International Journal of Research
Available at https://edupediapublications.org/journals
e-ISSN: 2348-6848
p-ISSN: 2348-795X
Volume 04 Issue 14
November 2017

Optimality of Manpower Reserve for Asymmetric Logistic-Distribution

Ms. Parul Saini,
Research Scholar, Department of Statistics M.D. University, Rohtak, Haryana

Abstract
For every organisation, its functioning is affected by many factors. Manpower is one of the main factors which strongly influences its functional. Every organization should have proper availability of manpower. Present study, focuses on recruitment and training of manpower in organizations using m and $\hat{S}$. It was observed that as the value of 'm' increases, a corresponding increase in $\hat{S}$ is observed in both cases, i.e. for, the first order statistic and the nth order statistic.
Keywords: manpower reserve, asymmetric logistic-distribution

Introduction

There are many factors which play a vital role for the smooth functioning of the organization and for its further progress, such as the capital investment, marketing strategies and industrial maintenance etc. Manpower is one of the major components in any organization, so it is necessary to maintain proper manpower availability in the organizations. The production and other activities of the organization, such as marketing and administrative may be affected, if enough of manpower is not available. So steps are needed to keep the enough manpower in other words the optimum manpower in the organization. Many organizations, for this purpose, do recruitment of sufficient manpower. It happens occasionally that the number of recruited persons or the availability of manpower in terms of man per hour may be more than the required one and the availability manpower above the required one will lead to loss of profit due to the excess introduction/inventory of manpower. This is called holding cost of manpower. If there is shortage of manpower it will again lead to loss of profit.

Review of literature

The idea of optimal reserve between two machines in series has been talked about in **Hanssmann (1961).** An extension of this model has been given by **Rajagopal and Sathiyamoorthi (2003). Muthaiyan et.al (2009)** have discussed about the estimation of expected time to recruitment using order statistics. Determination of optimal manpower stock has been studied by **G.Arivazhagan et.al (2010)**.

Arivazhagan et al., (2010) In view of the fluctuation in demand for manpower, recruitment is done frequently followed by training. This is called stage 1. Sometimes there may be a delay in the process of recruitment in stage 1 and hence shortage of manpower arises, in stage 2, which involves use of manpower. The breakdown of activities at stage 2 due to shortage of manpower is very costly. Hence a reserve of manpower or inventory is suggested.

Training may have a breakdown which influences the working of the organization. Because of shortage of manpower, which frequently occurs and pronounces in Indian industries, a reserved inventory of manpower is maintained by the organizations. Manpower is separated in two nodes, *viz*, requirement node and work spot node. The optimal reserve of manpower in terms of man per hours is determined under the following assumptions:

- There are two nodes in the maintenance of manpower in the organization, and are such as recruitment & training node, and the work spot node.
- If there is a break down at the first node, it will lead to shortage of manpower. It will lead to the breakdown of the work spot node. The cost of which is high and is prohibitive.
- A reserve inventory of trained personnel is maintained in between the two nodes.
- If the reserve is more than the required manpower then it leads to increase in holding cost. If there is insufficient stock, it may lead to breakdown of the second node which is highly prohibitive due to delayed work schedule. and the cost of this is very high.

Notations for study:

k : The cost of holding excess manpower per man per hour.

m : The shortage cost due to shortage of manpower per hour.

S : The stock level of manpower reserve in man per hours.

: a random variable denoting the demand for manpower in terms of man-hours, with p.d.f p(.), which follows skewed logistic distribution and c.d.f P( ).

CASE 1:

It is assumed that the demand for manpower is a random variable denoted as 'τ', and it follows first order statistics and it has probability density function $p(\tau)$ with cumulative distribution function $P_{(1)}(\tau)$.

The expected cost due to excess manpower or shortage of manpower is given by

$$E(C) = k\int_0^S (s-\tau)\,p_{(1)}(\tau)d\tau + m\int_S^\infty (\tau-s)\,p_{(1)}(\tau)d\tau$$

Where,

$$p_{(1)}(\tau) = n[1-P(\tau)]^{n-1}p_{(1)}(\tau)d\tau + m\int_S^\infty (\tau-s)\,n[1-P(\tau)]^{n-1}p_{(1)}(\tau)d\tau$$

$$= nk\int_0^S (s-\tau)[1-P(\tau)]^{n-1}\,p_{(1)}(\tau)d\tau + nm\int_S^\infty (\tau-s)\,[1-P(\tau)]^{n-1}p_{(1)}(\tau)d\tau$$

$$= nk(A) + nm(B) \qquad \text{...........(1.1)}$$

Where

$$A \Rightarrow \int_0^S (s-\tau)[1-P(\tau)]^{n-1} p_{(1)}(\tau)d\tau$$

$$B \Rightarrow \int_S^\infty (\tau - s)\,[1-P(\tau)]^{n-1} p_{(1)}(\tau)d\tau$$

Now $\frac{dE(C)}{ds}$ gives the optimum reserve of manpower

$$\frac{dA}{ds} = 0 \Rightarrow \frac{d}{ds}\int_0^S (s-\tau)[1-P(\tau)]^{n-1} p_{(1)}(\tau)d\tau$$

$$\emptyset(s) = 0, \emptyset'(s) = 0, \varphi(s) = s, \varphi(s) = 1$$

By Leibnitz rule for the differentiation of an integral

$$\frac{dA}{ds} = (s-s)[1-P(\tau)]^{n-1}p(s) + 0 + \int_0^s \frac{d}{ds}(s-\tau)[1-P(\tau)]^{n-1} p_{(1)}(\tau)d\tau$$

$$= \int_0^S [1-P(\tau)]^{n-1} p_{(1)}(\tau)d\tau$$

since the p.d.f of a skewed logistic distribution is given by

$$p(\tau) = \frac{ae^{-\tau}}{(1+e^{-\tau})^{-\alpha+1}}$$

$$P(\tau) = (1+e^{-\tau})^{-\alpha}$$

$$1-P(\tau) = 1-(1+e^{-\tau})^{-\alpha}$$

$$= \int_0^S 1-(1+e^{-\tau})^{n-1} \frac{ae^{-\tau}}{(1+e^{-\tau})^{-\alpha+1}} d\tau$$

$$let\ t = 1-(1+e^{-\tau})^{-\alpha}$$

$$dt = 0 - (-\alpha)(1+e^{-\tau})^{-\alpha-1}(-e^{-\tau})d\tau$$

$$dt = -\alpha e^{-\tau}(1+e^{-\tau})^{\alpha+1}$$

$$dt = -\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}$$

τ	0	S
$p(\tau)$	1-$2^{-\alpha}$	$1-(1+e^{-S})^{-\alpha}$

$$= \int_{1-2^{-\alpha}}^{1-(1+e^{-S})^{-\alpha}} t^{n-1}(-dt)$$

$$\frac{dA}{ds} = -\frac{1}{n}[[1-[1+e^{-S}]^{-\alpha}]^n - [1-2^{-\alpha}]^n] \dots\dots\dots\dots\dots (1.2)$$

$$B \Rightarrow \int_S^\infty (\tau - s)\,[1-P(\tau)]^{n-1} p_{(1)}(\tau)d\tau$$

$$= -\int_s^\infty [1-(1+e^{-\tau})^{\alpha}]^{n-1} \frac{ae^{-\tau}}{(1+e^{-\tau})^{-\alpha+1}} d\tau$$

$$\tau = 1-(1+e^{-\tau})^{\alpha}$$

$$dt = 0 - (-\alpha)(1+e^{-\tau})^{-\alpha-1}(-e^{-\tau})d\tau$$

$$dt = -\alpha e^{-\tau}(1+e^{-\tau})^{-(\alpha+1)}\ d\tau$$

$$dt = \frac{-\alpha e^{-\tau}}{(1+e^{-\tau})^{-(\alpha+1)}} d\tau$$

τ	S	∞
$p(\tau)$	$1-(1+e^{-S})^{-\alpha}$	0

$$= -\int_{1-(1+e^{-S})^{-\alpha}}^{0} t^{n-1}\,(-dt)$$

$$\frac{\mathrm{dB}}{\mathrm{ds}} = -\frac{1}{n}[1-(1+e^{-S})^{-\alpha}]^{n} \ldots\ldots\ldots\ldots(1.3)$$

Substituting (1.2) & (1.3) in (1.1).

$$nk\left(\frac{1}{n}[[1-[1+e^{-S}]^{-\alpha}]^{n}-[1-2^{-\alpha}]^{n}\right)+nm\left(-\frac{1}{n}[1-(1+e^{-S})^{-\alpha}]^{n}\right)=0$$

$$= -k[[1-[1+e^{-S}]^{-\alpha}]^{n}]+k[1-2^{-\alpha}]^{n}-m[1-(1+e^{-S})^{-\alpha}]^{n}=0$$

$$[[1-[1+e^{-S}]^{-\alpha}]^{n}][-k-m]=-k[1-2^{-\alpha}]^{n}$$

$$[1-[1+e^{-S}]^{-\alpha}]^{n}=\frac{k[1-2^{-\alpha}]^{n}}{[k+m]}$$

$$1-[1+e^{-S}]^{-\alpha}=\left[\frac{k[1-2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{1}{n}}$$

$$[1+e^{-S}]^{-\alpha}=\left[\frac{k[1-2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{1}{n}}-1$$

$$[1+e^{-S}]=\left[\left[\frac{k[1-2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{1}{n}}-1\right]^{\frac{-1}{\alpha}}$$

$$e^{-S}=\left[\left[\frac{k[1-2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{1}{n}}-1\right]^{\frac{-1}{\alpha}}-1$$

$$-S=log\left[\left[\left[\frac{k[1-2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{1}{n}}-1\right]^{\frac{-1}{\alpha}}-1\right]$$

$$\hat{S}=\ log\left[\left[\left[\frac{k[1-2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{1}{n}}-1\right]^{\frac{-1}{\alpha}}-1\right] \ldots\ldots\ldots\ldots\ldots\ldots..(1.4)$$

The optimal value of S namely $\hat{S}$ can be obtained from (1.4) given the values of k, m, n, α.

Case-II:

The demand'τ' is a random variable which has the distribution of n^{th} order statistic with p.d.f of $p_{(n)}(\tau)$

The p.d.f of the n^{th} order statistic is given as

$$p_{(n)}(\tau)=n[P(\tau)]^{n-1}p(\tau)$$

Using the p.d.f. of the skewed logistic distribution, we have,

$$E(C)=k\int_{0}^{S}(s-\tau)\,n[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$+m\int_{S}^{\infty}(\tau-s)\,n[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$

International Journal of Research
Available at https://edupediapublications.org/journals
e-ISSN: 2348-6848
p-ISSN: 2348-795X
Volume 04 Issue 14
November 2017

$$= nk\int_0^S (s-\tau)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$+ nm\int_S^\infty (\tau-s)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$nk(C)+nm(D)\ldots\ldots\ldots\ldots\ldots..(1.5)$$

Where

$$C \Rightarrow \int_0^S (s-\tau)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$

Now $\frac{dC}{ds}=\frac{d}{ds}\int_0^S (s-\tau)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$

$$\emptyset(s)=0,\emptyset'(s)=0,\varphi(s)=s,\varphi(s)=1$$
$$\frac{dC}{ds}=[s-s][(1+e^{-\tau})^{-\alpha}]^{n-1}-0+\int_0^S \frac{d}{ds}(s-\tau)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$\int_0^s [(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$t=(1+e^{-\tau})^{-\alpha}$$

$$dt=-\alpha(1+e^{-\tau})^{-\alpha-1}(-e^{-\tau})d\tau$$
$$=\alpha e^{-\tau}(1+e^{-\tau})^{-(\alpha+1)}d\tau$$
$$dt=\frac{\alpha e^{-\tau}}{(1+e^{-\tau})^{(\alpha+1)}}d\tau$$

τ	0	S
$p(\tau)$	$2^{-\alpha}$	$(1+e^{-S})^{-\alpha}$

$$=\int_{2^{-\alpha}}^{(1+e^{-S})^{-\alpha}} t^{n-1}dt$$
$$\frac{dC}{ds}=\frac{1}{n}[[[1+e^{-s}]^{-\alpha}]^n-[2^{-\alpha}]^n]\ldots\ldots\ldots\ldots.(1.6)$$

Also

$$D \Rightarrow \int_S^\infty (\tau-s)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$\frac{dD}{dS} \Rightarrow \frac{d}{dS}\int_S^\infty (\tau-s)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$\emptyset(s)=0,\emptyset'(s)=0,\varphi(s)=s,\varphi(s)=1$$
$$\frac{dD}{dS}=[s-s][(1+e^{-\tau})^{-\alpha}]^{n-1}-0+\int_S^\infty \frac{d}{ds}(\tau-s)\,[(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$=-\int_s^\infty [(1+e^{-\tau})^{-\alpha}]^{n-1}\frac{ae^{-\tau}}{(1+e^{-\tau})^{\alpha+1}}d\tau$$
$$t=(1+e^{-\tau})^{-\alpha}$$
$$dt=-\alpha(1+e^{-\tau})^{-\alpha-1}(0-e^{-\tau})d\tau$$
$$=\ \alpha e^{-\tau}(1+e^{-\tau})^{-(\alpha+1)}d\tau$$
$$dt=\frac{\alpha e^{-\tau}}{(1+e^{-\tau})^{(\alpha+1)}}d\tau$$

τ	S	∞
$p(\tau)$	$(1-e^{-S})^{-\alpha}$	1

$$= -\int_{(1-e^{-s})^{-\alpha}}^{1} t^{n-1} dt$$

$$\frac{dD}{dS} = -\frac{1}{n} + \frac{1}{n}[(1-e^{-s})^{-\alpha}]^{n} \dots\dots\dots (1.7)$$

Substituting (1.6) & (1.7) in (1.5)

$$nk\left(\frac{1}{n}[[[1+e^{-s}]^{-\alpha}]^{n} - [2^{-\alpha}]^{n}]\right) + nm\left(-\frac{1}{n} + \frac{1}{n}[(1-e^{-s})^{-\alpha}]^{n}\right) = 0$$

$$k[[1+e^{-s}]^{-\alpha}]^{n} - k[2^{-\alpha}]^{n} + nm\left(-\frac{1}{n}\right) + nm\left(\frac{1}{n}\right)[(1-e^{-s})^{-\alpha}]^{n} = 0$$

$$k[[1+e^{-s}]^{-\alpha}]^{n} + m[(1-e^{-s})^{-\alpha}]^{n} = k[2^{-\alpha}]^{n} + m$$

$$[[1+e^{-s}]^{-\alpha}]^{n}[k+m] = m + k[2^{-\alpha}]^{n}$$

$$[[1+e^{-s}]^{-\alpha}]^{n} = \frac{k[2^{-\alpha}]^{n}}{[k+m]}$$

$$[1+e^{-s}]^{-\alpha} = \left[\frac{k[2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{1}{n}}$$

$$[1+e^{-s}] = \left[\frac{k[2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{-1}{n\alpha}}$$

$$e^{-s} = \left[\frac{k[2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{-1}{n\alpha}} - 1$$

$$-s = \log\left[\left[\frac{k[2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{-1}{n\alpha}} - 1\right]$$

$$\hat{S} = \log\left[\left[\frac{k[2^{-\alpha}]^{n}}{[k+m]}\right]^{\frac{-1}{n\alpha}} - 1\right]$$

Conclusions:

Theoretically it could be inferred that as the cost of holding of the stock of manpower increases, it is preferred to have a smaller reserve inventory of manpower, the optimal reserve size, namely, $\hat{S}$ shows a decline.

If the shortage is tight then the loss incurred due to shortage of manpower will be higher. Hence an appropriate level of manpower inventory has to be maintained. If the shortage cost is higher than it will be preferable to keep a higher level of manpower inventory. It is observed that as the value of 'm' increases, then a corresponding increase in $\hat{S}$ is seen in both the cases.

References:

[1]Hanssmann (1961**).** In Progress in Operation Research, Vol 1 R.. sckoff,ed. Wiley, New York, pp 65-104.

[2]Rajagopal, k., R. Sathiyamoorthi.,(2003) "An extension of the Optimal Reserve Inventory level Between machines model", second National Conference on

Mathematical & Computational methods- PSG college of Arts & Science, Coimbatore, 2003

[3]Muthaiyan, A., A.Sulaiman and R. Sathiyamoorthi.(2009) "A Stochastic model based on order statistics for estimation of expected time to recruitment", International Journal of Agriculture Statistical Science. Vol.5 No: 2, pp -501-508, 2009.

[4]Arivazhagan, G., R. Elangovan and R. Sathiyamoorthi.,(2010) "Determination of Optimal Manpower reserve when demand for manpower has fluctuations", Bulletin of Pure and Applied Sciences. Vol.29 E, No.2, pp-243-248, 2010.