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Fibonacci Numbers and Golden Ratio- Teaching through the History of Mathematics

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Abstract

The Fibonacci sequence is named after Italian mathematician Leonardo Bonacci, commonly known as Fibonacci from Republic of Pisa. Fibonacci numbers are widely used in computer algorithms such as the Fibonacci search techniques and graphs termed as Fibonacci cubes used for interconnecting parallel and distributed systems. Mostly, the Fibonacci numbers are used in biological settings. The main aim of the present study is to explore the Fibonacci numbers through its historical view as it will not only be inspiring but more significant to a large group of students as they are concerned in knowing who, how and why about certain theorems and definitions that they use daily in Fibonacci sequences.

Keywords: - History of Mathematics, Fibonacci Numbers, Golden Ratio

Introduction

Augustus de Morgan, the earliest chairman of London Mathematical Society delivered the speech at the occasion of opening ceremony on 16, January, 1865: "except if a field of science and art is concerned to human history, human being can not be broad minded. It is rather astonishing that mathematicians teach mathematics even they do not know anything about the history of mathematics. Claiming that they know the reality, these people destroy the history of mathematics. Every human being has assured historical knowledge in his / her mind or imagination." In his speech, he (Augustus de Morgan) points out that everyone should know the mathematical history in order to have a significant knowledge about mathematics.

Recent studies in the mathematical history put stress on the fact that the mathematical history is basic to mathematics teaching. Thanks to the vast culture and broad knowledge held within the mathematical history, it becomes easier to understand the concepts of mathematics. According to Blom and Gulikers, the mathematical history can be applied in learning cases to create a lively classroom environment and to add liveliness to mathematics teaching. According to Perkins, through the mathematical history, lessons can be taught in a more fascinating way, which will thus bring about more accomplishment. The aim of this study is to establish the influence of teaching Fibonacci numbers and golden ratio and Fibonacci numbers through mathematical history on student attainment and the opinions of students regarding this matter. It is really distressing that students do mathematical operations through numbers, concept and symbols even they do not know what these operations serve for in practice. However, mathematics and science is an interdisciplinary approach. It can manipulate other branches of science, as well. Mathematics is that type of study which can be applied in daily life and that can be explained with examples from daily life. Another intention of this study is to take mathematics away from its traditional structure and to reveal that mathematics is a lively branch of science with its real appearance in nature. Actually, Fibonacci numbers are present everywhere in nature such as in the scales of a pineapple, daisies, sunflowers, a grain of wheat, tobacco plants, a hive of bees, etc. In the same way, the golden ratio can be seen in nature and even in all of mankind. It present on faces- both human and nonhuman, fingers, teeth, arms, flowers, sea shells, starfish, honey bees, sand dollars, snails, chairs, cars, etc. Through Fibonacci numbers and golden ratio, students will be able to understand the mathematics in an interesting and fascinating way.

The golden ratio

The golden numbers are the most natural real numbers, since it can be written as:

$$\phi = [\bar{1}]$$

This is standard short form for the continued fraction expansion

$$\phi = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\cdots}}}$$

From above, it is cleared that $\phi - 1 = \frac{1}{\phi}$, so that ϕ is also equal to $\frac{1+\sqrt{5}}{2}$. The golden number is also the limit of the convergent sequence

$(p_n/q_n)_{n=0}^{\infty} = \left(\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \dots \dots \dots \dots\right)$, which includes the Fibonacci numbers.

Fibonacci sequence

The well-known Fibonacci sequence such as 1,1,2,3,5,8,13,21,34,55,… with the recurrence

Formula given below:

$F_1 = F_2 = 1, F_n = F_{n-1} + F_{n-2}; n \geq 3$

$F_{m+n} = F_{n-1}F_m + F_nF_{m+1}$; Where m, n are natural numbers (1)

Generalized Fibonacci sequence

Fibonacci sequence has many promotional. A class of generalized Fibonacci sequence is given to discuss whether the process of the spread is inclined by a critical value.

If A_1, A_2, F_1, F_2 are all positive real numbers. This paper discusses the recurrence formula, as given below:

$F_1 = F_2 = 1$

$F_n = A_1F_{n-1} + A_2F_{n-2}, n = 3,4$ (2)

The derivation of the generalized Fibonacci sequence -based

Each node in the network is normally given a particular initial capacity and load. When a node for some reason the load beyond its capacity to produce failure, put the load on the node according to a definite percentage (with the previous time the load on the node) allocated to the net moment that point, it can be seen sometimes the point will be to rapid growth, that could produce an avalanche phenomenon; but sometimes only to grow to

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basically stop growing after a particular value, produce a very small possibility of an avalanche phenomenon.

There is a class of the propagation problem. Every node has load capability. Set the volume of the node on the moment of t as$F_i(t)$, then the volume of the first two moments $F_i(t-1)$ and $F_i(t-2)$ have particular relation, which is assumed as ω. Is there any impact on the type of the communication process? If so, how they change the network nodes with a capacity of spread? The paper has researched the process of the propagation which is based upon the generalized Fibonacci mode.

$$F_i(t) = F_i(t-1) + \omega[F_i(t-1) - F_i(t-2)] = [1+\omega]F_i(t-1) - \omega F_i(t-2) \quad \text{and } \omega > 0 \qquad (3)$$

So the formula (3) is according with the generalized Fibonacci sequence formula (2)

$$\begin{cases} A_1 = 1 + \omega \\ A_2 = \omega \end{cases}$$

If $F_n = F_i(t), F_{n-1} = F_i(t-1), F_{n-2} = F_i(t-2)$

According to the formula (3)

$$\begin{cases} F_n = (1+\omega)F_{n-1} - \omega F_{n-2} \\ F_0 = 0, F_1 = 1, F_2 = 1+\omega \end{cases} \qquad (4)$$

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$$\text{If } a_n = F_n, a_{n-1} = F_{n-1}, a_{n-2} = F_{n-2}$$

$$a_n = (1+\omega)a_{n-1} - \omega a_{n-2} \qquad (5)$$

If $a_n = a_2, a_{n-1} = a_1, a_{n-2} = a_0$, among that a is the constant to be determined, call it the test solution or special solution, substituted into the recursive formula, so :

$a_2 = (1+\omega)a - \omega$

i.e. $a_2 - (1+\omega)a + \omega = 0$

so
$$a = \frac{(1+\omega) \pm |1-\omega|}{2} \qquad (6)$$

So we have two solutions, such as:

$$\begin{cases} F_n[1] = a_{1n} = 1n \\ F_n[2] = a_{2n} = \omega n \end{cases} \qquad (7)$$

It can be proved that a combination of these two solutions (K1, K2 are arbitrary constants)

$$F_n = K_1 F_n[1] + K_2 F_n[2] = K_1 + K_2 \omega_n \qquad (8)$$

Still in line with the solution of the recursive formula, call it "general solution".

Solve the K_1, K_2 in the F_n, that make F_1=1, F_0=0.

$$\begin{cases} K_1 + K_2\omega_0 = 0 \\ K_1 + K_2\omega_1 = 1 \end{cases} \qquad (9)$$

So,

$$\begin{cases} K_1 = 1/(1-\omega) \\ K_2 = 1/(\omega-1) \end{cases} \tag{10}$$

Then the general formula of the Fibonacci sequence is derived by the formula (8)

$$F_n = \frac{1-\omega^n}{1-\omega} \tag{11}$$

To derive f (t) is a function about time.

$$f(t) = \frac{1-\omega^t}{1-\omega} \tag{12}$$

Powdiff $f(t)$

$$lnf(t) = -t\frac{ln\omega}{1-\omega}$$

$$\frac{f'(t)}{f(t)} = \frac{-ln\omega}{1-\omega}$$

$$f'(t) = \frac{-ln\omega}{(1-\omega)^2(1-\omega^t)} > 0 \tag{13}$$

So $f'(t)$ is the exponential function of time t.

If ω>1, $f(t)$ is rapid growth in avalanche, then if 0<ω<1, $f(t)$ is the slow growth to close a value.

Conclusion

This paper includes the Mathematical derivations and formulae based on the generalized Fibonacci sequence. These formulae are used to find a critical value, which is further used to make clear in what condition the avalanche happened to the node. The avalanche will happened in exponential order to the node and with this, the probability of system breakdown when ω, the value of key nodes Aihua Gu and Yingying Xu / Physics Procedia 24 (2012) 1737 – 1741 1741 Author name / Physics Procedia 00 (2011) 000–000 found at the time of the transition, is bigger than 1 and the entire system will be even when ω is between 0 and 1. In the course of generalized Fibonacci series mathematical derivation and numerical simulation, we get that electricity network and the set of connections etc, which is the load capacity of the network. We study the network, there is change in the capacity of the point due to a point change over time, whether the degree of rapid growth? Or steady in the vicinity of a value? Only in reality, it is very important, but how to find the critical value of ω and how to control, is more important. We must try to manage the range of ω between 0 and 1, so as to successfully control the devastating avalanche incidents.

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