Applications of the Pythagoras Theorem

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Abstract:

Investigate the history of Pythagoras and the Pythagorean Theorem. We analyzed the information on the Pythagorean Theorem including not only the meaning and application of the theorem, but also the proofs.

Keywords:

Pythagoras, Greek, hypotenuse, Pythagorean Theorem.

I. Motivation

You're locked out of your house and the only open window is on the second floor, 25 feet above the ground. You need to borrow a ladder from one of your neighbours. There's a bush along the edge of the house, so you'll have to place the ladder 10 feet away from the house. What length of ladder do you need to reach the window?

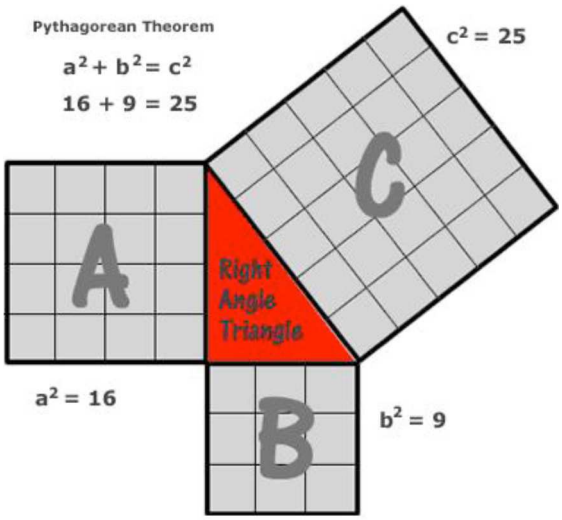

Figure 1: Logic behind the Pythagorean Theorem.

II. History

Pythagoras lived in the 500's BC, and was one of the first Greek mathematical thinkers. Pythagoreans were interested in Philosophy, especially in Music and Mathematics. The statement of the Theorem was discovered on a Babylonian tablet circa 1900−1600 B.C. Professor R. Smullyan in his book 5000 B.C. and Other Philosophical Fantasies tells of an experiment he ran in one of his geometry classes. Then he asked,"Suppose these three squares were made of beaten gold, and you were offered either the one large square or the two small squares. Which would you choose?" Interestingly enough, about half the class opted for the one large square and half for the two small squares. Both groups were equally amazed when told that it would make no difference.

2.1 Statement of Pythagorean Theorem

The famous theorem by Pythagoras defines the relationship between the three sides of a right triangle. Pythagorean Theorem says that in a right triangle, the sum of the squares of the two right-angle sides will always be the same as the square of the hypotenuse (the long side). In symbols: $A2 + B2 = C2$.

2.2 The Converse of Pythagorean Theorem

The converse of Pythagorean Theorem is also true. That is, if a triangle satisfies Pythagoras' theorem, then it is a right triangle. Put it another way, only right triangles will satisfy Pythagorean Theorem. Now, on a graph paper ask the students to make two lines. The first one being three units in the horizontal direction, and the second being four units in perpendicular (i.e. vertical) direction, with the two lines intersect at the end points of the two lines. The result is right angle. Ask the

students to connect the other two ends (open) of the lines to form a right triangle. Measure this distance with a ruler, see the Figure below.

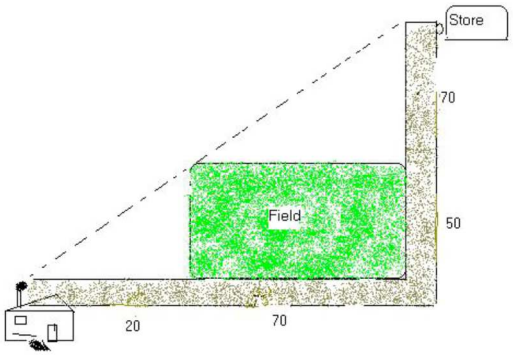

Figure 2. Finding the shortest distance.

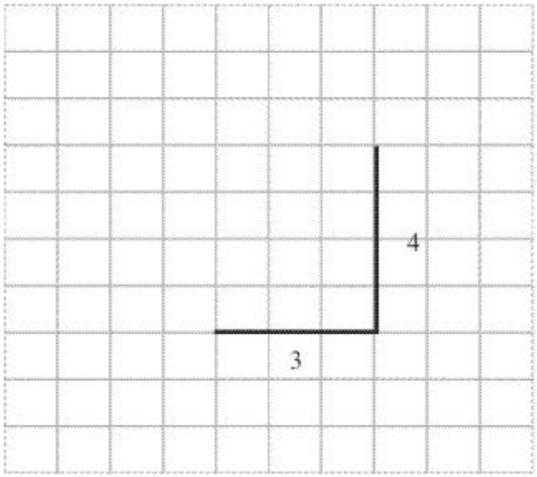

Figure 3: Converse of Pythagorean Theorem.

III. Indian Proof of Pythagorean Theorem

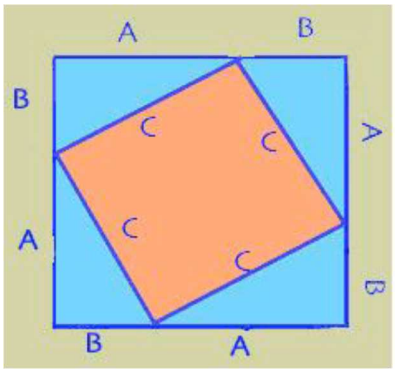

Figure 4: Indian Proof of Pythagorean Theorem.

The area of the inner square if Figure 4 is $C \times C$ or C^2, where the area of the outer square is, $(A + B)^2 = A2 + B2 + 2AB$. On the other hand one may find the area of the outer square as follows: The area of the outer square = the area of inner square + the sum of the areas of the four right triangles around the inner square, therefore.

$$A2 + B2 + 2AB = C2 + 41$$
$$2AB, \text{ or } A2 + B2 = C2.$$

IV. Applications of Pythagorean Theorem

In this segment we will consider some real life applications to Pythagorean Theorem: The Pythagorean Theorem is a starting place for trigonometry, which leads to methods, for example, for calculating length of a lake. Height of a Building, length of a bridge. Here are some examples.

1. To find the length of a lake, we pointed two flags at both ends of the lake, say A and B. Then a person walks to another point C such that the angle ABC is 90. Then we measure the distance from A to C to be 150m, and the distance from B to C to be 90m. Find the length of the lake.

2. A television screen measures approximately 15 in. high and 19 in. wide. A television is advertised by giving the approximate length of the diagonal of its screen. How should this television be advertised?

3. AD = 3, BC = 5 and CD = 8. The angle ADC and BCD are right angle. The point P is on the line CD. Find the minimum value of AP + BP

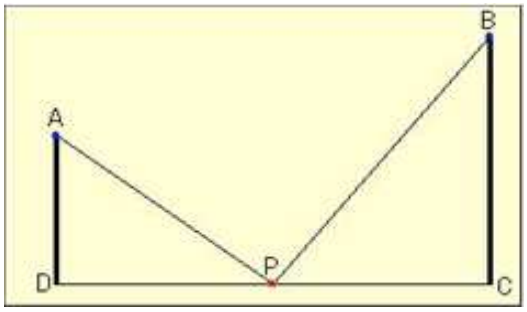

Figure 5: Minimum value of $AP + BP$.

V. Real Life Applications

Some real life applications to introduce the concept of Pythagoras' theorem to your middle school students are given below:

5.1 Road Trip: Let's say two friends are meeting at a playground. One friend is located on the south-west corner of playground and other is located on the north-east corner of the playground. There are two ways to go let us see how you can take the help of Pythagoras' theorem to calculate the shortest distance between the meeting points of two friends. If you follow a road 3 miles east and then 4 miles north. Your total distance covered will be 3+4 = (7) miles but if you apply Pythagoras' theorem to calculate the distance you will get:

$(3)^2 + (4)^2 =$

$9 + 16 = C^2$

$\sqrt{25} = C$

5 Miles. = C

So this will save them 2 miles distance.

5.2 Painting on a Wall: Painters use ladders to paint on high buildings and often use the help of Pythagoras' theorem to complete their work. Take for example a painter who has to paint a wall which is about 8 m high. The painter has to put the ladder 6 m away to avoid a rack in between. What will be the length of the ladder required by the painter to complete his work? You can calculate it using Pythagoras' theorem:

$(8)^2 + (6)^2 =$

$64 + 36 = C^2$

$\sqrt{100} = C$

10 Mts. = C

Thus, the painter will need a ladder 10 meters high.

5.3 Buying a Suitcase: Mr. Harry wants to purchase a suitcase. The shopkeeper tells Mr. Harry that he has a 30 inch of suitcase available at present and the height of the suitcase is 18 inches. Calculate the actual length of the suitcase for Mr. Harry using Pythagoras' theorem. It is calculated this way:

$(18)^2 + (b)^2 = (30)^2$

$324 + b^2 = 900$

$B^2 = 900 – 324$

$b= \sqrt{576}$

= 24 inches

5.4 What Size TV Should You Buy? Mr. James saw an advertisement of a T.V.in the newspaper where it is mentioned that the T.V. is 16 inches high and 14 inches wide. Calculate the diagonal length of its screen for Mr. James. By using Pythagoras' theorem it can be calculated as:

$(16)^2 + (14)^2 =$

$256 + 196 = C^2$

$\sqrt{452} = C$

21 inches approx. = C

5.5 Finding the Right Sized Computer: Mary wants to get a computer monitor for her desk which can hold a 22 inch monitor. She has found a monitor 16 inches wide and 10 inches high. Will the computer fit into Mary's cabin? Use Pythagoras' theorem to find out:

$(16)^2 + (10)^2 =$

$256 + 100 = C^2$

$\sqrt{356} = C$

18 inches approx. = C

5.6 Architecture and Construction: The most obvious application of the Pythagorean Theorem is in the world of architecture and building construction, particularly in reference to triangular-shaped roofs and gables. The theorem applies only when dealing with right triangles or triangles with a 90-degree angle.

5.7 Earthquake Location: Geologists also use the Pythagorean Theorem when tracking earthquake activity. Earthquakes result in two different types of waves -- one that is slower than the other. By triangulating the distance travelled by the faster waves with that travelled by the slower waves, geologists can determine the centre or source of the earthquake.

5.8 Crime Scene Investigation: Forensic investigators use the Pythagorean Theorem to determine bullet trajectory. Bullet trajectory shows the path the bullet took before impact. This trajectory tells police the area from which the bullet originated. Investigators can also tell how close the shooter was to the victim. This can help police determine whether or not a death is a suicide or a homicide. Blood spatter evidence can also be analyzed with the Pythagorean Theorem. Blood spatter is the spray of blood from a victim after an assault. Police use these calculations to determine the angle of impact and the positions of both the victim and the assailant during the assault.

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