
Eco-Epidemiological Models – Ancient to Modern Era

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Abstract

The population dynamics is one of the important fields of mathematical biology. Thomas Malthus was the first researcher in this field. His idea that population growth rate is directly proportional to its current size seems to be fairly true. Later on Pierre-Francois Verhulst, a Belgian Mathematician extended the Malthusian model by considering the fact that populations suffer internal competition and this competition in fact effects their growth. Later on, in the twenties of the past century, the revolutionary work by Lotka and Volterra extended the mathematical modeling to the prey-predator interactions. A variety of models have been formed on disease caused by pathogens. The present study is an attempt to study some ecological models along with epidemiological models.

Keywords: - Prey, Predator, Epidemiology, Stability.

Introduction

Population ecologists use a variety of mathematical methods to model population dynamics (how populations change in size and composition over time). Some of these models represent growth without environmental constraints, while others include "ceilings" depending upon the resources. Mathematical models can accurately describe changes occurring in a population and, importantly, to predict future changes. We consider here the different population models used in ecology and eco-epidemiology. An epidemic model simply describes the transmission of infectious disease through individuals. Modelling of infectious diseases is a tool with the help of which we study the features regarding the spread of disease in a particular population and then predict the future course. There are mainly two types of epidemic models: stochastic and deterministic. Stochastic models are used in small populations and deterministic models are used in those areas where the population is large. Moreover the real world is nonlinear; fitting the components together is a much harder puzzle.

Ecological Models

I. Malthusian Growth Model

A Malthusian growth model, sometimes called a simple exponential growth model, is essentially exponential growth based on a constant rate. The model is named after Thomas Robert Malthus, who wrote an Essay on the principle of Population (1798), one of the earliest and most influential books on population.

Malthusian model has the following form:

$P(t) = P_0 e^{rt}$ where

$P_0 = P(0)$ is the initial population size,

r = the population growth rate, sometimes called *Malthusian parameter*,

t = time.

Malthusian model is commonly referred to as the *exponential law*. It is widely regarded in the field of population ecology as the first principle of population dynamics with Malthus as the key researcher in the field. The exponential law is sometimes referred to as the *Malthusian Law*. Malthus wrote that all life forms, including humans, have a propensity to exponential population growth when resources are abundant but he did not consider the fact that actual growth of a particular population depends upon the available resources. Malthus considered that a population's *per capita* (per individual) growth rate stays the same regardless of population size, making the population grow faster and faster as it gets larger. Malthus model main limitation is that it completely neglects the fact that populations cannot grow indefinitely as there are always limited resources in every population.

II. *Logistic Growth Model*

Logistic growth model is modification of Malthusian growth model as it considered that populations can grow substantially but cannot grow indefinitely. He considered a particular population of size 'N'. The logistic growth model equations are given by

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

Where N is the population size and r is recruitment rate and K is the carrying capacity of the system.

The logistic growth model is more realistic than the Malthusian model as it limits the population growth depending upon the number of available resources and carrying capacity of the system.

Epidemiological Models

1. *SIR Model*

In SIR model we use the three compartments viz; susceptible S(t), infected I(t), and removed R(t).

S (t) represents the number of individuals not yet infected with the disease at any time t. I (t) represent the number of individuals who have been infected with the particular disease and they are also becoming the cause of spreading the disease to those who are in susceptible category.

R (t) represents the number of individuals who have been infected and then recover their disease either due to immune or death. This category includes the individuals who are not supposed to be infected again.

So the SIR Equations are given by

$$\frac{dS}{dt} = -\alpha SI$$

$$\frac{dI}{dt} = \alpha SI - \beta I$$

$$\frac{dR}{dt} = \beta I$$

2. SIS Model

You get sick, then recover, but without immunity. For example the common cold. To get SIS equation we eliminate the equation of the recovered population from the SIR model and adding those removed individuals from the infected population into the susceptible population gives the following equations:

$$\frac{dS}{dt} = \beta I - \alpha SI$$

$$\frac{dI}{dt} = \alpha SI - \beta I$$

3. SI Model

Some epidemic processes are non recoverable. So if some individual gets infected, he will remain infected forever. Here r is the growth rate of susceptible population which is assumed to be constant. So the SI equations are given by

$$\frac{dS}{dt} = r - aSI$$

$$\frac{dI}{dt} = aSI$$

Conclusion

The present paper considers some ecological and epidemiological models. These models are further discussed based upon their classifications. The detailed study of these models is essential to find the main cause, which is spreading the disease. The detailed study can also be helpful in finding the control measures for the spread of disease. We cannot find the best models, only better models can be formed which are more realistic depending upon the situation of the population considered. One can make the models more perfect by taking the least number of assumptions which best match with the characteristics of the population under consideration. So the extensive research is the need of the hour.

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