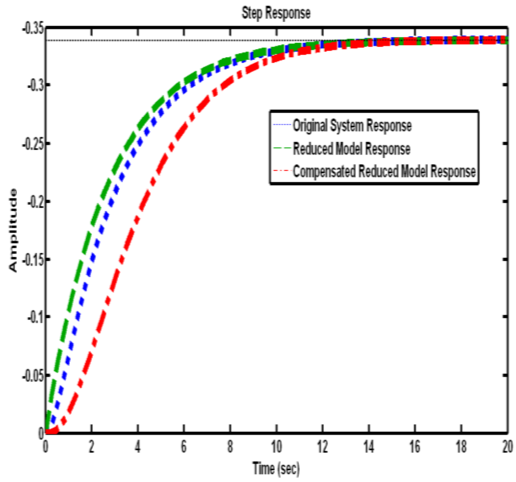

Figure 5: Step response of Compensated 2^{nd} order model using Modal Analysis Approach method

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