Digital Filter

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Abstract:

*In signal processing, a **digital filter** is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. This is in contrast to the other major type of electronic filter, the analog filter, which is an electronic circuit operating on continuous-time analog signals. A digital filter system usually consists of an analog-to-digital converter to sample the input signal, followed by a microprocessor and some peripheral components such as memory to store data and filter coefficients etc. Finally a digital-to-analog converter to complete the output stage. Program Instructions (software) running on the microprocessor implement the digital filter by performing the necessary mathematical operations on the numbers received from the ADC. In some high performance applications, an FPGA or ASIC is used instead of a general purpose microprocessor, or a specialized DSP with specific paralleled architecture for expediting operations such as filtering. Digital filters may be more expensive than an equivalent analog filter due to their increased complexity, but they make practical many designs that are impractical or impossible as analog filters. When used in the context of real-time analog systems, digital filters sometimes have problematic latency (the difference in time between the input and the response) due to the associated analog-to-digital and digital-to-analog conversions and anti-aliasing filters, or due to other delays in their implementation. Digital filters are commonplace and an essential element of everyday electronics such as radios, cellphones, and AV receivers.*

Introduction:

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialised DSP (Digital Signal Processor)chip. The analog input signal must first be sampled and digitised using an ADC (analog to digital converter). Theresulting binary numbers, representing successive sampled values of the input signal, are transferred to the processor, which carries out numerical calculations on them. These calculations typically involve multiplying the input values by constants and adding the products together. If necessary, the results of these calculations, which now represent sampled values of the filtered signal, are output through a DAC (digital to analog converter) to convert the signal back to analog form. Note that in a digital filter, the signal is represented by a sequence of numbers, rather than a voltage or current.

Characterization: A digital filter is characterized by its transfer function, or equivalently, its difference equation. Mathematical analysis of the transfer function can describe how it will respond to any input. As such, designing a filter consists of developing specifications appropriate to the problem (for example, a second-order low pass filter with a specific cut-off frequency), and then producing a transfer function which meets the specifications.

The transfer function for a linear, time-invariant, digital filter can be expressed as a transfer function in the *Z*-domain; if it is causal, then it has the form:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}$$

where the order of the filter is the greater of *N* or *M*. See *Z*-transform's LCCD equation for further discussion of this transfer function.

This is the form for a recursive filter with both the inputs (Numerator) and outputs (Denominator), which typically leads to an IIR infinite impulse response behaviour, but if the denominator is made equal to unity i.e. no feedback, then this becomes an FIR or finite impulse response filter.

Analysis techniques: A variety of mathematical techniques may be employed to analyze the behaviour of a given digital filter. Many of these analysis techniques may also be employed in designs, and often form the basis of a filter specification. Typically, one characterizes filters by calculating how they will respond to a simple input such as an impulse. One can then extend this information to compute the filter's response to more complex signals.

Impulse response: The impulse response, often denoted $h[k]$ or h_k, is a measurement of how a filter will respond to the Kronecker delta function. For example, given a difference equation, one would set $x_0 = 1$ and $x_k = 0$ for $k \neq 0$ and evaluate. The impulse response is a characterization of the filter's behaviour. Digital filters are typically considered in two categories: infinite impulse response (IIR) and finite impulse response (FIR). In the case of linear time-invariant FIR filters, the impulse response is exactly equal to the sequence of filter coefficients:

$$y_n = \sum_{k=0}^{n-1} h_k x_{n-k}$$

IIR filters on the other hand are recursive, with the output depending on both current and previous inputs as well as previous outputs. The general form of an IIR filter is thus:

$$\sum_{m=0}^{M-1} a_m y_{n-m} = \sum_{k=0}^{n-1} b_k x_{n-k}$$

Plotting the impulse response will reveal how a filter will respond to a sudden, momentary disturbance.

Difference equation: In discrete-time systems, the digital filter is often implemented by converting the transfer function to a linear constant-coefficient difference equation (LCCD) via the Z-transform. The discrete frequency-domain transfer function is written as the ratio of two polynomials. For example:

$$H(z) = \frac{(z+1)^2}{(z-\frac{1}{2})(z+\frac{3}{4})}$$

This is expanded:

$$H(z) = \frac{z^2+2z+1}{z^2+\frac{1}{4}z-\frac{3}{8}}$$

and to make the corresponding filter causal, the numerator and denominator are divided by the highest order of z:

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}} = \frac{Y(z)}{X(z)}$$

The coefficients of the denominator, a_k, are the 'feed-backward' coefficients and the coefficients of the numerator are the 'feed-forward' coefficients, b_k. The resultant linear difference equation is:

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

or, for the example above:

$$\frac{Y(z)}{X(z)} = \frac{1+2z^{-1}+z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}}$$

rearranging terms:

$$\Rightarrow (1+\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2})Y(z) = (1+2z^{-1}+z^{-2})X(z)$$

then by taking the inverse *z*-transform:

$$\Rightarrow y[n]+\frac{1}{4}y[n-1]-\frac{3}{8}y[n-2] = x[n]+2x[n-1]+x[n-2]$$

and finally, by solving for $y[n]$:

$$y[n] = -\frac{1}{4}y[n-1]+\frac{3}{8}y[n-2]+x[n]+2x[n-1]+x[n-2]$$

This equation shows how to compute the next output sample, $y[n]$, in terms of the past outputs, $y[n-p]$, the present input, $x[n]$, and the past inputs, $x[n-p]$. Applying the filter to an input in this form is equivalent to a Direct Form I or II realization, depending on comthe exact order of evaluation.

Comparison of analog and digital filter: Digital filters are not subject to the component non-linearities that greatly complicate the design of analog filters. Analog filters consist of imperfect electronic components, whose values are specified to a limit tolerance (e.g. resistor values often have a tolerance of ±5%) and which may also change with temperature and drift with time. As the order of an analog filter increases, and thus its component count, the effect of variable component errors is greatly magnified. In digital filters, the coefficient values are stored in computer memory, making them far more stable and predictable. Because the coefficients of digital filters are definite, they can be used to achieve much more complex and selective designs – specifically with digital filters, one can achieve a lower passband ripple, faster transition, and higher stopband attenuation than is practical with analog filters. Even if the design could be achieved using analog filters, the engineering cost of designing an equivalent digital filter would likely be much lower. Furthermore, one can readily modify the coefficients of a digital filter to make an adaptive filteror a user-controllable parametric filter. While these techniques are possible in an analog filter, they are again considerably more difficult. Digital filters can be used in the design of finite impulse response filters. Analog filters do not have the same capability, because finite impulse response filters require delay elements. Digital filters rely less on analog circuitry, potentially allowing for a better signal-to-noise ratio. A digital filter will introduce noise to a signal during analog low pass filtering, analog to digital conversion, digital to analog conversion and may introduce digital noise due to quantization. With analog filters, every component is a source of thermal noise (such as Johnson noise), so as the filter complexity grows, so does the noise. However, digital filters do introduce a higher fundamental latency to the system. In an analog filter, latency is often negligible; strictly speaking it is the time for an electrical signal to propagate through the filter circuit. In digital systems, latency is introduced by delay elements in the digital signal path, and by analog-to-digital and digital-to-analog converters that enable the system to process analog signals. In very simple cases, it is more cost effective to use an analog filter. Introducing a digital filter requires considerable overhead circuitry, as previously discussed, including two low pass analog filters.

Types of digital filter: Many digital filters are based on the fast Fourier transform, a mathematical algorithm that quickly extracts the frequency spectrum of a signal, allowing the spectrum to be manipulated (such as to create band-pass filters) before converting the modified spectrum back into a time-series signal. Another form of a digital filter is that of a state-space model. A well used state-space filter is the Kalman filter published by Rudolf Kalman in 1960.Traditional linear filters are usually based on attenuation. Alternatively nonlinear filters can be designed, including energy transfer filters [10] which allow the user to move energy in a designed way. So that unwanted noise or effects can be moved to new frequency bands either lower or higher in frequency, spread over a range of frequencies, split, or focused. Energy transfer filters complement traditional filter designs and introduce many more degrees of freedom in filter design. Digital energy transfer filters are relatively easy to design and to implement and exploit nonlinear dynamics.

Advantages of using digital filters:

1. A digital filter is programmable, i.e. its operation is determined by a program stored in the processor's memory. This means the digital filter can easily be changed without affecting the circuitry (hardware).An analog filter can only be changed by redesigning the filter circuit.

2. Digital filters are easily designed, tested and implemented on a general-purpose computer or

Workstation.

3. The characteristics of analog filter circuits (particularly those containing active components) are subject to drift and are dependent on temperature. Digital filters do not suffer from these problems, and so are extremely stable with respect both to time and temperature.

4. Unlike their analog counterparts, digital filters can handle low frequency signals accurately. As the speed of DSP technology continues to increase, digital filters are being applied to high frequency signals in the RF (radio frequency) domain, which in the past was the exclusive preserve of analog technology.

5. Digital filters are very much more versatile in their ability to process signals in a variety of ways; this includes the ability of some types of digital filter to adapt to changes in the characteristics of the signal.

6. Fast DSP processors can handle complex combinations of filters in parallel or cascade (series), making the hardware requirements relatively simple and compact in comparison with the equivalent nalog circuitry.

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