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Comparison of Problem-Solving Strategies in Middle-Grade Mathematics Textbooks

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Abstract:
This study compared the problem-solving activities imbedded in two selected mathematics textbooks of middle-grade. The purpose in this study was to analyze the quality and adequacy of proficiency-based instructional methods that are appropriate for the development of problem-solving skills in students. A validated framework designed for the analysis of proficiency-based textbook materials was used to determine the evidences for appropriateness and quality activities in Geometry portion of selected textbooks. The results showed that the NSM contained significantly more quality tasks with a variety of complexities, more open-ended and non-routine problems as compared to M8. The findings also revealed that NSM was seen to be effective and appropriate for development of problem-solving skills in students whereas, M 8 mostly used routine procedures with few connections to daily life. The results of this study will have greater significance for curriculum developers, textbook authors, textbook analysis experts and school administrators.

Keywords: Curriculum, Geometry, Instructional Methods, Mathematics Proficiency

1.Introduction

The textbooks and other materials are developed primarily to provide instructional support to teachers in their daily classroom teaching (Hussain & Shaheen, 2017). Majority teachers across subjects rely on the textbook material as the only resource for their teaching. Although the government clearly provides guidelines (MOE, 2006) for incorporation of key instructional features in textbooks before publishing; many mathematics educators feel that there is significant gap between the curriculum guidelines and the published textbooks of mathematics (Hussain S; 2017). It is evident that classroom teachers of mathematics use textbooks as instructional guide for teaching mathematics in the classroom (Thomson & Fleming, 2004; Van Den Heuvel-Panhuizen & Wijers, 2005). No doubt, textbooks are considered to have great impact on both teaching and students' learning in mathematics. (Remillard J. T., 2000; Reys, Reys, & Chávez, 2004) noted that there is a complex relationship between teachers, students and textbooks and the use of textbooks is mostly depends on the context where it is used. Many studies on impacts of textbook have shown that students' achievement rate was positively correlated with the instructional content embedded in mathematics textbooks (Linor, 2017; Törnroos, 2005) . (Vincent & Stacey, 2008) used the criteria from the TIMSS Video Study, to analyze the exercises questions and other tasks contained in mathematics textbooks. They argued that mathematics textbooks must provide opportunities for students to develop mathematical representations, problem-solving abilities, and reflection (p. 102). On the other hand, many researchers found that students' demonstrated low performance using poorly designed instructional activities in mathematics textbooks (Jones, Langrall, Thornton, & Nisbet, 2002). In fact, the textbooks that are incorporated with student-centered activities and that emphasize on problem-solving strategies have significant influence on students learning and such textbooks are adopted by many schools around the world (Remillard, 2005). Therefore, it is logical for the schools to analyze the quality of instructional methods embedded in mathematics textbooks before adopting it for the students. This study analyzed the salient features of problem-solving strategies in selected mathematics textbooks using the indicators reported in Proficiency-based Framework (Hussain & Shaheen, 2017). Geometry portion that represent the highest weightage (40%) in the curriculum (MOE, 2006), was focused to examine whether the selected mathematics textbooks provide quality and sufficient instructional activities that are aligned with the instructional goals and indicators of the framework.

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2.Review of Research Literature

2.1. Promotion of Problem-Solving

During 20th century, research on mathematical problem-solving has greatly focused to promote students learning (Pehkonen 1991). Polya (1949) was one of the pioneer researchers focusing on heuristic strategies in problem solving. During 1960s and the 1970s, Polya's models became popularized among many researchers; heuristic strategies have widely been used in studies that placed emphasis on problem-solving. In the 1980s, problem solving has been the focus of school mathematics textbooks (NCTM 1980). This Polya's model has been reviewed by many researchers and has experienced many revisions. The National Council of Teachers of Mathematics (NCTM) included problem solving in NCTM's Principles and Standards for School Mathematics (2000). The National Mathematics Advisory Panel (NMAP, 2008) found that explicit instruction in problem-solving is suitable for students, especially students who may struggle with learning mathematics. Gersten, et al., (2009) analyzed six studies in which explicit instruction was used with students with special needs and found that "explicit instruction can expressively develop proficiency in solving word problems and operations across grade levels and students of multiple learning needs." (p. 21).

2.2. Problem-Solving Designs

The NCTM (2000) defines problem-solving as "engaging in a task for which the solution method is not known in advance" (p. 52). Polya (1965) defines problem-solving as "using an approach to reach a solution to the problem, determining a way out to an unattainable obstacle" (p. vii). Polya (1957) emphasized on the importance of teaching students how to think through problem-solving. He developed a four stage heuristics in order to engage students in problem-solving: Understanding the problem, Devise planning, Implementing of the plan, Looking Back (Polya, 1957; Wilson et al., 1993). Polya's (1957) four stages problem-solving is an interconnected process. Students must possess adequate knowledge base to solve the problems (Polya, 1957; Schoenfeld, 1985, 1987). Polya (1957) also emphasized on the significance of metacognition for the organization and extraction of important data for problem-solving.

Schoenfeld refined the principles of problem-solving practically as oppose to the Polya (1949) who treated theme at a theoretical level. (Schoenfeld 1985, 1992) developed a framework for better understanding for the teaching of problem-solving and how to solve problems. In Schoenfeld's view, problems should be solved through a process of discourse that must focus the previous knowledge of problem solver, his efforts, and the thoughts he possesses along the way (Schoenfeld 1982). Schoenfeld (1985) proposed six stages for students to be good problem solvers in mathematics:

> 1) A good base in mathematical knowledge, 2) Effective organization of mathematical knowledge, 3) Algorithms, 4) Repertoire of heuristics, 5) Decision making mechanism to select from among the available heuristics, or be able to develop new ones, as problem situations are encountered, 6) Reflection on what he/she has done and ability to decide if it is a reasonable answer or way of solving the problem (Schoenfeld, 1985).

(Kilpatrick et al. 2001) used the term strategic competence for problem-solving. According to Kilpatrick strategic competence refers to the "students ability to formulate mathematical problems, how to represent them, and solve them". The teacher should provide variety of problem solving techniques to students so that they should be able to formulate the problems of different situations out of school. In "How to Solve It", Polya explained powerful problem-solving strategies such as making generalizations, making use of analogy, re-formulating a problem, manipulating the solution of related problems, exploiting symmetry, and look back at the solution. Mental representations of problems, developing mathematical relationships and making innovative solution strategies are also key for students to become proficient problem solvers. Kilpatrick et al. (2001) referred flexibility as the fundamental characteristic for problem-solving process. Contrary to the routine problems, non-routine problems require flexibility because students do not have prior workable solution methods for non-routine problems based on their previous practice. Learners need productive thinking to formulate and solve non-routine problems (p.126).

3.Methodology

3.1. Criteria of Textbook Selection

Two books, one in use of public schools and other in private schools, were selected to examine the quality of instructional materials. According to (I-SAPS, 2010), 31.11% of all students in Pakistan in the middle-level were enrolled in private intuitions in 2007-08. These textbooks were chosen due to the maximum usage in public and private schools. *Mathematics 8* (M8), developed by Punjab Text Book board, and used by majority teachers as instructional guide for teaching mathematics at grade-8 level, was selected from textbooks in use of public schools. M 8 adopted by more than 60 % of all students in public schools across Pakistan (PBS, 2017). *New Syllabus Mathematic 2* (NSM2), published by Oxford University Press and authorized by the Singapore Ministry of Education and the University of Cambridge International Examinations (CIE), was selected because of its maximum adoption in private schools for teaching grade-8 equivalent students. The publisher of both textbooks claims that their textbooks contain effective strategies designed to enhance students' problem-solving skills.

3.2. Textbook Analysis Procedure

The selected topics from geometry *Surface area and Volume of the Cone and Sphere* were focused to conduct in-depth analysis rather than selecting too many topics and doing a superficial examination of textbook material. The focused topics were thoroughly studied page by page to determine the evidences of effective strategies related to the instructional goals that promote students' problem-solving abilities in mathematics. Three goals were used in order to check whether the embedded activities, tasks, strategy or exercise problems in the textbooks aligned with the goals that address indicators' rigorous expectations.

Goal 1. The material provides explorative tasks and guide teachers to engage students in exploring mathematical investigation.

This goal examines whether the textbook material guide teachers to assist students to formulate and construct their own strategies in solving problems. Ineffective strategy choices may results in less acquisition of higher order thinking and mental computation (Wu, 1999). Thus, explicit instruction can work to assist students to select more appropriate and effective strategies. Does the textbook include problem-solving lessons throughout in each content domain? Does the material of the textbooks provide opportunity to apply problem-solving skills in real-life situations? Are scaffolded models including sequence of example problems provided before students are asked to solve problems on their own? Does the material contain exploratory questions that help students make sense of their experiences?

Goal 2. The materials encourage students to explain/communicate their investigations and mathematical thinking orally and in writing with others.

This goal examines whether the following questions are incorporated in selected textbooks: Whether the textbook includes such problems that help students express their thinking and explanations? The textbook material must provide opportunity for teachers to involve students compare their understanding with those of others so that they should reflect and review their ideas if needed. Whether the material assist teacher for alternative justifications after students explains their ideas?

Goal 3. Materials make connections within the subject and in contexts outside mathematics to develop strong mathematical understanding.

The textbook material should focus on strengthening conceptual connections between new and previous experiences. Students should be engaged and encouraged to use learning to explain new ideas. Assist teachers to probe into students" explanations of mathematical investigations that help students draw reasonable conclusions from evidence and data. To respond this goal also involve examining whether the material provide an opportunity to engage students in new situations and problems that require the application of identical or similar explanations and generalize the concepts, processes, and skills.

Each activity, task, and exercise problem was rated on a 4-point scale from 0 to 3 (3= High potential, 2= Medium potential, 1= Low potential, 0= No potential). The score for each instructional goal was calculated by averaging the ratings of indicators for each topic.

4.Results and Discussions

Four indicators for each three goals of "Strategic Competence" were used to measure the quality and relevancy of instructional activities of textbook. Table 1 show that NSM contained numerous activities that provide 58.33% alignment with indicators for the first goal of strategic competence. The instructional activities in the textbook "M 8" included 37.50% strategies that were likely to support student's strategic competence in Geometry. Likewise, for the second goal, NSM fulfilled 41.67% indicators to addresses the key ideas of strategic competence in Mathematics. In contrast M 8 contained 29.17% strategies that provide quality instructional support to meet the indicators. For the third goal, NSM provided 41.67% quality instructional activities that can support develop strong mathematical understanding. NSM and M8

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provided a variety of context and manipulative activities including explorative tasks, hand-on-activities and verbal statements as pedagogical content tools. For example, on page number 221 in NSM and page number 160 in M8, two explorative activities have been provided. The common purpose of these explorative activities in both textbooks is to involve students to investigate the volume of the cone but there is a considerable difference strategy between tasks with regard to the development of problem-solving competence in students. The explorative prompt in M8, student have been given guided instruction to fill up the cone three times with sand and asked to pour it into the cylinder and then a conclusion has also drawn that three times volume of a cone is equal to the volume of the cylinder of equal base and height. This investigation confined the level of students' thinking and students are less likely to use their own problem-solving strategies to reach the conclusion. On the other hand, the exploration activity given in NSM created an inquiring and challenging environment for students. Two questions have been asked from students: 1) if we are to fill the cone with sand and pour it into the cylinder, what fraction of the cylinder will be filled? 2) If we repeat the process, how many times it will take to fill the cylinder completely? (p. 211). This is a hands-on material activity that may get students to think about the relationship between a cone and cylinder. The solution of this kind of investigation tasks requires problem-solving heuristics, and that can help to develop students' problem-solving skills.

Table 1. Two textbooks series showing average scores with percentage on each of the learning Goals

Std	Learning Goals	Average rating	
		NSM	M 8
SC	1. The materials guide teachers to engage students in exploring mathematical investigation.	1.75 (58.33)	1.12 (37.50)
	2. The materials encourage students to explain/ communicate their investigations and mathematical thinking orally and in writing to peers, teachers, and others.	1.25 (41.67)	0.87 (29.17)
	3. Materials make connections within the subject and in contexts outside mathematics to develop strong mathematical understanding	1.25 (41.67)	0.87 (29.17)
	Overall	1.56 (51.85)	0.81 (27.31)

Note. SC-=strategic competence

Students can start thinking about the exploration task in groups and decide on how to set up the problem (Montague, 2005). Students (in both cases) can begin with the formation of nets of cone and cylinder of the same circular base and same height and construct the objects from those nets (Boyd, Burrill, Cummins, Timothy, & Malloy, 1998). Students can also experience hands-on exploration and may have a lot of fun filling these shapes up with sand and comparing. Then they proceed to analyze and generalize their finding until they can apply, explain and draw conclusions about the concept (Van & M, 1986). During the investigation, students maximize their interactions with teachers and peers in the groups, and this interaction and communication is the foundation of cognitive strategy instruction. (Council, 2013) claimed that in such kind of tasks students use integrated process skills such as writing hypotheses, designing and carrying out mathematical investigations, constructing data tables and graphs, and analyzing relationships between geometric concepts. Instead of memorizing the formula, students can know and be able to find through hands-on experience and this whole practice help students with their confidence in tackling the problem-solving task in any situation.

It is also interesting to note that there was the greater difference that existed between routine and Non-routine tasks in both textbook series. NSM provided considerably more such tasks in exercises whose solutions required thinking skills and problem-solving heuristics than M8. Similarly, NSM provided students with more opportunities to experience problems with a variety of complexities, contextual tasks and open-ended problems as compared to M8. (Stein & Smith, 1998), argued that open-ended problems and contextual tasks help students solve mathematical tasks having implicit procedures. (Kwon, Park, & & Park, 2006), argued that providing students with more opportunities to practice open-ended problems can promote fluency, flexibility, and originality.

5.Summary and Conclusion

We compared the quality of Instructional methods of two textbook series of middle-grade used in public and private schools in Pakistan. This procedure of analysis was used to identify, to what extent the selected textbooks provided quality instructional strategies that address the important concept of

Geometry which is most likely to assist students to gain proficiency in mathematics problem-solving. The instructional strategies used in selected series were analyzed using author's previously published and validated framework which was based on mathematics proficiency strands plus assessment. The analysis was limited to focus on the important concept of Geometry in middle year's mathematics series.

Overall from this analysis, NSM2 appeared to stand out M8. According to performance grading criteria, NSM falls in the "Medium potential" category while M8 fallS in "Low potential" category. When we looked at how instructional strategies in textbooks addressed and provided instructional support across all instructional goals, it evidences a significant gap between textbooks with regards to quality tasks and instructional approach. The results call for urgent consideration from the curriculum and instruction developers and textbook authors to the issue of the wider instructional gap between textbook and the national curriculum goals. The national curriculum lays its much emphasis on problem-solving strategies which we termed here as "Strategic Competence". The study reveals that in both textbooks needs more improvement to fill the gap between curriculum standards and the instructional approach embedded in the textbooks.

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