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Stratified Fluid and Perturbation State of Suspended Particles

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Abstract:

We will discuss the perturbation state of stratified fluid in the presence of suspended particles and variable magnetic field confined between free boundaries at $z=0$ *and* $z=d$. *In the presence of factors such as suspended particles, magnetic field, and magnetic viscosity introduces oscillatory modes in the system which are otherwise non-existent. When instability sets in an stationary convection, the suspended particles and magnetic viscosity are found to have, respectively destabilizing and stabilizing effects on the thermal instability of the system.*

Keywords: Perturbation, magnetic field, convection ,thermal diffusivity, pressure.

The basic equations of momentum and continuity of non-viscous incompressible, non-homogeneous stratified fluid particles between two free boundaries at uniform but different temperature T_0 and T_1 $(T_0 > T_1)$ of lower and upper boundaries, respectively, in the presence of a horizontal variable magnetic field are,

$$\rho\left[\frac{\partial v}{\partial t}+(v.\nabla)v\right]=-\nabla P+\rho X_i+KN(U-V)+\frac{\mu_e}{4\pi}[(\nabla\times H)\times H]$$

$$\nabla . v = 0$$

$$\nabla . H = 0$$

$$\frac{\partial T}{\partial t} + (v.\nabla)\, T = K_T \nabla^2 T$$

$$\rho = \rho_0 [1 + \alpha (T_0 - T)]$$

where ρ, P, μ_e, $v(u, v, w)$ and K_T stand for the density, pressure, magnetic permeability, fluid velocity component and thermal diffusivity of the fluid, ρ_0 be the density of the fluid at lower boundary. Further, $u(x, t)$ and $N(x, t)$ denote velocity and number density of the suspended particles and $K = 6\pi\mu a$, a the radius of the particle, is constant. Further, we assume that the fluid layer of variable density, which is function of co-ordinate z only, is of zero resistivity.

Let the initial state of the fluid layer, whose stability we wish to examine, has the velocity, temperature and magnetic field as

$$v = (0,\ 0,\ 0)$$

$$T_1 = T_0 - \beta_1 z$$

$$H = (H(z), 0,\ 0)$$

where $\beta_1 = \left| dT/dz \right|$ is the uniform temperature gradient. In the equation of motion, the pressure of particles adds an extra force term, proportional to the velocity difference between particles and fluid. Since the force exerted by the fluid on the particles is equal and opposite to the force exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion of the particles. The buoyancy force on the particles is neglected. Further the interparticle reactions are also not considered as we assume that the distance between particles are quite large compared with their diameter. The equation of motion and continuity, under the above assumptions are :

$$mN\left[\frac{\partial u}{\partial t}+(u.\nabla)u\right]=mNX_i+KN(v-u)$$

$$\frac{\partial N}{\partial t}+\nabla.(Nu)=0$$

where mN is the mass of particles per unit volume. Since the density of a particle moving with the fluid remains unchanged, we have

$$\frac{\partial \rho}{\partial t}+\rho(v.\nabla)=0$$

Perturbation State :

Let the initial state be slightly perturbed, so that the perturbed state is characterized by

$$\bar{v}=(u,v,w)$$

$$\bar{u}=(\rho,r,s)$$

$$\overline{T_1} = T_1 + \theta$$

$$\overline{P} = P + \delta P$$

$$\overline{\rho} = \rho_0 \left[1 + \frac{\delta\rho}{\rho_0} + (T_0 - T_1 - \theta) \right]$$

$$\overline{H} = (H(z) + h_x, h_y, h_z)$$

where (u, v, w), $\theta, \delta P, \delta\rho, (h_x, h_y, h_z)$ and $u(\rho, r, s)$ respectively, denote the perturbation in fluid velocity $(0, 0, 0)$, temperature, pressure, density, magnetic field $(H(z), 0, 0)$ and particle velocity $(0, 0, 0)$.

Then the linearized perturbation equations are

$$\rho \frac{\partial v}{\partial t} = -\nabla \delta_p - g\delta\rho + g\alpha\rho\theta + KN(U - v) + \frac{\mu_e}{4\pi} \int ((\nabla \times h) \times H + (\nabla \times H) \times h) \qquad (1)$$

$$\nabla . v = 0 \qquad (2)$$

$$\nabla . H = 0 \qquad (3)$$

$$\frac{\partial \theta}{\partial t} - \beta_1 w = K_T \nabla^2 \theta \qquad (4)$$

$$\frac{\partial h}{\partial t} = (H.\nabla) v - (v.\nabla) H \qquad (5)$$

$$mN \frac{\partial u}{\partial t} = KN(v - u) \qquad (6)$$

$$\frac{\partial(\delta\rho)}{\partial t} = -\frac{d\rho}{dz} w \qquad (7)$$

Analyzing the perturbations into normal modes, we seek solution whose dependence on X, Y and t is given by

$$\exp[i(K_x X + K_y Y) + nt] \qquad (8)$$

where $K^2 = K_x^2 + K_y^2$ is the resultant wave number and n is a complex constant.

Eliminating u between equations (1), (6) and using equation (8), then equations (1) to (7) give

$$n'\rho u = -iK_x\delta p + \frac{\mu_e}{4\pi} h_z \Delta H \qquad (9)$$

$$n'\rho v = -iK_y\delta p + i\frac{\mu_e}{4\pi} H(K_x h_y - K_y h_x) \qquad (10)$$

$$n'\rho w = -D\delta p - g\delta\rho + g\alpha\rho\theta + \frac{\mu_e}{4\pi} H[iK_x h_z - Dh_x - h_x DH/H]$$

(11)

$$K_x u + K_y v = iDw$$

$$K_x h_y + K_y h_y = iDh_z$$

$$[n - K_T(D^2 - K^2)]\theta = \rho_1 w$$

$$nh_x = iK_x Hu - wDH$$

$$nh_y = i K_x H v$$

$$nh_z = iK_x H w$$

$$n\delta\rho = -(D\rho)\,w$$

where $D = \frac{d}{dz}$ and

$$n' = n\left[1 + \frac{mN}{\rho\left(1 + \frac{mn}{k}\right)}\right]$$

Now we multiply equation (9) by K_x, equation (10) by K_y, and adding we get

$$D(n'\rho Dw) = -K^2 D\delta P + \frac{\mu_e}{4\pi n} D(K^2 Hw DH) \qquad (12)$$

Multiply equation (11) by K^2 and subtracting it from equation (12) we get

$$D(n'\rho\, Dw) - n'\rho\, K^2 w + \frac{g}{n}(D\rho)K^2 w + g\alpha K^2 \rho\theta$$

$$+ \frac{\mu_e}{4\pi n} K_x^2 [H^2(D^2 - K^2)w + DH^2 Dw] = 0$$

Now substituting the value of n' and simplifying, we get the required result.

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