# Theory and Applications for Diffraction of Light 

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#### Abstract

: The duality of light waves can sometimes be a difficult concept for high school level students to understand. To help demonstrate this dual personality the photoelectric effect often is used to show light acting as a particle (photons) and interference and diffraction can depict light's wave nature. However, at a high school level these principles can be difficult to apply and describe in the classroom. This lesson plan focuses on the wave nature of light and utilizes real-world examples and straightforward activities to explain the complexities of interference patterns and the diffraction of electromagnetic waves


## Introduction

Diffraction refers to various phenomena that occur when a wave encounters an obstacle or a slit. It is defined as the bending of light around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle. In classical physics, the diffraction phenomenon is described as the interference of waves according to the Huygens-Fresnel principle. These characteristic behaviors are exhibited when a wave encounters an obstacle or a slit that is comparable in size to its wavelength. Similar effects occur when a light wave travels through a medium with a varying refractive index, or when a sound wave travels through a medium with varying acoustic impedance. Diffraction has an impact on the acoustic space. Diffraction occurs with all waves, including sound waves, water waves, and electromagnetic waves such as visible light, X-rays and radio waves.

Since physical objects have wave-like properties (at the atomic level), diffraction also occurs with matter and can be studied according to the principles of quantum mechanics. Italian scientist Francesco Maria Grimaldi coined the word "diffraction" and was the first to record accurate observations of the phenomenon in 1660.

While diffraction occurs whenever propagating waves encounter such changes, its effects are generally most pronounced for waves whose wavelength is roughly comparable to the dimensions of the diffracting object or slit. If the obstructing object provides multiple, closely spaced openings, a complex pattern of varying intensity can result. This is due to the addition, or interference, of different parts of a wave that travel to the observer by different paths, where different path lengths result in different phases (see diffraction grating and wave superposition). The formalism of diffraction can also describe the way in which waves of finite extent propagate in free space. For example, the expanding profile of a laser beam, the beam shape of a radar antenna and the field of view of an ultrasonic transducer can all be analyzed using diffraction equations.

## Review of Literature

The effects of diffraction of light were first carefully observed and characterized by Francesco Maria Grimaldi, who also coined the term diffraction, from the Latin diffringere, 'to break into pieces', referring to light breaking up into different directions. The results of Grimaldi's observations were published posthumously in 1665.Isaac Newton studied these effects and attributed them to inflexion of light rays. James Gregory (1638-1675) observed the diffraction patterns caused by a bird feather, which was effectively the first diffraction grating to be discovered. Thomas Young performed a celebrated experiment in 1803 demonstrating interference from two closely spaced slits. Explaining his results by interference of the waves emanating from the two different slits, he deduced that light must propagate as waves. AugustinJean Fresnel did more definitive studies and calculations of diffraction, made public in 1815. and 1818,.and thereby gave great support to the wave theory of light that had been advanced by Christiaan Huygens and reinvigorated by Young, against Newton's particle theory.

## Theory of Diffraction of Light

## Basically phenomenon of diffraction is divided into two main groups-

| S. NO. | Fraunhofer Diffraction | Fresnel Diffraction |
| :---: | :---: | :---: |
| 1 2. | The Diffraction device is at infinite distance from the source. <br> The wavefront is a plane one. | The Diffraction device is at finite distance from the source and the screen. <br> The wavefront is either spherical or cylindricl |
| 3. | The centre of diffraction pattern is always bright. | The centre of diffraction pattern may be bright ordark. |
| 4. | Two convex lenses are used to observe diffraction effect. | Lenses or mirrors are not used to observe diffraction effect. |
| 5. | The diffraction pattern is the image of source itself. | The diffraction pattern is the image of the obstacle or aperture. <br> In this distance are important. |
| $\begin{aligned} & 6 . \\ & 7 . \end{aligned}$ | In this the inclinations are important. <br> In this the effects of all diffracting devices are added. | single diffraction device i.e., the diffraction effects of devices are not added. |

## Difference between Interference and Diffraction

| S.NO. | INTERFERENCE | DIFFRACTION |
| :---: | :---: | :---: |
| 1. | It occurs between two different wavefronts originating from two coherent sources | It occurs due to the secondary wavelets originating from infinite different points of the same wavefront. |
| 2. | In an interference pattern, the regions of minima are usually perfectly dark. | In the diffraction pattern, they are not perfectly dark. |
| 3. | The interference fringes are usually (not always) of the same width. | The diffraction fringes are never of the same width. |
| 4. | All maxima are of same intensity. | They are of varying intensity. |
| 5. | Condition for maxima <br> (a) Path difference $\Delta=2 n \lambda / 2$ <br> (a) Phase difference $\delta=2 n \pi$ | Condition for maxima <br> (a)Path difference $\Delta=(2 n+1) \lambda / 2$ <br> (b)Phase difference $\delta=(2 n \pm 1) \pi$ |
| 6. | Condition for minima <br> (a) Path difference $\Delta=(2 n \pm 1) \lambda / 2$ <br> (b) Phase difference $\delta=(2 \mathrm{n} \pm 1) \pi$ | Condition for minima <br> (a)Path difference $\Delta=2 \mathrm{n} \lambda / 2$ <br> (b)Phase difference $\delta=2 n \pi$ |

## Single Slit Diffraction (Franuhofer's Diffraction)



Fig. represent a section AB of a narrow slit of width a perpendicular to the plane of the paper. Let a plane waveform ww' of monochromatic light of wavelength propagating normally to the slit be incident on it. Let the diffracted light be focused by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygen's-Fresnel; every point of the wavefront in the plane of the slit is a source of
secondary spherical wavelets, which spread out of the right in all directions. The secondary wavelets traveling normally to the slit, i.e., along the direction $\mathrm{OP}_{\mathrm{o}}$ are brought to focus at $\mathrm{P}_{\mathrm{o}}$ by the lens. Thus, $\mathrm{P}_{\mathrm{o}}$ is bright central image. The secondary wavelets traveling at an angle $\theta$ with the normal are focused at a point $P_{1}$ on the screen. The point $P_{1}$ is the minimum intensity depending upon the path difference between the secondary waves originating form the corresponding points of the wavefront. In order to find out intensity at $P_{1}$, draw a perpendicular $A C$ on $B R$. The path difference between secondary wavelets from $A$ and $B$ in the direction $\theta=\mathrm{BC}=\mathrm{AB} \sin \theta=\mathrm{e} \sin \theta$

$$
\text { and corresponding phase difference }=\frac{2 \pi}{\lambda} \mathrm{e} \sin \theta
$$

Let us consider that the width of the slit is divided into n equal parts and the amplitude of the waveform each part is a (because width of each part is same). The Phase difference between any two consecutive waves form these parts would be-

$$
\frac{1}{n}(\text { Total Phase })=\frac{1}{n}\left(\frac{2 \pi}{\lambda} e \sin \theta\right)=\mathrm{d}(\text { say })
$$



Let there be n vibration of the same period, same amplitude a and same phase difference d between successive vibrations which act on a particle simultaneously. Our aim is to consider the resultant amplitude of these vibrations. For this Purpose, We construct the Polygon of amplitudes as shown In the fig.

The closing side OP and angle $\theta$ then gives the resultant amplitude R and Phase of the resultant vibration respectively. To evaluate R and $\theta$, we resolve the amplitudes along and perpendicular to OA and write

$$
\begin{align*}
& \mathrm{R} \cos \theta=\mathrm{a}[1+\cos \mathrm{d}+\cos 2 \mathrm{~d}+\ldots \ldots \ldots \ldots \ldots+\cos (\mathrm{n}-1) \mathrm{d}] \\
& \mathrm{R} \sin \theta=\mathrm{a}[1+\sin \mathrm{d}+\sin 2 \mathrm{~d}+\ldots \ldots \ldots \ldots \ldots+\sin (\mathrm{n}-1) \mathrm{d}]
\end{align*}
$$

Multiplying equation (1) by $2 \sin \mathrm{~d} / 2$ we get -
$2 R \cos \theta \cdot \sin \mathrm{~d} / 2=\mathrm{a}[2 \sin \mathrm{~d} / 2+2 \cos \mathrm{~d} \cdot \sin \mathrm{~d} / 2+\ldots \ldots+2 \cos (\mathrm{n}-1) \mathrm{d} \cdot \operatorname{sind} / 2)]$
$=\mathrm{a} \quad\left[2 \sin \frac{d}{2}+\left\{\sin \frac{3 d}{2}-\sin \frac{d}{2}\right\}+\left[\sin \left(n-\frac{1}{2}\right) d-\sin \left(n-\frac{3}{2}\right) d\right]\right]$ and
$=\mathrm{a} \quad\left[\sin \frac{d}{2}+\sin \left(n-\frac{1}{2}\right) d\right]$
$=2 \mathrm{a} \sin \frac{n d}{2} \cos \frac{(n-1) d}{2}$
or R $\cos \theta=\mathrm{a} \frac{\sin n d / 2}{\sin d / 2} \cos \frac{(n-1) d}{2}$

Similarly , multiplying equation (2) by $2 \operatorname{sind} / 2$ and simplifying, we get-
$\mathrm{R} \sin \theta=\mathrm{a} \frac{\sin n d / 2}{\sin d / 2} \sin \frac{(n-1) d}{2}$

Squaring equation (3) and (4) and adding we get-

$$
\begin{align*}
& \mathrm{R}^{2}=\mathrm{a}^{2} \frac{\sin ^{2} n d / 2}{\sin ^{2} d / 2} \\
& \text { or } \mathrm{R}=\mathrm{a} \frac{\sin n d / 2}{\sin d / 2} \tag{5}
\end{align*}
$$

$=\mathrm{a} \frac{\sin (\pi e \sin \theta / \lambda)}{\sin (\pi e \sin \theta / \lambda)}$

$$
=\mathrm{a} \frac{\sin \alpha}{\sin \alpha / n} \text { where } \alpha=\lambda e \sin \theta / \lambda
$$

$=\mathrm{a} \frac{\sin \alpha}{\alpha / n}\left(\frac{\alpha}{n}\right.$ isverysmal $)$
$=\mathrm{na} \frac{\sin \alpha}{\alpha}$
$=\mathrm{A} \frac{\sin \alpha}{\alpha}$ (where $\mathrm{n} \quad \alpha, \mathrm{a} \quad 0$ but $\quad$ A remains finite $)$
Now the intensity is givin by-
$\mathrm{I}=\mathrm{R}^{2}=\mathrm{A}^{2}\left(\frac{\sin \alpha}{\alpha}\right)^{2}$
(a) Position of maxima / minima

## Principal Maxima

The expression for the resultant amplitude R can be written in ascending power of $\alpha$ as

$$
\left.\begin{array}{rl} 
& \mathrm{R}=\frac{A}{\alpha}\left[\alpha-\frac{\alpha^{3}}{3}+\frac{\alpha^{5}}{5}-\frac{\alpha^{7}}{7}+\ldots . . . . . . . . . . .\right.
\end{array}\right]
$$

If the negative terms vanish, the value of R will be maximum. i.e., $\alpha=0$
$\therefore \alpha=\frac{\pi e \sin \theta}{\lambda}=0$ or $\sin \theta=0$ or $\theta=0$

Now maximum value of $R$ is $A$ and intensity is proportional to $A^{2}$. The condition $\theta=0$ means that this maximum is formed by those secondary wavelets which travel normally to the slit. The maximum is known as principal maxima.

## Minimum intensity position-

The intensity will be minimum, when $\sin \alpha=0$. The value of $\alpha$ which satisfy this equation are
$\alpha= \pm \pi, \pm 2 \pi, 3 \pi, \pm 4 \pi, \ldots \ldots .$. etc. $= \pm m \pi$
or $\frac{\pi e \sin \theta}{\lambda}= \pm \mathrm{m} \pi$ or e $\sin \theta= \pm \mathrm{m} \lambda$
where $\mathrm{m}=1,2,3,4, \ldots \ldots \ldots \ldots$.etc.

In this way we obtain the points of minimum intensity on the either sideof the principal maximum. The value of $\mathrm{m}=0$ is not admissible, because for this value $\theta=0$ and this corresponds to principal maximum.

## $\underline{\text { Secondary Maxima - }}$

In addition to principal maximum at $\alpha=0$, there are weak secondary maxima between equally spaced minima. The position can be obtained with the rule of finding maxima and minima of a given function in calculus. Differentiating the expression of I with respect to $\alpha$ and equating to zero, we have

$$
\begin{aligned}
& \frac{d I}{d \alpha}=\frac{d}{d \alpha}\left[A^{2}\left(\frac{\sin \alpha}{\alpha}\right)^{2}\right]=0 \\
& A^{2} \cdot \frac{2 \sin \alpha}{\alpha} \cdot \frac{(\alpha \cos \alpha-\sin \alpha)}{\alpha^{2}}=0
\end{aligned}
$$

Either $\sin \alpha=0$ or $(\alpha \cos \alpha-\sin \alpha)=0$

The equation $\sin \alpha=0$ gives the values of (except $\theta$ )for which the intensity is zero on the screen. Hence, the position of maxima is given by the roots of the equation.

$$
(\alpha \cos \alpha-\sin \alpha)=0 \text { or } \alpha=\tan \alpha
$$

The values of $\alpha$ satisfying the above equation are obtained graphically by plotting the curves $\mathrm{y}=\alpha$ and y $=\tan \alpha$ on the graph.

The point of intersection of two curves gives the values of $\alpha$ which satisfy equation( 4). The plots of $y=\alpha$ and $\mathrm{y}=\tan \alpha$ are shown in fig.


The points of intersection are
$\alpha=0, \pm 3 \pi / 2, \pm 5 \pi / 2, \ldots \ldots .$. etc
$\alpha=0$ gives principal maximum
Substituting approximate value of $\alpha$ in equation (6), we get the intensities in the various maxima as $\mathrm{I}_{0}=\mathrm{A}^{2}$ (principal maximum)
$\mathrm{I}_{1}=A^{2}[\sin (3 \pi / 2) /(3 \pi / 2)]^{2}=4 \mathrm{I}_{0} / 9 \pi^{2}=\mathrm{A}^{2} / 22$ approx.
[ Ist subsidiary maximum]
$\mathrm{I}_{2}=A^{2}[\sin (5 \pi / 2) /(5 \pi / 2)]^{2}=4 \mathrm{I}_{0} / 25 \pi^{2}=\mathrm{A}^{2} / 62$ approx.
[ IInd subsidiary maximum]
and son on . from the expression of $\mathrm{I}_{0}, \mathrm{I}_{1}$, and $\mathrm{I}_{2}$ it is evident that most of the incident light in concentrated in the principal maximum.

## Plane Transmission Diffraction Grating.

## Construction

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the grating consisted of a large number of parallel wires placed very closely side-by-side at regular intervals. Now grating is constructed by ruling equidistant parallel lines on a transparent material such as glass, with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as slit. This is known as plane transmission grating.

## Theory of Plane Transmission Diffraction Grating



Fig. represent the section of a plane transmission grating placed perpendicular to the plane of the paper. Let e be the width of each slit and $d$ the width of each opaque part. Then (e+d) is known as grating element. XY is the screen placed perpendicular to the plane of the paper. Suppose a parallel beam of monochromatic light of wavelength $\lambda$ be incident normally on the grating.

The intensity at a point $\mathrm{P}_{1}$ may be considered by applying the theory of Fraunhofer diffraction at a singlr slit. The wavelets proceeding from all points in a slit along the direction $\theta$ are equivalent to a single wave of amplitude[A $(\sin \alpha / \alpha)$ ],
where $\alpha=\pi \mathrm{e} \sin \theta / \lambda$.
If there are N slits, then we have diffracted waves, one each from the middle points of the slits. The path difference between two consecutive slits is $(\mathrm{e}+\mathrm{d}) \sin \theta$. Therefore, there is a corresponding phase difference $2 \pi(\mathrm{e}+\mathrm{d}) \sin \theta / \lambda$ between two consecutive wave. The phase difference is constant and it be $2 \beta$

By the method of vector addition of amplitudes-
$\mathrm{MP}_{1}=\mathrm{P}_{1} \mathrm{P}_{2} \ldots \ldots \ldots \mathrm{P}_{\mathrm{N}-1} \mathrm{P}_{\mathrm{N}}=\mathrm{R}=\mathrm{A}(\sin \alpha / \alpha)$
Representing wave amplitude by a phase vector, let us draw vectors $\mathrm{MP}_{1}, \mathrm{P}_{1} \mathrm{P}_{2}, \mathrm{P}_{2} \mathrm{P}_{3}$ $\qquad$ $\mathrm{P}_{\mathrm{N}-1,} \mathrm{P}_{\mathrm{N}}$ of equal length A but each increasing in phase by constant amount $2 \beta$. We thus end up with a polygon of vectors of N side whose resultant $\mathrm{OP}_{\mathrm{N}}$. Say it has amplitude $\mathrm{R}_{\mathrm{N}}$. If O be the centre of polygon, then from the geometry of figure. It is seen that
$\mathrm{MP}_{1}=2 \mathrm{OM} \sin \beta$
$\mathrm{MP}_{\mathrm{N}}=2 \mathrm{OM} \sin \mathrm{N} \beta$
Dividing we have-
$\mathrm{MP}_{\mathrm{N}}=\mathrm{MP}_{1} \sin \mathrm{~N} \beta / 2 \sin \beta=\mathrm{R} \sin \mathrm{N} \beta / 2 \sin \beta$
$\mathrm{R}_{\mathrm{N}}=\mathrm{A}(\sin \alpha / \alpha)(\sin \mathrm{N} \beta / \sin \beta)$
Hence resultant intensity is given by-

$$
I=A^{2}\left(\sin ^{2} \alpha / \alpha^{2}\right)\left(\sin ^{2} N \beta / \sin ^{2} \beta\right)
$$

The factor $\left[[A(\sin \alpha / \alpha)]^{2}\right.$ gives the distribution of intensity due to a single slit while the factor $\left(\sin ^{2}\right.$ $N \beta / \sin ^{2} \beta$ ) gives the distribution of intensity as a combined effect of all the slits.

## Principal Maxima

The intensity would be maximum when $\sin \beta=0$
OR $\beta= \pm n \pi$ where $n=0,1,2,3 \ldots \ldots$.
But at the same time $\sin N \beta=0$, so that the factor $(\sin N \beta / \sin \beta)$ becomes indeterminate. It may be evaluated by applying the usual method of differentiating the numerator and the denominator i.e., by applying the Hospital's rule. Thus-

$$
\begin{aligned}
& \operatorname{Lim}_{\beta \rightarrow n \pi} \sin N \beta / \sin \beta=\operatorname{Lim}_{\beta \rightarrow n \pi} d / d \beta(\sin N \beta) / d / d \beta(\sin \beta) \\
& =\operatorname{Lim}_{\beta \rightarrow n \pi} N \cos N \beta / \cos \beta \\
& = \pm N
\end{aligned}
$$

Hence, $\operatorname{Lim}_{\beta \rightarrow n \pi}(\sin N \beta / \sin \beta)^{2}=N^{2}$

The resultant intensity as $[\mathrm{A}(\sin \alpha / \alpha)]^{2} \mathrm{~N}^{2}$.
The maxima are obtained for $\beta= \pm n \pi$
$\pi(\mathrm{e}+\mathrm{d}) \sin \theta / \lambda= \pm \mathrm{n} \pi$
$(\mathrm{e}+\mathrm{d}) \sin \theta= \pm \mathrm{n} \lambda \quad \mathrm{n}=0,1,2,3, \ldots .$.
$\mathrm{n}=0$ corresponds to zero order maximum.

## Minima

A series of minima occur, when $\sin N \beta=0$ but $\sin \beta \neq 0$
For minima $\sin N \beta=0$ or $N \beta= \pm m \pi$
$\mathrm{N} \pi / \lambda(\mathrm{e}+\mathrm{d}) \sin \theta== \pm \mathrm{m} \pi$
$\mathrm{N}(\mathrm{e}+\mathrm{d}) \sin \theta= \pm \mathrm{m} \pi$
Where m has all integral values except $0, \mathrm{~N}, 2 \mathrm{~N}, \ldots . \mathrm{nN}$ because for these values
$\sin \beta$ becomes zero and we get principal maxima. Thus $m=1,2,3, \ldots .(\mathrm{N}-1)$ hence, there are adjacent principal maxima.

## Secondary Maxima

$$
\frac{d I}{d \beta}=\left[\frac{A \sin \alpha}{\alpha}\right]^{2} \cdot 2\left[\frac{\sin N \beta}{\sin \beta}\right] X\left[\frac{N \sin N \beta \sin \beta-\sin N \beta \cos \beta}{\sin ^{2} \beta}\right]=0
$$

OR $N \cos N \beta \sin \beta-\sin N \beta \cos \beta=0$
$N \tan \beta=\tan N \beta$

The roots of this equation other than those for which $\beta= \pm n \pi$ (which correspond to principal maxima) gives the position of secondary maxima.

To find out the value of $\left(\sin ^{2} N \beta / \sin ^{2} \beta\right)$ from equation $N \tan \beta=\tan N \beta$, we make use of the triangle shown in fig.


$$
\sin N \beta=\frac{N \tan \beta}{\sqrt{1+N^{2} \tan ^{2} \beta}}
$$

$$
\frac{\sin ^{2} N \beta}{\sin ^{2} \beta}=\frac{N^{2} \tan ^{2} \beta}{() 1+N^{2} \tan ^{2} \beta \sin ^{2} \beta}
$$

$$
=\frac{N^{2}}{\cos ^{2} \beta+N^{2} \sin ^{2} \beta}
$$

$$
=\frac{N^{2}}{1+\left(N^{2}-1\right) \sin ^{2} \beta}
$$

$$
\frac{\text { Intensity of secondarymax ima }}{\text { int } \text { ensityofprincipal } \max \text { ima }}=\frac{N^{2}}{1+\left(N^{2}-1\right) \sin ^{2} \beta}
$$

As N increases, the intensity of secondary maxima relative to principal maxima decrease and becomes negligible when N become large.

## \# Missing spectra with a diffraction grating

The principal maxima in the grating spectrum are obtained in the direction given by
$(\mathrm{e}+\mathrm{d}) \sin \theta= \pm \mathrm{n} \lambda$
The minima in a single slit diffraction pattern (assumed a grating as N single slits) are obtained in the direction given by-

$$
\begin{equation*}
(\mathrm{e}+\mathrm{d}) \sin \theta= \pm \mathrm{m} \lambda \tag{14}
\end{equation*}
$$

If the equation (13) and (14) are simultaneously satisfied, a particular maximum or spectrum of order $n$ will be absent in the grating spectrum.

Now dividing equation (13) by equation (14) we get

$$
\begin{equation*}
\frac{e+d}{a}=\frac{n}{m} \tag{15}
\end{equation*}
$$

This is the condition for the spectrum of the order n to be missing in the diffraction pattern.
(i)If $\mathrm{e}=\mathrm{d}$ then $\mathrm{n}=2 \mathrm{~m}$

$$
\mathrm{n}=2,4,6 \ldots \ldots \ldots \ldots \text {. } \mathrm{for} \mathrm{~m}=1,2,3, \ldots \ldots \ldots \ldots . .]
$$

which is the $2^{\text {nd }}, 4^{\text {th }}, 6^{\text {th }} \ldots \ldots$. order spectrum will be missing.
(ii)If $2 \mathrm{e}=\mathrm{d}$ then $\mathrm{n}=3 \mathrm{~m}$

$$
n=3,6,9 \ldots \ldots \ldots \ldots[\text { for } m=1,2,3, \ldots \ldots \ldots \ldots .]
$$

Which is $3^{\text {rd }}, 6$ th, 9 th........... order spectrum will be missing.
Thus, the relative between width of ruling (d) and width of slits(e) determine the order of absent spectra.
The relation between grating element and wavelength of light also determine the order of absent spectra. According to equation (13)

$$
\begin{equation*}
\sin \theta=\frac{n \lambda}{(a+b)} \tag{1}
\end{equation*}
$$

If $(\mathrm{e}+\mathrm{d})<\lambda$ then $\sin \theta>1$ for $\mathrm{n} \geq 1$

Hence, the first and higher order spectra are missing.

If $(\mathrm{e}+\mathrm{d})<2 \lambda$ then $\sin \theta>1$ for $\mathrm{n} \geq 2$

Hence, the second and higher order spectra are missing similarly.

If $(\mathrm{e}+\mathrm{d})<\mathrm{n} \lambda$ then $\sin \theta>1$ for $\theta \geq \mathrm{n}$

Hence, $\mathrm{n}^{\text {th }}$ and higher order spectra are missing.

Therefore, grating element (e+d) should be greater than $n$-times of wavelength of light $\lambda$, to see $n$ order spectra.

## \# Overlapping of spectral lines

If the light incident on the grating surface consist of a large range of wavelengths, then the spectral lines of shorter wavelength and of higher order overlap on the spectral lines of longer wavelength and of lower order.

Let the angle of diffraction $\theta$ be the same for-
(I) the spectral line of wavelength $\lambda_{1}$ in the first order
(II) the spectral line of wavelength in. $\lambda_{2}$ in the second order
(III) the spectral line of wavelength in $\lambda 3$ in the third order
then
$(\mathrm{e}+\mathrm{d}) \sin \theta=1 \cdot \lambda_{1}=2 \cdot \lambda_{2}=3 \cdot \lambda_{3}$
as $\left(\lambda_{1}>\lambda_{2}>\lambda_{3}\right)$

## Resolving power of grating.

## Rayleigh Criterion of resolutions

In order to express the resolving power of an optical instruments lord Raleigh proposed a universal criterion known as Raleigh's Criterion of resolution. This criterion is the generally accepted criterion for the minimum resolvable detail. According to Raleigh two nearby point source images are said to be just resolved if the position of the central maxima of one point source coincides (overlaps) with the minimum of another point source. It is possible to resolve the two objects as long as the central maxima of the two diffraction patterns do not overlap. If the two central maxima overlap the two objects look like one.


Let P and Q be principle (central) maxima of diffraction pattern of two spectral lines of wavelength say $\lambda$ and $\lambda+\mathrm{d} \lambda$. If the difference in the angle of diffraction is quite large then two maxima can be seen distinctly and both the spectral lines will appear well resolved. As shown in fig.
position of spectral line of wavelength or vice-versa. Then the resultant intensity of both diffraction patterns will only be observed. This is known as condition of just-resolution. The resultant intensity curve is shown in fig. A dip in the intensity curve in the middle of enctral maxima of P and central maxima of Q is found, where the intensity is approximately $20 \%$ less than that of any intensity peak individually. This position of spectral lines where the intensity in the middle is $80 \%$ approximately of any spectral lines is known as condition of just resolution for spectral line, also known as spectral resolution.
we are not able to resolve these spectral lines. As shown in fig.

## (b) Expression for resolving power of a grating

Let $A B$ is the grating surface and $X Y$ is the screen. $P_{1}$ is the $n^{\text {th }}$ principle maxima of a spectral line of wavelength $\lambda$ at an angle of diffraction $\theta_{\mathrm{n}} . \mathrm{P}_{2}$ is the $\mathrm{n}^{\text {th }}$ principal wavelength $(\lambda+\mathrm{d} \lambda)$ at angle of diffraction $\left(\theta_{\mathrm{n}}+\mathrm{d} \theta_{\mathrm{n}}\right)$ position of principal maximum given by-

$$
\begin{equation*}
(\mathrm{e}+\mathrm{d}) \sin \theta_{\mathrm{n}}=\mathrm{n} \lambda \tag{21}
\end{equation*}
$$

$$
\begin{equation*}
(\mathrm{e}+\mathrm{d}) \sin \left(\theta_{\mathrm{n}}+\mathrm{d} \theta_{\mathrm{n}}\right)=\mathrm{n}(\lambda+\mathrm{d} \lambda) \tag{22}
\end{equation*}
$$

According to reyleigh criterion, these spectral lines will appear just resolved if the principal maxima due to $(\lambda+d \lambda)$ or vice- versa.

First minimum of wavelength $\lambda$ formed in the direction $\left(\theta_{n}+d \theta_{n}\right)$ it at $P_{2}$.
$\mathrm{N}(\mathrm{e}+\mathrm{d}) \sin \left(\theta_{\mathrm{n}}+\mathrm{d} \theta_{\mathrm{n}}\right)=\mathrm{m} \lambda$
The first minimum adjacent to the $\mathrm{n}^{\text {th }}$ principal maxima in the direction $\left(\theta_{\mathrm{n}}+\mathrm{d} \theta_{\mathrm{n}}\right)$ will be obtained for $\mathrm{m}=\mathrm{nN} \pm 1$, where N is the total number of rulings on the grating.

Maxima of another spectral lines of
(e+d) $\sin \left(\theta_{\mathrm{n}} \pm \mathrm{d} \theta_{\mathrm{n}}\right)=\frac{(n N \pm 1) \lambda}{N}= \pm \frac{n \lambda+\lambda}{N}$

Comparing equation (3) and (4) we get-
$n \lambda+\frac{\lambda}{N}=n(\lambda+d \lambda) ; n d \lambda=\frac{\lambda}{N}$
Resolution limit of grating $\frac{\lambda}{d \lambda}=\frac{1}{N n}$
And resolution power of grating $\mathrm{R}=\frac{d \lambda}{\lambda}=N n$
Thus, the resolving power of grating is equal to the product of the order of the spectrum and the total number of rulings on the grating.

## Gratings for Industrial Applications

For several decades HORIBA Scientific's OEM division has developed a range of manufacturing processes to produce diffraction gratings that are used in a variety of industrial applications. These established processes are optimised to ensure high volume production capacity and the expected level of industrial component quality, at competitive prices.

Our gratings are replicas which offer consistent optical performance that is often essential for industrial applications, where the same product is manufactured for many years. It also allows more flexibility to match the range of expected environmental conditions (temperature, humidity etc.). Although we have a standard range of gratings we can also customise these to offer different sizes or coatings. Contact us to discuss your specific requirements

Our gratings are typically used in Biomedical devices, HPLC systems, Telecommunication active \& passive modules, Sensing equipment, Control process monitors, Laboratory spectroscopy systems, Colour analysis instruments, Lasers, Life science products, Photovoltaic production and control machines, Mineralogy controllers, Analytical chemistry application, to name a few.

Aberration Corrected Concave Spectrograph (Flat Field) Gratings
Aberration Corrected Monochromator Gratings (Type IV)
Holographic Concave Roland Circle Gratings
Holographic Plane Gratings
Blazed Holographic Plane Gratings
Ruled Plane Gratings

## Reference:

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