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Condition on an Element of p-Group That Determines Inner Automorphism of Order p

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Abstract: Let G be a p-group. We give some important results and define inner automorphism, and find the condition on an element of a p-group that determines an inner automorphism of order p.

Keywords: automorphism, centre, inner automorphism

1. Introduction

Let G be a group. Aut(G) denote the set of all automorphisms of G. We know that Aut(G) is a group under the resultant composition. The automorphism φ_a: G → G given by $f_a(x) = axa^{-1}$ ∀ x ε G is called an inner automorphism of G determined by a. Inn(G) denotes the set of all inner automorphisms of G. A longstanding conjecture is that every finite non-abelian p-group admits a noninner automorphism of order p. This conjecture has been settled for p-group G s.t. (a) G is regular (b) G is nilpotent of class 2 or 3 (c) G' is cyclic (d) G/Z(G) is powerful (e) G is of coclass 2. Coclass of a p-group of order p^n, we mean the number n-c, where c is the nilpotency class of G.

2. Some Important Results

Theorem 2.1. For any group G, Inn(G) is a normal subgroup of Aut(G). Further Inn(G) ≅ G/Z(G), where Z(G) denotes the centre of G.

Theorem 2.2. Let G ≠ {e} be a cyclic group generated by b. Then any endomorphism φ of G is an automorphism of G If and only If f(b) is a generator of G. Further (i) of G is infinite then Aut(G) is of order 2. (ii) If G is of finite order n, then Aut(G) is isomorphic to the group of those positive integers ≤n, which are relatively prime to n, with binary operation as multiplication modulo n and order of Aut(G) is φ(n).

3. Inner automorphism of order p

Let G be a p-group If a ε Z(G), then clearly the inner automorphism f_a is trivial.

Now If a ∉Z(G) but o(a) = p, then first we prove that $f_a^p(x) = a^{-p}\, x\, a^p$

Let $f_a^2(x) = f_a(f_a(x)) = f_a(a^{-1} x\, a) = a^{-1}(a^{-1}x\, a)\, a = a^{-2}\, x\, a^2$

In the similar way we arrived at $f_a^p(x) = a^{-p}\, x\, a^p$, Now since 0(a) = p ∴ $a^p = e$, e the identity of G, so $f_a^p(x) = x$

Since p is the smallest prime, therefore order of f_a is p.

Now Let a ∉ Z(G) and o(a) ≠ p. Since G/Z(G) is a p-group, therefore let order of aZ(G) is p. So $(aZ(G))^p = Z(G)$, $a^p Z(G) = Z(G)$. We see that $a^p \varepsilon Z(G)$ and $f_a^p(x) = a^{-p} x a^p = x$

Because p is prime, so we conclude that If order of aZ(G) is p then f_a^p is identity. Therefore order of f_a is p. This result is open for discussion.

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