
Ancient Indian Mathematicians

Sanjay Gupta

Assistant Professors, P.G. Department of Mathematics, Dev Samaj College For Women, Ferozpur City, Punjab, India.

Poonam

Assistant Professors, P.G. Department of Mathematics, Dev Samaj College For Women, Ferozpur City, Punjab, India.

Sandeep Sandhu

Assistant Professors, P.G. Department of Mathematics, Dev Samaj College For Women, Ferozpur City, Punjab, India.

Abstract

The aim of this paper is to present a brief history of ancient Indian mathematicians. We want to summarize the results and contributions of Indian mathematicians which were made in the period from the first civilization in Indian subcontinent to 5th century AD when classical era of Indian mathematics began.

Keywords : History, Ancient, Mathematicians, Civilization, Classical.

1. Introduction

In this paper, we have to present enormous work and contributions made by Ancient Indian Mathematicians. It is no doubt that the world today is greatly thankful to the contributions made by Indian mathematicians. One of the most important contributions made by them was the invention of zero as well as the introduction of decimal system. Indian mathematics emerged in the Indian subcontinent from 1200 BC until the end of the 18th century. In the classical period of Indian mathematics (400 AD to 1200AD), important contributions were made by scholars like Aryabhata, Brahmagupta and Bhaskara II. The negative numbers, arithmetic, algebra and trigonometry were further advanced in India and in particular, the modern definitions of "sine" and "cosine" were developed there.. Some of the famous Indian mathematicians from Indus Valley civilization and Vedas are as follow

2. Baudhayana

Baudhayana was a mathematician who lived around 800 BCE in what is now modern day India. One of the most important contributions by Baudhayana was the theorem nowadays known as "Pythagoras theorem" that has been credited to Greek mathematician Pythagoras. This might come as a surprise to many, but it's true that Pythagoras theorem was known much before Pythagoras and it was Indians who actually discovered it at least 1000 years before Pythagoras was even born.



Fig. 1: Baudhayana

Baudhayana listed Pythagoras theorem in his book called *Baudhayana Sulbasutra* (800BCE) which also contains calculation of value of pi, calculating square root of 2, circling the square and geometric solutions of a linear equation in a single unknown. Incidentally, Baudhayana Sulabutra is also one of the oldest books on advanced Mathematics.

The actual shloka (verse) in Baudhayana Sulbasutra that describes Pythagoras theorem is given below :

“ दीर्घचतुरश्रस्याक्षण्या रज्जुः पार्श्वमानी तिर्यग् मानी च यत् पृथग् भूते कुरुतस्तदुभयं करोति ॥ ”

Interestingly, Baudhayana used a rope as an example in the above shloka which can be translated as :

“ A rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides make together ”.

To elaborate more clearly, the shloka is to be translated as :

“ The diagonal of a rectangle produces by itself both (the areas) produced separately by its two sides.”

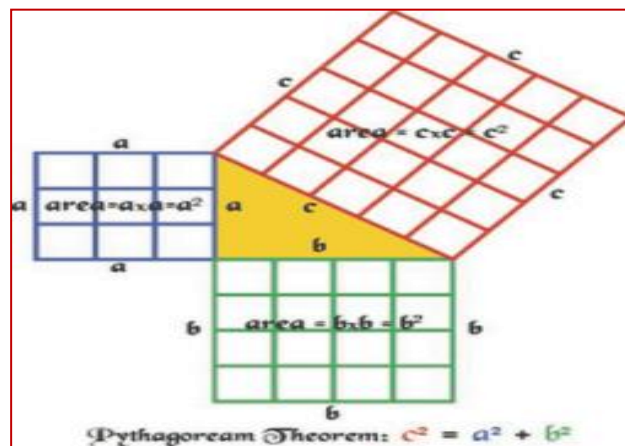


Fig. 2: Pythagoras Theorem

Therefore, little else is known about Baudhayana except that he was the author of one of the earliest Sulbasutras. Not even the exact date of death of this great mathematician is recorded

but historians attach the date 800 BCE. Some believe that he was not just a mathematician but in fact, he was also a priest and an architect of very high standards.

3. Aryabhata

Aryabhata was one of the first Indian mathematicians and astronomers belonging to the classical age. He was born in 476 BC in Tarenaga (a town in Bihar), India. It is however definite that he travelled to Kusumapara (modern day Patna) for studies and even resided there for some time. It is mentioned in a few places that Aryabhata was the head of the educational institute in Kusumapara. The University of Nalanda had an observatory in its premises so it is hypothesized that Aryabhata was the principal of the university as well. On the other hand some other commentaries mention that he belonged to Kerala.

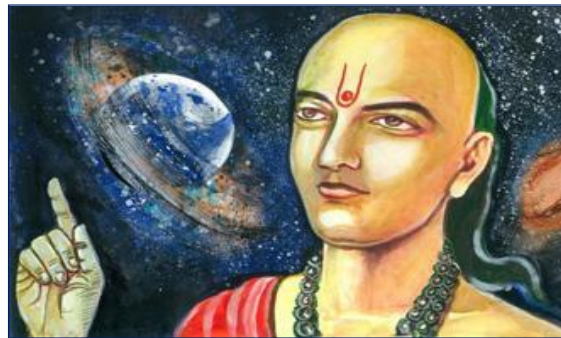


Fig. 3: Aryabhata

Aryabhata wrote many mathematical and astronomical treatises. His chief work was the '*Aryabhatiya*' which was a compilation of mathematics and astronomy. The name of this treatise was not given to it by Aryabhata but by later commentators. A disciple by him called the '*Bhaskara*' names it '*Ashmakatantra*' meaning '*treatise from the Ashmaka*'. This treatise is also referred to as '*Arya-shatas-ashta*' which translates to '*Aryabhata's 108*'. This is a very literal name because the treatise did in fact consist of 108 verses. It covers several branches of mathematics such as algebra, arithmetic, plane and spherical trigonometry. Also included in it are theories on continued fractions, sum of power series, sine tables and quadratic equations.

Aryabhata worked on the place value system using letters to signify numbers and stating qualities. His most significant contributions to mathematics include approximation of the value of pi up to five decimal places. Aryabhata was the one who calculated the area of the triangle as perpendicular multiplied by the half side. He introduced the concept of sine in his work called '*Ardha-jya*' which is translated as '*half-chord*'. In algebra, he summed series of squares and cubes and solved equations of the type $ax - by = c$.

Aryabhata also did a considerable amount of work in astronomy. He knew that the earth is rotating on an axis around the sun and the moon rotated around it. He also discovered the position of nine planets and stated that these also revolved around the sun. He pointed out the eclipses; both lunar and solar. Aryabhata stated the correct number of days in a year that is 365. He was the first person to mention that the earth was not flat but in fact a spherical

shape. He also gave the circumference and diameter of the earth and the radius of the orbits of 9 planets.

Aryabhata was a very intelligent man. The theories that he came up with at that time present a wonder to the scientific world today. His works were used by the Greeks and the Arabs to develop further. A commentary by Bhaskara I a century later on Aryabhata says:

“Aryabhata is the master who, after reaching the furthest shores and plumbing the inmost depths of the sea of ultimate knowledge of mathematics, kinematics and spherics, handed over the three sciences to the learned world.”

Aryabhata was an immense influence to mathematics and astronomy. Many of his works inspired Arabs more particularly. His astronomical calculations helped form the ‘*Jalali Calendar*’. He has been honored in many ways. The first Indian satellite is named after him as ‘*Aryabhata*’ and an Indian research center as ‘*Aryabhata Research Institute of Observational Sciences*’.

4. Brahmagupta

Brahmagupta was an Indian Mathematician, born in 598 AD in Bhinmal, a state of Rajasthan, India. He spent most of his life in Bhinmal which was under the rule of King Vyaghramukha. He was the head of the astronomical observatory at Ujjain which was the center of mathematics in India witnessing the work of many extraordinary mathematicians.

Brahmagupta wrote many textbooks for mathematics and astronomy while he was in Ujjain. These include ‘*Durkamyarnada*’ (672), ‘*Khandakhadyaka*’ (665), ‘*Brahmasphutasiddhanta*’ (628) and ‘*Cadamakela*’ (624). The ‘*Brahmasphutasiddhanta*’ meaning the ‘*Corrected Treatise of Brahma*’ is one of his well-known works. It contains a lot of criticism on the work of his rival mathematicians. Brahmagupta had many discrepancies with his fellow mathematicians and most of the chapters of this book talked about the loopholes in their theories.

One of the most significant contribution of Brahmagupta to mathematics was the introduction of ‘*zero*’ to the number system which stood for ‘*nothing*’. He gave the world the concept of negative numbers. His work the ‘*Brahmasphutasiddhanta*’ contained many mathematical findings written in verse (shaloka) form. It had many rules of arithmetic which is part of the



Fig. 4: Brahmagupta

mathematical solutions now. These are 'A positive number multiplied by a positive number is positive.', 'A positive number multiplied by a negative number is negative', 'A negative number multiplied by a positive number is negative' and 'A negative number multiplied by a negative number is positive'. The book also consisted of many geometrical theories like the 'Pythagorean Theorem' for a right angle triangle. Brahmagupta was the one to give the area of a triangle and the important rules of trigonometry such as values of the sine function. He gave the value of 'Pi' as square root ten to be accurate and 3 as the practical value. He also proposed rules for solving simultaneous and quadratic equations. He calculated the area of a cyclic quadrilateral with semi-perimeter. Brahmagupta is the founder of "Numerical Analysis", a branch of higher mathematics. He was the one to identify that $x^2 - y^2 = (x - y)(x + y)$.

Brahmagupta argued that the Earth and the universe are round and not flat. He was the first to use mathematics to predict the positions of the planets, the timings of the lunar and solar eclipses. Though all this seems like obvious and simple solutions it was a major improvement in science at that time. He also calculated the length of the solar year which was 365 days, 5 minutes and 19 seconds which is quite accurate based on today's calculation of 365 days, 5 hours and 19 seconds. He also talked about 'gravity' in one of his statements saying:

"Bodies fall towards the earth as it is in the nature of the earth to attract bodies, just as it is in the nature of water to flow".

5. Madhava

Madhava of Sangamagrama was an important Indian mathematician who also endeavoured the field of astronomy. He belonged to Irinnalakkūṭa which was a town close to Cochin (in modern day India). Many progressions have been attributed to this great mathematician, for instance he is labelled as being responsible for the founding of Kerala School of Astronomy and Mathematics.

Many of his writings have been known to be lost, but some of his contributions in astronomy have managed to survive. There have been reports that much of his work was discovered by Keralese mathematicians who were alive till around 100 years after him.

He was the pioneer in formulating infinite series approximations for trigonometric functions. Many people believe that his work helped open a gateway towards 'Mathematical Analysis'. This well-known astronomer-mathematician contributed in numerous fields which included infinite series, calculus, trigonometry, geometry and algebra. Madhava was a revolutionary mathematical figure of the Middle Ages and a few scholars have reason to believe that his work reached through to Europe due to the presence of Jesuit missionaries and traders at an ancient port relevant to his surroundings at that time.

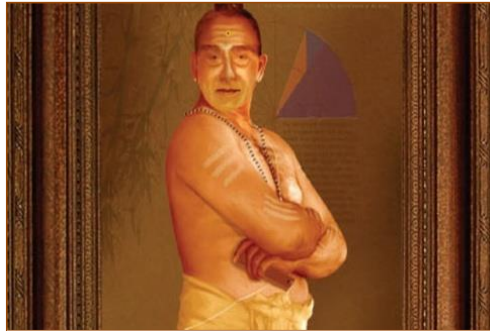


Fig. 5: Madhava

Something of peculiar interest regarding this mathematician includes the fact that he discovered the series equivalent to the Maclaurin expansions ($\sin x$, $\cos x$ and $\arctan x$), more than two hundred years ahead of Europe. He determined the explanations of a few 'transcendental' equations by a process of repetition. A procedure was unfolded by him to determine the sitting of the Moon every 36 minutes, and techniques to evaluate the motions of the planets. His contributions to infinite series and infinitesimal quantities attest his writing existence. A lot of his work has been transmitted through other mathematicians after him. To suggest his popularity, there is a formula named "Madhava - Leibnitz" which many students are taught during their college years.

6. Varahamihira

Varahamihira was an Indian philosopher, astronomer and mathematician, author of the *Pancha-siddhantika* ("Five Treatises"), a compendium of Greek, Egyptian, Roman, and Indian astronomy.

Varahamihira's knowledge of Western astronomy was thorough. In five sections, his monumental work progresses through native Indian astronomy and culminates in two treatises on Western astronomy, showing calculations based on Greek and Alexandrian reckoning and even giving complete Ptolemaic mathematical charts and tables. But his greatest interest lay in astrology. He repeatedly emphasized its importance and wrote many treatises on *shakuna* ("augury") as well as the *Brihaj-jataka* ("Great Birth") and the *Laghu-jataka* ("Short Birth"), two works on the casting of horoscopes.

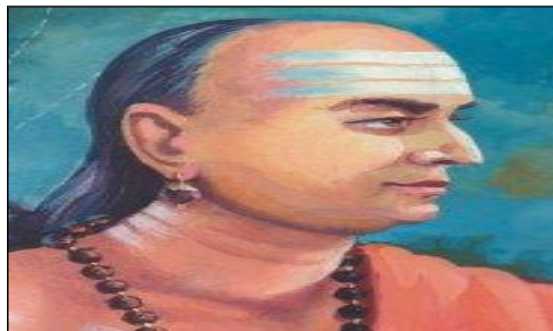


Fig. 6: Varahamihira

Varahamihira also improved the accuracy of Aryabhata's sine tables. Varahamihira also defined the algebraic properties of zero and negative numbers. He was among the first mathematicians to discover a version of Pascal's triangle as we know it today.

7. Bhaskara

Bhaskara (commonly called Bhaskara I to avoid confusion with the 12th century mathematician Bhaskara II) was a 7th century mathematician who was the first to write numbers in the Hindu decimal system with a circle for the zero, and who gave a unique and remarkable rational approximation of the sine function in his commentary on Aryabhata's work. This commentary '*Aryabhatiyabhasya*' written in 629 CE, is among the oldest known prose works in Sanskrit on mathematics and astronomy. He also wrote two astronomical works in the line of Aryabhata's school, the *Mahabhaskariya* and the *Laghubhaskariya*.

On 7th June, 1979, the Indian Space Research Organisation launched Bhaskara-1 honouring the mathematician.

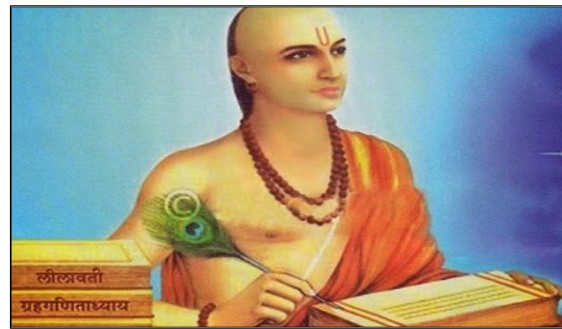


Fig. 7: Bhaskara

Little is known about Bhaskara's life. He was probably a Marathai astronomer. He was born at Bori, in Parbhani district of Maharashtra state in India in 7th century. His astronomical education was given by his father. Bhaskara is considered the most important scholar of Aryabhata's astronomical school. He and Brahmagupta are two of the most renowned Indian mathematicians who made considerable contributions to the study of fractions.

Bhaskara's probably most important mathematical contribution concerns the representation of numbers in a positional system. The first positional representations were known to Indian astronomers about 500 years ago. However, the numbers were not written in figures, but in words and were organized in verses (shalokas). For instance, the number 1 was given as *moon*, since it exists only once; the number 2 was represented by *wings, twins, or eyes* as they always occur in pairs; the number 5 was given by the (5) *senses*. Similar to our current decimal system, these words were aligned such that each number assigns the factor of the power of ten corresponding to its position, only in reverse order: the higher powers were right from the lower ones.

His system is truly positional, since the same words representing, can also be used to represent the values 40 or 400. Quite remarkably, he often explains a number given in this

system, using the formula *ankair api* ("in figures this reads"), by repeating it written with the first nine Brahmi numerals, using a small circle for the zero. Contrary to his word number system, however, the figures are written in descending valuedness from left to right, exactly as we do it today. Therefore, at least since 629 the decimal system is definitely known to the Indian scientists. Presumably, Bhaskara did not invent it, but he was the first having no compunctions to use the Brahmi numerals in a scientific contribution in Sanskrit.

Bhaskara expanded on the work of Aryabhata and found an approximation of the sine function. Bhaskara laid the foundation of differential calculus and gave an example of the differential coefficient and discussed the idea of what we know as Rolle's Theorem today. He told the world that sum of any number and infinity is infinity and any number divided by zero is infinity. He was the one to introduce the cyclic method of solving algebraic equations. The "inverse cyclic" method that we know today stems from this.

8. Mahavira

Mahavira (or Mahaviracharya_Mahavira the Teacher) was a 9th century Jain Mathematician from Karnataka, India. He was the author of *Ganita Sara Samgraha* (c.850), which revised the Brahmasphutasiddhanta. He was patronised by the Rashtrakuta king Amoghavarsha. He separated astrology from mathematics. It is the earliest Indian text entirely devoted to mathematics. He expounded on the same subjects on which Aryabhata and Brahmagupta contended, but he expressed them more clearly. His work is a highly syncopated approach to algebra and the emphasis in much of his text is on developing the techniques necessary to solve algebraic problems. He is highly respected among Indian mathematicians because of his establishment of terminology for concepts such as equilateral and isosceles triangle, rhombus, circle and semicircle. Mahavira 's eminence spread in all South India and his books proved inspirational to other mathematicians in Southern India. It was translated into Telugu language by Pavuluri Mallana as *Saar Sangraha Ganitam*.

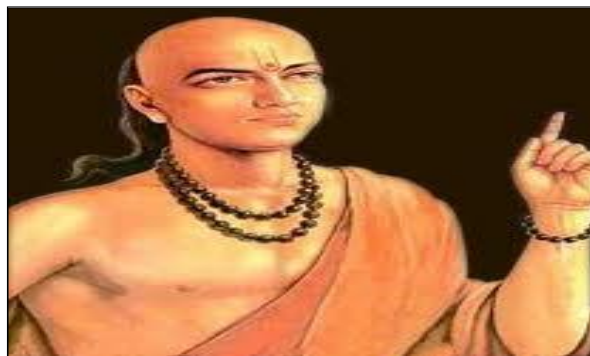


Fig. 8: Mahavira

He discovered algebraic identities like $a^3 = a(a + b)(a - b) + b^2(a - b) + b^3$. He also found out the formula for n_{C_r} as $\frac{n(n-1)(n-2)\dots(n-r+1)}{r(r-1)(r-2)\dots 2.1}$. He devised formula which approximated area and perimeters of ellipses and found methods to calculate the square of a number and cube roots of a number. He asserted that the square root of a negative number did not exist.

Mahavira *Ganita Sara Samgraha* gave systematic rules for expressing a fraction as the sum of unit fractions. This follows the use of unit fractions in Indian mathematics in the Vedic period and the Sulbasutras giving an approximation of $\sqrt{2}$ equivalent to $1 + \frac{1}{3} + \frac{1}{3.4} - \frac{1}{3.4.34}$.

9. Bhaskara II

Bhaskara II, also known as Bhaskara or Bhaskaracharya, was a 12th century Indian mathematician. He was also a renowned astronomer who accurately defined many astronomical quantities, including the length of the sidereal year. A brilliant mathematician, he made the significant discovery of the principles of differential calculus and its application to astronomical problems and computations centuries before European mathematicians like Newton and Leibnitz made similar discoveries. It is believed that Bhaskara II was the first to conceive the differential coefficient and differential calculus. The son of a mathematician and astronomer, he was trained by his father in the subjects. Following in his father's footsteps the young man too became a renowned mathematician and astronomer and was considered the lineal successor of the noted Indian mathematician Brahmagupta as head of an astronomical observatory at Ujjain. Bhaskara II wrote the first work with full and systematic use of the decimal number system and also wrote extensively on other mathematical techniques and on his astronomical observations of planetary positions, conjunctions, eclipses, cosmography and geography. In addition, he also filled many of the gaps in his predecessor Brahmagupta's work. In recognition of his invaluable contributions to mathematics and astronomy, he has been called the "*greatest mathematician of medieval India*"

Bhaskara himself gave the details of his birth in a verse in the Arya metre according to which he was born in 1114 near Vijjadavida (believed to be Bijjaragi of Vijayapur in modern Karnataka). His father was a Brahmin named Mahesvara. He was a mathematician, astronomer and astrologer who passed on his knowledge to his son.

Bhaskara followed in his father's footsteps and became a mathematician, astronomer and astrologer himself. He went on to become the head of an astronomical observatory at Ujjain, the leading mathematical centre of ancient India. The centre was a famous school of mathematical astronomy.



Fig. 9: Bhaskara II

He made many significant contributions to mathematics throughout his career. He is credited to have given a proof of the Pythagorean theorem by calculating the same area in two different ways and then canceling out terms to get $a^2 + b^2 = c^2$.

His work on calculus was groundbreaking and much ahead of his times. He not only discovered the principles of differential calculus and its application to astronomical problems and computations, but also determined solutions of linear and quadratic indeterminate equations (Kuttaka).

His major work '*Siddhanta Siromani*' ("Crown of treatises") was completed in 1150 when he was 36 years old. Composed in Sanskrit Language, the treatise consists of 1450 verses. The work is divided into four parts called '*Lilavati*', '*Bijaganita*', '*Grahaganita*' and '*Goladhyaya*', which are also sometimes considered four independent works. The different sections deal with different mathematical and astronomical fields.

The first part '*Lilavati*' consists of 13 chapters, mainly definitions, arithmetical terms, interest computation, arithmetical and geometrical progressions, plane geometry, and solid geometry among others. It also has a number of methods of computing numbers such as multiplications, squares, and progressions.

His work '*Bijaganita*' ("Algebra") was a work in 12 chapters. This book covered topics like positive and negative numbers, zero, surds, determining unknown quantities, and elaborated the method of '*Kuttaka*' for solving indeterminate equations and Diophantine equations. He also filled many of the gaps in his predecessor Brahmagupta's work.

The sections '*Ganitadhyaya*' and '*Goladhyaya*' of '*Siddhanta Shiromani*' are devoted to astronomy. He used an astronomical model developed by Brahmagupta to accurately define many astronomical quantities, including the length of the sidereal year. These sections covered topics such as mean longitudes of the planets, true longitudes of the planets, solar and lunar eclipses, cosmography and geography, etc.

Bhaskara II was especially well-known for his in-depth knowledge of trigonometry. Discoveries first found in his works include computation of sines of angles of 18 and 36 degrees. He is credited to have discovered spherical trigonometry, a branch of spherical geometry which is of great importance for calculations in astronomy, geodesy and navigation.

Bhaskara II's major work was the treatise '*Siddhanta Siromani*' which was further divided into four parts, each of them dealing with diverse topics on arithmetic, algebra, calculus, trigonometry, and astronomy. He is considered to be a pioneer in the field of calculus as it is probable that he was the first to conceive the differential coefficient and differential calculus.

Bhaskara II was married with children. He passed his mathematical knowledge to his son Loksamudra and years later Loksamudra's son helped to set up a school in 1207 for the study of Bhaskara's writings. It is believed that Bhaskara's book '*Lilavati*' was named after his daughter. He died around 1185.

10. Srinivasa Ramanujan Lyengar

Srinivasa Ramanujan Iyengar, an Indian mathematical wizard, occupies a unique position in the history of mathematics. Although he had little formal education, Ramanujan has contributed a lot to the world of mathematics. His life is an inspiration to the students and researchers of India. On December 22, 2012 the world celebrates the 125th birth anniversary of Srinivasa Ramanujan. The year 2012 was declared as the “National Mathematics Year (NMY)” by the Indian government.

Ramanujan was born on December 22, 1887 in an orthodox Brahmin family at Erode, a town in Tamilnadu of South India. His parents are Mr. K. Srinivas Iyengar and Komalatammal. His schooling started in Kumbakonam. He passed his Matriculation from Town High School, Kumbakonam by 1903. He studied S.L Loney’s Trigonometry which was then recommended as a textbook in colleges. After schooling he joined First Arts Class in The Government Arts College, Kumbakonam, a Cambridge in South India.

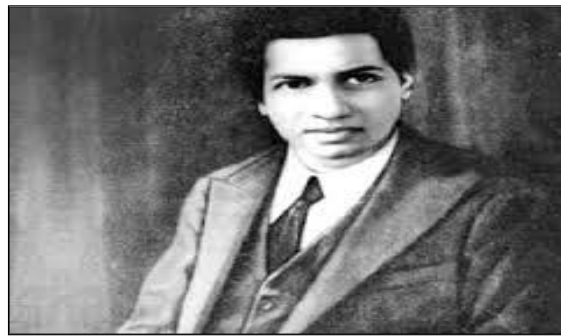


Fig. 10: Srinivasa Ramanujan Iyengar

On July 14, 1909, he married a nine-year-old girl Janaki Ammal. However, Ramanujan did not live with his wife until she was 12-years-old. During this period, he published many papers and was becoming well known in Chennai as a mathematical genius. In 1913, while he worked as a clerk in the Indian Mathematical Society, Ramanujan wrote a letter to Cambridge mathematician, G.H. Hardy, and told him about his work. In 1914, Ramanujan sailed for London. He started his work under the guidance of Hardy. He published 21 papers there, five of which were in collaboration with Hardy.

In 1916, Ramanujan graduated from Cambridge with a Bachelor of Science by Research. (This degree was recognised as a Ph.D. after 1920). But a year later he fell seriously ill and his doctors feared that he would die. In 1919, He reached India. In bed also, he never left his mathematics. It was a great shock, when he was diagnosed with pulmonary tuberculosis (T.B. of the lung). Over the months, he began to show signs of acute breathing problems and pain in the body. Finally, on April 26, 1920, India’s greatest mathematician of the twentieth century left his breath.

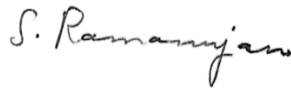
His original Note Books have been edited in a series of five volumes by Bruce C. Berndt cover the results and theorems about Hyper geometric series, Elliptic functions, Bernoulli’s

numbers, Divergent Series, Continued fractions, Elliptic modular equations, Highly Composite numbers, Riemann Zeta functions, Partition of numbers, Mock-theta functions. We can learn a little bit from Ramanujan's life. The seeds from Ramanujan's garden have been blowing on the wind and have been sprouting all over the landscape.

"Ramanujan did mathematics not for University degrees, but for his infinite thirst for mathematics.

Whatever profession one take up for livelihood, but needs not to sacrifice one's passion for which one like."

"An equation means nothing to me unless it expresses a thought of God".



S. Ramanujan

11. Conclusion

Mathematics owes a great deal to the contributions made by Indian mathematicians over many centuries. Indian mathematicians of the early Indus Valley Civilization to the scholars of the 5th to 12th century AD made contributions in the field of algebra, arithmetic, geometry, trigonometry, and differential equations. Later on, in the 14th to 16th AD, Indian mathematicians derived infinite series and expansion of trigonometric functions. In a nutshell, Indian scholars have always led the development of mathematics from the front. Apart from this, Indian mathematicians are also responsible for the creation and refinement of the current decimal place-value system including the number zero without which higher mathematics would not be possible. These mathematical concepts were transmitted to the Middle East, China, and Europe and led to further developments that now form the foundations of many areas of mathematics.

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