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Effect of Variable Suction on Unsteady Mhd Oscillatory Flow of Jeffrey Fluid in An Inclined Channel

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Abstract

In this work, we investigated the effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in an inclined channel. Closed form analytical solution method was constructed for the problem and the solutions for the velocity and temperature profiles were obtained. Graphs were plotted using MATLAB in order to depict the effects of the pertinent flow parameters that govern the fluid flow. It was found that the velocity $u(x,t)$ *increases with increase in radiation parameter* N*, angle of inclination* α*, Jeffrey parameter* λ_1 *and Grashof number* G_r*. While the temperature distribution is enhanced with increase in radiation parameter* N*. The velocity also decreases with increase in Hartmann number* H *and Reynolds number* Re*. This work has wide range of engineering and industrial applications such as recovery extraction of crude oil, molten iron flow, geothermal systems etc.*

Keywords: *MHD, Jeffrey fluid, Oscillatory flow, Variable suction.*

1. Introduction

The effect of heat transfer on unsteady MHD oscillatory flow of fluid in horizontal media are encountered in a wide range of engineering and industrial applications such as molten iron flow, recovery extraction of crude oil, geothermal systems. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched in a cooling system; the fluid mechanical properties of penultimate product depend mainly on the cooling liquid used and the rate of stretching.

Some polymers fluids like polyethylene oxide and polysobutylene solutions in a cetane, having better electromagnetic properties are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. Also, the radiative heat transfer is an important factor of thermodynamics of very high temperature systems such as electric furnaces, solar collectors, storage of nuclear wastes packed bed catalytic reactors, satellites, steel rolling, cryogenic engineering etc. The study of such flow under the influence of magnetic field and heat transfer has attracted the interest of many investigators and researchers.

Kamala et al (2016) [1] investigated the effect of heat transfer on unsteady MHD and radiative oscillatory flow of a Jeffrey fluid in a horizontal channel filled with saturated porous medium and non-uniform walls temperature, Cogley et al (1968) [2]discussed differential approximation for radiative heat transfer in non-linear equations-grey gas heat equilibrium.Joseph et al (2016)[3] studied the effect of variable suction on unsteady magneto hydrodynamic free convective couette flow of a viscous incompressible electrically conducting fluid in the slip flow regime in presence of variable suction with heat and mass transfer was analyzed. Joseph et al(2016)[3] investigated the effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer. The temperature prescribed at the plates is uniform and asymmetric. A perturbation method is employed to solve the momentum and energy equations. Dheia Gaze salih Al-khafajy (2016)[4] investigated the influence of heat transfer on MHD oscillatory flow of Jeffrey fluid with variable viscosity through porous medium. The velocity is assumed to vary as an exponential function of temperature. Ayuba et al (2016)[5] investigated the effect of variable suction on magneto Hydrodynamic couette flow through a porous medium in slip flow regime with heat and mass transfer. Uwanta and Omokhuale (2014)[6] studied the effect of thermal conductivity on heat and mass transfer with jeffrey fluid.

2. Formulation of the Problem

Consider the flow of a conducting optimally thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as it is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Take the Cartesian coordinates system (x, y) where ox lies along the center of the channel, y is the distance measured in the normal section .Then, assuming a Bossiqness incompressible fluid model, the equations governing the motion are given as:

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{v}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-\frac{v}{(1+\lambda_1)k}u-\frac{\sigma_e B_o{}^2}{\rho}u+g\beta(T-T_o)\sin\alpha \quad (1)$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_\rho}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_\rho}\frac{\partial q}{\partial y} \quad (2)$$

The boundary conditions are:

$$u=0, T=T_w \quad \text{On y=1} \quad (3)$$

$$u=0, T=T_o \quad \text{on y=0} \quad (4)$$

Following cogley et al.(1968)[2], it is assumed that the fluid is optimally thin with a relatively low density and the radiative heat flux is given by:

$$\frac{\partial q}{\partial y}=4\lambda^2(T_o-T) \quad (5)$$

Where λ is the mean radiative absorption co-efficient. The following dimensionless variable and parameters are introduced:

$$\mathrm{Re}=\frac{Ua}{v},\bar{x}=\frac{x}{a},\bar{u}\frac{u}{U},\bar{y}=\frac{y}{a},\theta=\frac{T-T_o}{T_w-T_o},H^2=\frac{a^2\sigma_e B_0{}^2}{\rho v},\bar{t}=\frac{tU}{a}$$

$$\bar{P}=\frac{aP}{\rho vU},Da=\frac{k}{a^2},Gr=\frac{g\beta(T_w-T_o)}{vU},Pe=\frac{Ua\rho c_\rho}{k},N^2=\frac{4\alpha^2 a^2}{k} \quad (6)$$

Where U is the mean velocity. The dimensionless governing equations together with the appropriate boundary conditions,(negating the bars for clarity) can be written as

$$\mathrm{Re}\left(\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial \bar{y}^2}\left(\frac{s^2}{1+\lambda}+H^2\right)U+Gr\theta\sin\alpha \quad (7)$$

$$Pe\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+N^2\theta \quad (8)$$

The boundary conditions are

$$u=0,\theta=1, \quad \text{on y=1} \quad (9)$$

$$u=0,\theta=0, \quad \text{on y=0} \quad (10)$$

Where $s^2 = \frac{1}{Da}$

3. Method of Solution

In order to solve equations (7) and (8) subject to the boundary conditions in (9) and (10), we use closed form solution method.

For purely oscillatory flow,

$$-\frac{\partial p}{\partial x} = \lambda^{iwt},$$

$$\theta(y,t) = \theta_o(y)e^{iwt}$$

$$u(y,t) = u_o(y)e^{iwt} \qquad (11)$$

Where λ is a constant and ω is the frequency of oscillation. Substituting the above expressions into equations (7) – (10), we have

$$L_1 u_0^{11}(y) - \mathrm{Re}\, u_0^{1}(y) - L_3 u_0 = -\lambda - Gr\theta_o sin\alpha \qquad (12)$$

Where $L_3 = L_2 + iw\mathrm{Re}$

$$\theta_o^{11}(y) - Pe\theta_o^{1}(y) + L_4\theta_o(y) = 0 \qquad (13)$$

Where $L_4 = N^2 - iwPe$

The boundary conditions now becomes

$u_o = 0, \theta_o = 1$ on y = 1

$u_o = 0,\ \theta_o = 0$ on y=0 (14)

The solution for fluid velocity and temperature are given as

$$u(y,t) = \left(c_3 e^{m_3 y} + c_4 e^{m_4 y} + K_1 + K_2 e^{m_1 y} + K_3 e^{m_2 y}\right)e^{iwt} \qquad (15)$$

$$\theta(y,t) = \left(c_1 e^{m_1 y} + c_2 e^{m_2 y}\right)e^{iwt} \qquad (16)$$

Where,

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$m_1 = \frac{Pe+\sqrt{Pe^2-4L_4}}{2}$,$m_2 = \frac{Pe-\sqrt{Pe^2-4L_4}}{2}$, $m_3 = \frac{Re+\sqrt{Re^2+4L_1L_3}}{2L_1}$,$m_3 = \frac{Re-\sqrt{Re^2+4L_1L_3}}{2L_1}$, $K_1 = \frac{\lambda}{L_3}$

$K_1 = -\frac{c_1 Grsin\alpha}{L_1 m_1^2 - Re m_1 - L_3}$, $K_3 = -\frac{c_2 Grsin\alpha}{L_1 m_2^2 - Re m_2 - L_3}$, $c_1 = \frac{1}{e^{m1} - e^{m2}}$, $c_2 = -\frac{1}{e^{m1} - e^{m2}}$, $c_4 = \frac{L_5 - L_6 e^{m3}}{e^{m3} - e^{m4}}$, $c_3 = L_6 - c_4$,
$L_5 = K_1 + K_2 e^{m_1 y} + K_3 e^{m_2 y}$, $L_6 = -(K_1 + K_2 + K_3)$

Shear Stress

The shear stress at the upper wall

$$\tau_u = \left.\frac{\partial u}{\partial y}\right|_{y=1} = (c_3 m_3 e^{m_3} + c_3 m_3 e^{m_4} + K_2 m_1 e^{m_1} + +K_3 m_2 e^{m_2}) e^{i\omega t} \qquad (17)$$

The shear stress at the lower wall of the channel;

$$\tau_L = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(c_3 m_3 + c_4 m_4 + K_2 m_1 + K_3 m_2\right) e^{iwt} \qquad (18)$$

Nusselt Number

The Nusselt number i.e. the rate of heat transfers on the upper wall

$$Nu_u = \left.\frac{\partial \theta}{\partial y}\right|_{y=1} = \left(c_1 m_1 e^{m_1} + c_2 m_2 e^{m_2}\right) e^{iwt} \qquad (19)$$

The Nusselt number at the lower plate

$$Nu_L = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = \left(c_1 m_1 + c_2 m_2\right) e^{iwt} \qquad (20)$$

4. Discussion of Results

In this research work, the effect of variable suction on an unsteady MHD oscillatory flow Jeffrey fluid in an inclined channel are investigated, and the results are discuss for various physical parameters Peclet number P_e , Grashof numberG_r Reynolds number R_e ,time variable t ,frequency of Oscillation ω ,porous medium shape factor s , Hartmann number H , Radiation parameter N , angle of inclination α and Jeffrey parameterλ_1. Taking the real part of the result obtained for the skin friction and Nusselt number, and made use of the following parameter values$P_e = 0.71, G_r = 1, R_e = 1, \lambda_1 = 0.3, \lambda = 1, t = 0.5, \omega = 1, S = 1, N = 1\ and\ \alpha = \frac{\pi}{4}$. These values kept as constant in entire calculation except when a particular parameter is varied.The variation of velocity $u$with $y$is calculated for different values of $N, \alpha, \lambda_1\ and\ G_r$ and shown in figures 1-4. It is observed that velocity increases with increasing $N, \alpha, \lambda_1\ and\ G_r$.

The variation of velocity $u$with $y$is calculated with different values of $H\ and\ R_e$ and depicted in figures 5 and 6. It is seen that velocity decreases with increasing $H\ and\ R_e$.

The variation of temperature $\theta$with y is calculated for different values of $N$and shown in figure 8. It is seen that temperature increases with increasing$N$

Fig. 2 depicts the radiation parameter$N$on velocity. The velocity increases with increase in radiation parameter$N$

Fig. 3 illustrates the effect of the angle of inclination αon velocity. The velocity increase with increasing angle of inclinationα

Fig. 4 demonstrates the effect of Jeffrey parameter λ_1 on velocity. The velocity increases with increasing Jeffrey parameterλ_1.

Fig. 5illustrates the effect of Grashof number $G_r$on velocity. The velocity increases with increasing Grashof number G_r.

Fig. 6 depicts the effect of Hartmann number Hon velocity. The velocity decreases with increasing Hartmann number$H$.

Fig. 7 shows the effect of Reynolds number $R_e$on velocity. The velocity decreases with increasing Reynolds numberR_e.

Fig. 8 illustrates the effect of radiation parameter$N$on temperature. The temperature increases with increasing radiation parameter$N$.

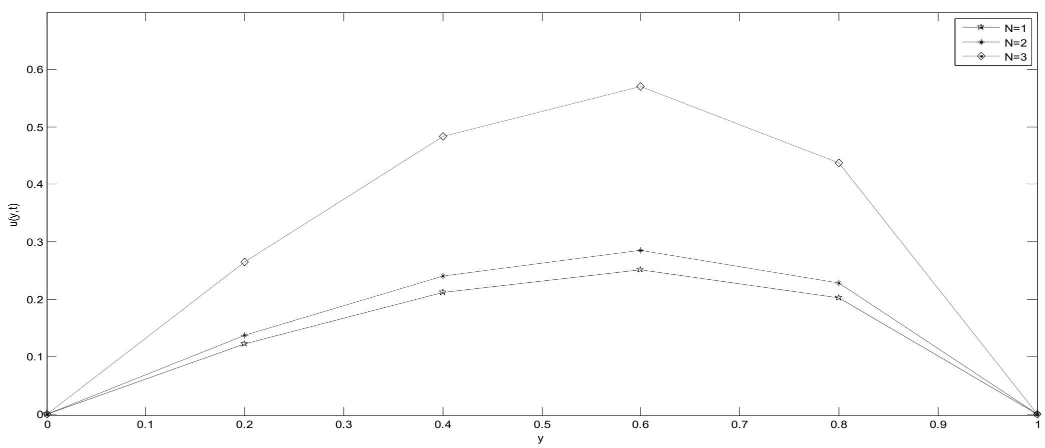

Figure 2: velocity profile for different values of N

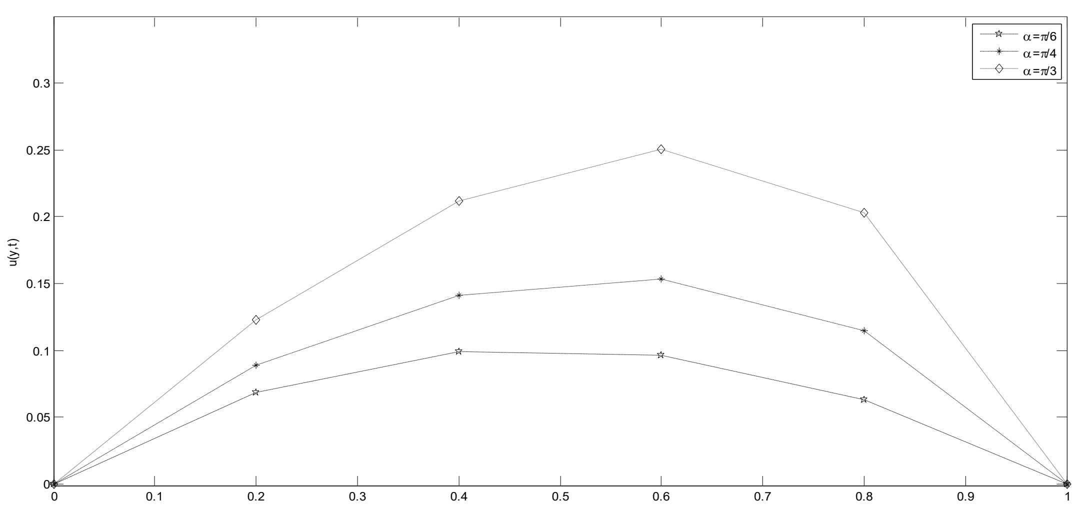

Figure 3: velocity profile for different values of α

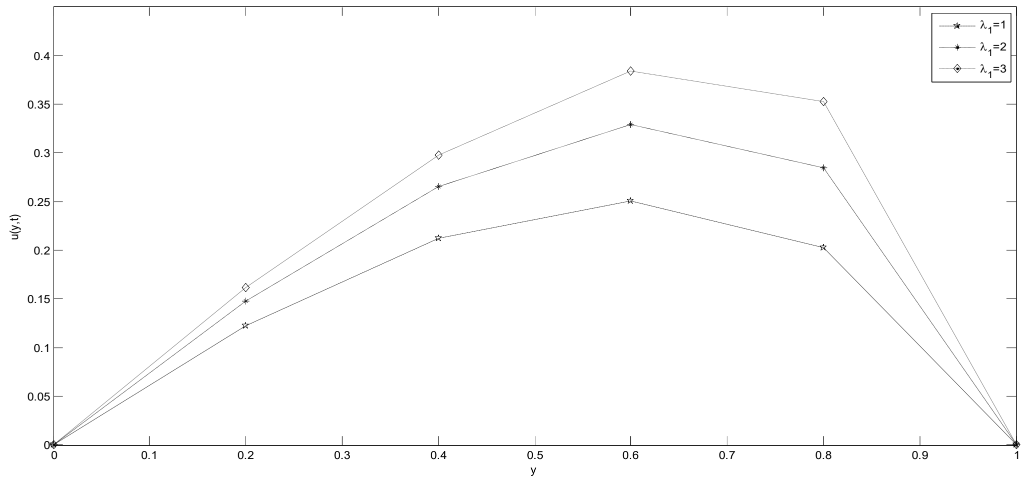

Figure 4: velocity profile for different values of λ_1

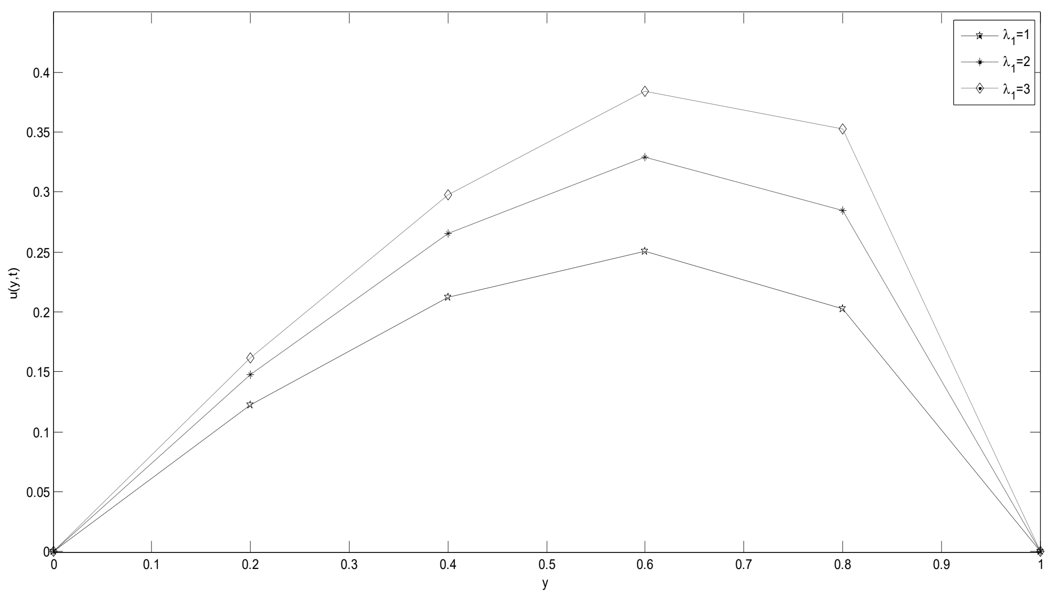

Figure 5: velocity profile for different values of Gr

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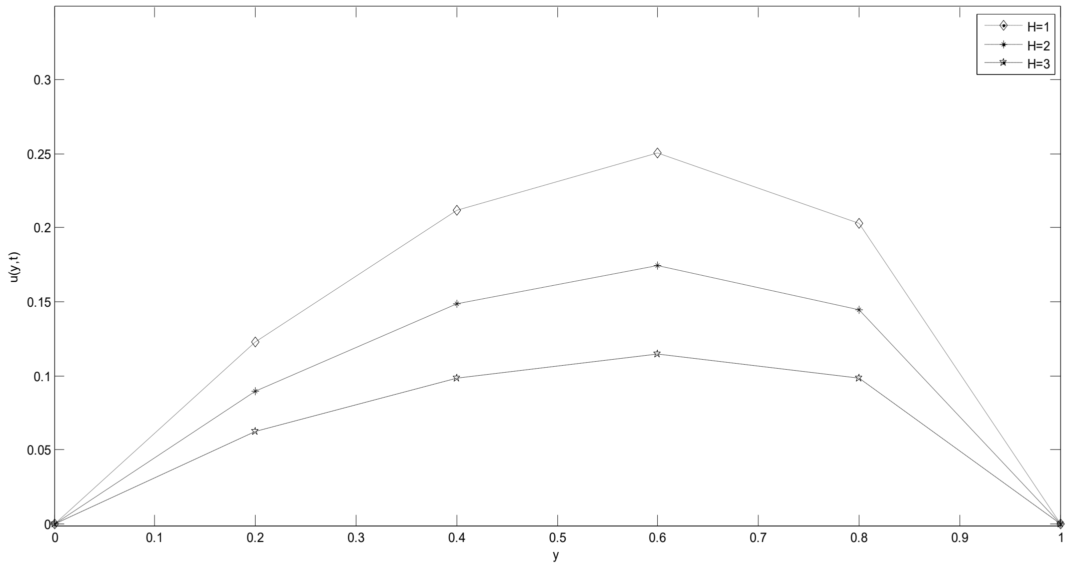
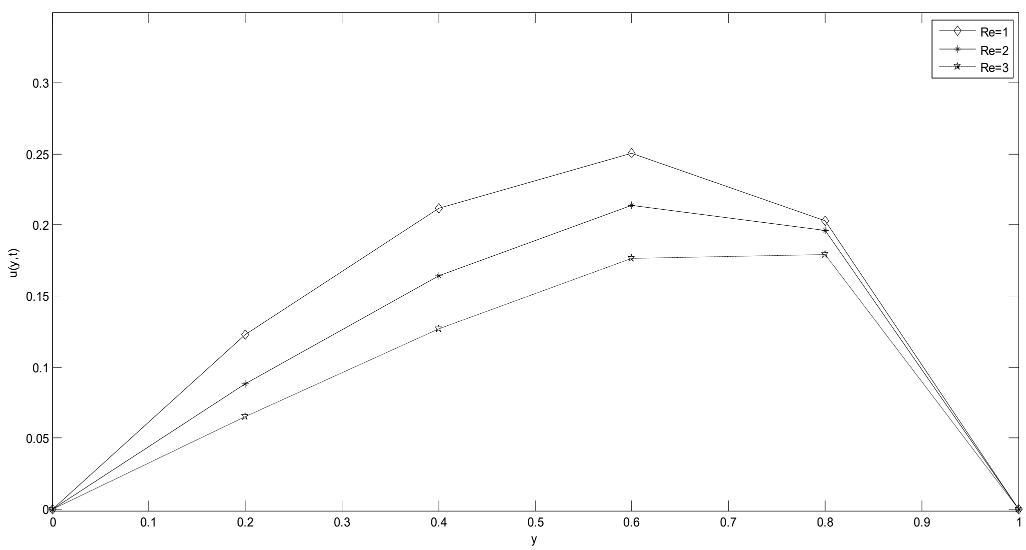


Figure 6: velocity profile for different values of *H*



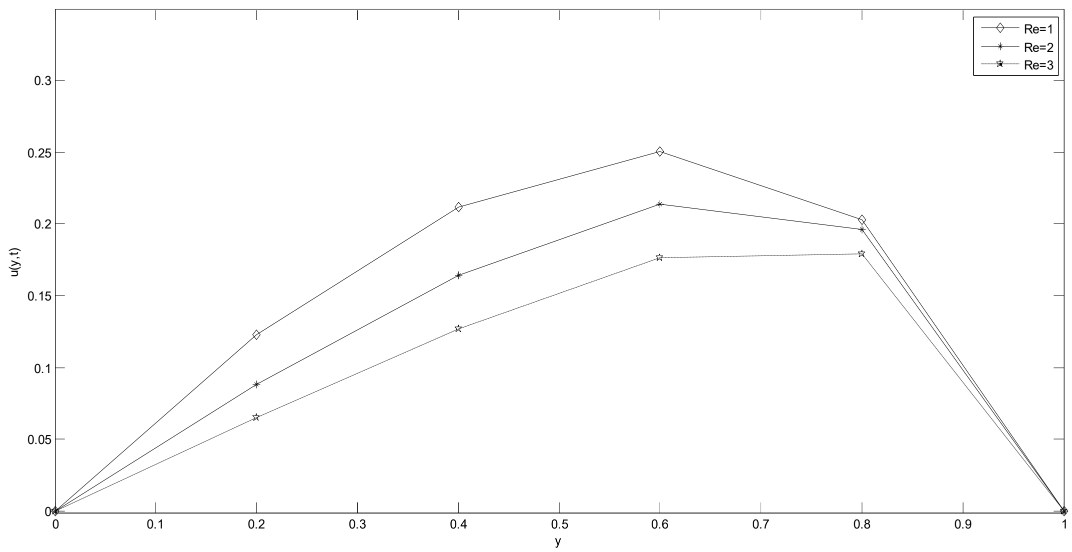


Figure 7: velocity profile for different values of Re

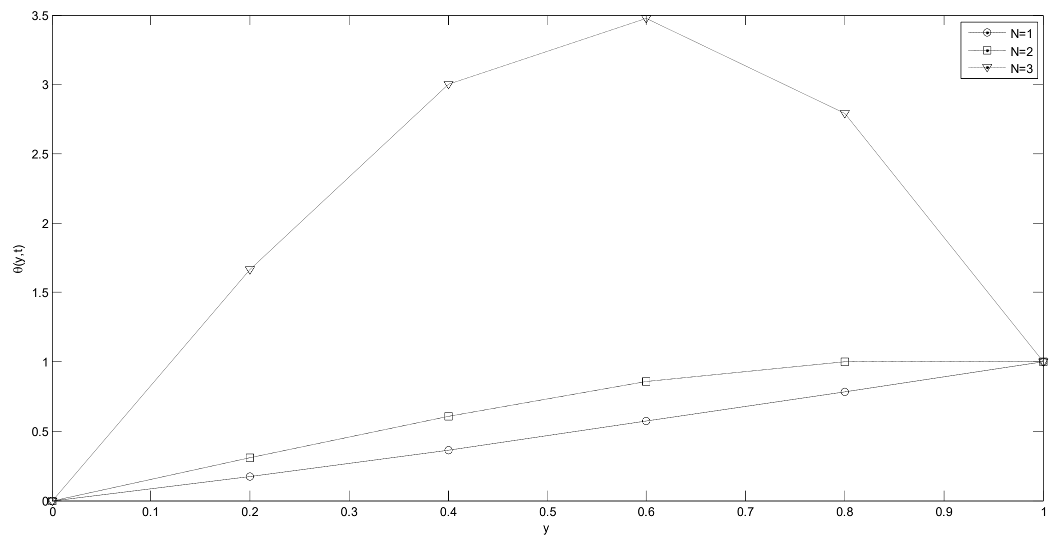


Figure 8: temperature profile for different values of N

5. Summary and Conclusion

The effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in an inclined channel. Closed form analytical solution method was constructed for the problem and the solutions for the velocity and temperature profiles were obtained. Graphs were plotted using MATLAB in order to depict the effects of the pertinent flow parameters that govern the fluid flow. It was found that the velocity $u(x,t)$ increases with increase in radiation parameter N, angle of inclination α, Jeffrey parameter λ_1 and Grashof number G_r. While the temperature distribution is enhanced with increase in radiation parameter N. The velocity also decreases with increase in Hartmann number H and Reynolds number Re. This work has wide range of engineering and industrial applications such as recovery extraction of crude oil, molten iron flow, geothermal systems etc.

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