

# Adaptive Wavelet Thresholding for Image Denoising Using Various Shrinkage under Different Noise Conditions

N.Gopala Krishna & Dr.A.Senthil Rajan

<sup>1</sup> Associate Professor (Department of CSE-HOD) <sup>2</sup> Director, Computer Center  
P.N.C & VIET College of engineering and technology, Repudi, Guntur, A.P, INDIA  
<sup>1</sup>[student@gmail.com](mailto:student@gmail.com) <sup>2</sup>[guide@gmail.com](mailto:guide@gmail.com)

## Abstract

*This paper presents a comparative analysis of different image denoising thresholding techniques using wavelet transforms. There are different combinations that have been applied to find the best method for denoising. Visual information transmitted in the form of digital images is becoming a major method of communication, but the image obtained after transmission is often corrupted with noise. The search for efficient image denoising methods is still a valid challenge at the crossing of functional analysis and statistics. Wavelet algorithms are useful tool for signal processing such as image compression and denoising. Image denoising involves the manipulation of the image data to produce a visually high quality image. The main aim is to modify the wavelet coefficients in the new basis, the noise can be removed from the data. In this paper, we analyzed several methods of noise removal from degraded images with gaussian noise and speckle noise by using adaptive wavelet threshold (neigh shrink, sure shrink, bivariate shrink and block shrink) and compare the results in term of psnr and mse.*

**Keywords:** wavelet thresholding, Neigh Shrink, Sure Shrink, Bivariate Shrink and Block Shrink.

## 1. INTRODUCTION

In the past two decades, many noise reduction techniques have been developed for removing noise and retaining edge details. Most of the standard algorithms use a defined filter window to estimate the local noise variance of a noise image and perform the individual unique filtering process. The result is generally a greatly reduced noise level in areas that are homogeneous. But the image is either blurred or over smoothed due to losses in detail in non-homogenous areas like edges or lines. This creates a barrier for sensing images to classify, interpret and analyze the image accurately especially in sensitive applications like medical field.

The primary goal of noise reduction is to remove the noise without losing much detail contained in an image. To achieve this goal, we make use of a mathematical function known as the wavelet transform to localize an image into different frequency components or useful sub bands and effectively reduce the noise in the sub bands according to the local statistics within the bands. The main advantage of the wavelet transform is that the image fidelity after reconstruction is visually lossless. The wavelet de-noising scheme thresholds the wavelet coefficients arising from the wavelet transform. The wavelet transform yields a large number of small coefficients and a small number of large coefficients. Wavelets are especially well suited for studying nonstationary signals and the most successful applications of

wavelets have been in compression, detection and denoising. The method consists of applying the DWT to the original data, thresholding the detailed wavelet coefficients and inverse transforming the set of thresholded coefficients to obtain the denoised signal. Given a noisy signal  $y = x + n$ ; where  $x$  is the desired signal and  $n$  is independent and identically distributed (i.i.d) Gaussian noise  $N(0, \sigma^2)$ ,  $y$  is first decomposed into a set of wavelet coefficients  $w = W[y]$  consisting of the desired coefficient  $\theta$  and noise coefficient  $n$ . By applying a suitable threshold value  $T$  to the wavelet coefficients, the desired Coefficient  $\theta = T[w]$  can be obtained; lastly an inverse transform on the desired coefficient  $\theta$  will generate the denoise signal  $x = WT[\theta]$ .

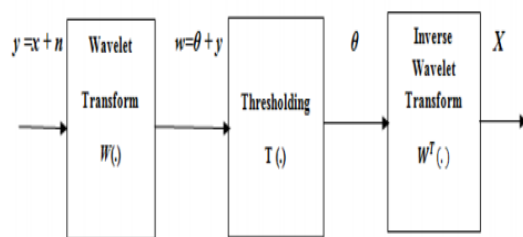


Figure 1: Block Diagram for DWT based denoising framework

In this paper, we propose a new denoising method which delivers better performance than the results of the previous works based on neighboring pixels. The results compared with standard denoising methods: VisuShrink, SureShrink, BayesShrink, and motivated by improving the performances of image denoising via wavelet-based thresholding that often induces the smoothing effect of the processed image mainly due to the thresholding of all detail sub-bands we introduce a new alternative that overcomes the shortcoming quoted previously by

adopting an interesting combination of spatial and transform domains that allow well benefiting from the advantages of both wavelet thresholding and spatial filtering. So, the acquired image from the transmission channel being degraded by Gaussian noise is decomposed into wavelet coefficients via a two-level 2D-DWT. Then, the finest detail sub-bands (LH1, HL1, HH1) being nearly constituted by only the noise coefficients, an adaptive thresholding is applied to cancel nearly the entire effect of the noise in the processed image. Moreover, the residual noise being mixed within the true coefficients of the coarse detail subbands (LH2, HL2, HH2); an adaptive wiener filtering is adopted to clean those coefficients from the residual noise instead of thresholding them; avoiding so the smoothing effect of the resulting image. Finally, and in order to well preserve the different features of the processed image, a joint bilateral filter (JBF) is performed to the resulting image.

### 1.1 WAVELET DENOISING

Wavelet transform is the mathematical tool used for various image processing applications such as noise removal, feature extraction, compression and image analysis. The general method of wavelet based denoising is that, the noisy image may first be transformed to wavelet domain [2] [6]. The transformed image appears as four sub-bands (A, V, H, and D) as shown in Fig 1 based on the level of decomposition 'j'. 2D discrete wavelet transform leads to decomposition of approximate coefficients at level 'j' into four components i.e. the approximation at level 'j+1' and details in three orientations (Horizontally, Vertically and diagonally). Since the noisy components are of high

frequency, the three higher bands may contain the noisy components, and proper threshold may be applied to smooth the noisy wavelet coefficients followed by the inverse 2D-DWT may be applied to reconstruct the denoised image. Selection of optimal threshold is crucial for the performance of denoising algorithm. Threshold is selected based on the image and noise priors such as mean and variance [10]. Selection of optimal threshold along with various types of wavelet threshold methods is presented in the next section.

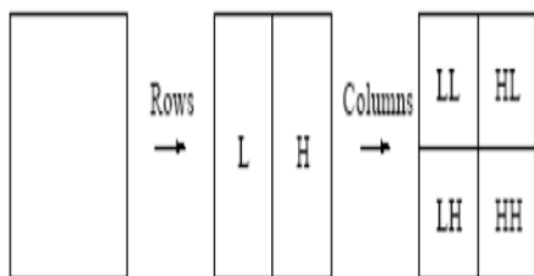


Fig.1 One DWT decomposition step

## 2. Literature survey

In “Image Denoising in the wavelet domain using wiener filtering”, the author Jacob and Martin[33] explained the importance of wavelet transforms in the field of image processing. Donoho presented thresholding of wavelet coefficients method for image denoising. The authors projected a new method based on the wiener filtering the wavelet coefficients. The results obtained surpass those of the other methods. They further discussed about the improvement along the edges and better optimality criteria similar to Mean Square Error.

In the paper “Adaptive wavelet thresholding for image denoising and compression”, the author S. Grace Chang et al, [34][35] proposed an adaptive

thresholding algorithm for image denoising. The experimental results illustrate excellence in performance. Then a specific coder is designed for simultaneous compression and denoising. The proposed Bayes shrink threshold also removes noise. Adaptive thresholding method with wavelet transform can be extended to other transform domains such as Discrete Cosine Transform (DCT), which also relies on energy compaction. The author demonstrated that a spatially adaptive thresholding method improves greatly in denoising over other thresholds.

In the papers “Translation-Invariant Denoising Using Multi wavelets”, and “Multi wavelets Denoising Using Neighboring Coefficients”, the authors Tien D.Bui and Guangyi Chen implemented signal denoising using Translation Invariant(TI) multi wavelets. Instead of using univariate thresholding they have experimented with multivariate thresholding and showed that the Translation invariant multi wavelet denoising gives excellent results than conventional translation invariant univariate wavelet denoising. They suggested carrying out the future work by selecting a better threshold value in multiwavelet denoising scheme.

In the paper “Image processing with complex wavelets”, the author Kingsbury discussed about the usage of wavelets for multi resolution for image processing and filter bank implementation of DWT, the perfect reconstruction conditions, problems with common wavelets like a shift dependencies , poor directional selectivity etc., Introduction of complex wavelets and its properties, Dual Tree Complex Wavelet Transform, its filter design and applications of complex

Wavelet Transform like Denoising, restoration, texture modelling, steerable filtration, registration, object segmentation image classification, video processing etc.

In the paper “A new complex-directional wavelet transform and its application to Image Denoising”, Ivan W. Selesnick describes a new expansive, perfect reconstruction complex-directional discrete wavelet transform based on the complex dual tree DWT. He also explained a simple subband dependent data driven denoising procedure in which the magnitudes of the complex wavelet coefficients are processed by soft thresholding.

In the Paper, “Complex wavelet transforms with allpass filters”, the authors Felix C.A. Fernandes et al, explained the advantages of Complex Discrete Wavelet Transforms such as reduced shift sensitivity improved directional selectivity and explicit phase information. They also discussed about the projection based approach to complex DWTs, which allows low redundancy and non-redundant complex DWTs implementation in the areas of image and video compression.

In the paper “Elimination Noise by Adaptive Threshold”, the author Iman Elyasi et al, proposed various methods for image denoising with the adaptive thresholding. This adaptive threshold removes additive Gaussian noise and under low noise conditions Normal Shrink gives better results. Modified Bayes Shrink gives better results under high noise conditions because it has minimum mean square error and maximum signal to noise ratio.

In the paper, “Image Denoising Using Block thresholding”, the author Zhou

Dengwen et al, Proposed an easy method Block Shrink for image denoising. It is based on the block thresholding which utilises the information of neighbor wavelet coefficients. They compared Block Shrink with Sure Shrink and Neigh Shrink and the investigational results proved that the high PSNR can be obtained.

In the paper “Image Denoising using wavelet thresholding”, the author Kaur proposed an adaptive subband threshold to recover from the noisy data. The image denoising algorithm uses soft thresholding to provide edge preservation and smoothness. The experimental results showed that better performance compared to Sure shrink, Oracle Shrink, Bayes Shrink and Wiener. They have suggested that their proposed method may be extended to another application like compression.

### 3. Proposed method

#### 3.0 Wavelet Thresholding

Let  $A = \{A_{ij}, i, j = 1, 2, \dots, M\}$  denote the  $M \times M$  matrix of the original image to be recovered, where  $M$  is some integer power of 2. During transmission the image is corrupted by independent, white Gaussian Noise  $Z_{ij}$  with standard deviation  $\sigma$  i.e.  $n_{ij} \sim N(0, \sigma^2)$  and at the receiver end, the noisy observations  $B_{ij} = A_{ij} + Z_{ij}$  is obtained. The goal is to estimate the signal  $A$  from noisy observations  $B_{ij}$  such that Mean Squared error (MSE) is minimum. Let  $W$  and  $W^{-1}$  denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then  $Y = WB$  represents the matrix of wavelet coefficients of  $B$  having four sub bands (LL, LH, HL and HH).

The sub-bands HHk, HLk, LHk are called details, where k is the scale varying from 1, 2 ..... J and J is the total number of decompositions. The size of the sub band a scale k is  $N/2k \times N/2k$ . The sub band LLJ is the lower resolution residue. The wavelet thresholding denoising method processes each coefficient of Y from the detail sub bands with a soft threshold function to obtain X . The denoised estimate is inverse transformed to  $A= W^{-1} X$ . In the experiments, soft thresholding has been used over hard thresholding because it gives more visually pleasant images as compared to hard thresholding; reason being the latter is discontinuous and yields abrupt artifacts in the recovered images especially when the noise energy is significant.

### 3.1. Sure Shrink

Sure Shrink is a thresholding by applying sub-band adaptive threshold, a separate threshold is computed for each detail sub-band based upon SURE (Stein's unbiased estimator for risk), The goal of Sure Shrink is to minimize the mean squared error, defined as,

$$MSE = \frac{1}{n^2} \sum_{X,Y-1}^n (Z(X,Y) - S(X,Y))^2$$

Where Z(X,Y) is the estimate of the signal, S(X,Y) is the original signal without noise and n is the size of the signal. Sure Shrink suppresses noise by threshold the empirical wavelet coefficients. The Sure Shrink threshold  $t^*$  is defined as

$$t^* = \min(t, \sigma\sqrt{2\log n})$$

Where t denotes the value that minimizes Stein's Unbiased Risk Estimator,  $\sigma$  is the

noise variance and an estimate of the noise level  $\sigma$  was defined based on the median absolute deviation given by

$$\hat{\sigma} = \frac{\text{median}(\{|g_{j-1,k}|: k = 0,1 \dots 2^{j-1} - 1\})}{0.6745}$$

and n is the size of the image. It is smoothness adaptive, which means that if the unknown function contains abrupt changes or boundaries in the image, the reconstructed image also does.

### 3.2. Bivariate Shrinkage

New shrinkage function which depends on both coefficient and its parent yield improved results for wavelet based image denoising. Here, we modify the Bayesian estimation problem as to take into account the statistical dependency between a coefficient and its parent.

Let  $w_2$  represent the parent of  $w_1$  ( $w_2$  is the wavelet coefficient at the same position as  $w_1$ , but at the next coarser scale.) Then,

$$y_1 = w_1 + n_1$$

$$y_2 = w_2 + n_2$$

Where  $y_1$  and  $y_2$  are noisy observations of  $w_1$  and  $w_2$  and  $n_1$  and  $n_2$  are noise samples. Then we can write

$$Y = w + n$$

$$y = (y_1, y_2)$$

$$w = (w_1, w_2)$$

$$n = (n_1, n_2)$$

Standard MAP estimator for w given corrupted y is



$$\hat{w}(y) = \text{argmax}_w P_{w/y}(W/y)$$

This equation can be written as

$$\hat{w}(y) = \text{argmax}_w [P_{w/y}(W/y) \cdot P_w(w)]$$

$$\hat{w}(y) = \text{argmax}_w [P_N(y - W) \cdot P_w(w)]$$

According to bayes rule allows estimation of coefficient can be found by probability densities of noise and prior density of wavelet coefficient. We assume noise is Gaussian then we can write noise as

$$P_n(n) = \frac{1}{2\pi\sigma_n^2} * \exp(-n_1^2 + n_2^2 + 2\sigma_n^2)$$

Joint of wavelet coefficients

$$P_w(w) = \frac{3}{2\pi\sigma_n^2} * \exp\left(-\sqrt{3\sqrt{W1^2} + W2^2}\right)/\sigma$$

We know from above equation

$$\hat{w}(y) = \text{argmax}_w [\log(P_N(y - W)) + \log(P_w(w))]$$

Let us define  $f(w) = \log(P_w(w))$

$$\hat{w}(y) = \text{argmax}_w \left[ -\frac{(y_1 - w_1)^2}{2\sigma_n^2} - \frac{(y_2 - w_2)^2}{2\sigma_n^2} + f(w) \right]$$

This equation is equivalent to solving following equations

$$y_1 - \frac{\hat{w}_1}{\sigma_n^2} + f_1(\hat{w}) = 0 \quad y_2 - \frac{\hat{w}_2}{\sigma_n^2} + f_2(\hat{w}) = 0$$

Where  $f_1$  and  $f_2$  represents the derivatives of  $f(w)$  with respect to  $w_1$  and  $w_2$  respectively. We know can be written as

$$\begin{aligned} f(w) &= \log(P_w(w)) \\ &= \log\left(\frac{3}{2\pi\sigma_n^2}\right) \\ &\quad * \exp\left(-\sqrt{3\sqrt{W1^2} + W2^2}\right)/\sigma \\ &= \log\left(\frac{3}{2\pi\sigma_n^2}\right) \\ &\quad - \left(-\sqrt{3\sqrt{W1^2} + W2^2}\right)/\sigma \end{aligned}$$

### 3.3. Neigh Shrink

Neigh Shrink thresholds the wavelet coefficients according to 3th magnitude of the squared sum of all the wavelet coefficients, i.e., the local energy, within the neighborhood window. The neighborhood window size may be, 3×3, 5×5, 7×7, 9×9 etc. A 3×3 neighboring window centered at the wavelet coefficient to be shrinked is shown in Fig 1.

In the spatial domain, it is well known that an adaptive Wiener method based on estimation from local information is very efficient for digital image enhancement. In the wavelet domain, despite the de-correlating properties of the wavelet transform, as pointed out in the introduction, there still exist significant residual statistical dependencies between neighbor wavelet coefficients. Our goal is to exploit this dependency to improve the estimation of a coefficient given its noisy observation and a context One of the simplest wavelet shrinkage rules for an N x N image is the universal threshold

$$\lambda = \sqrt{2\sigma^2 \log N^2}$$

The universal threshold grows asymptotically and removes more noise coefficients as  $N$  tends to infinity. The universal threshold is designed for smoothness rather than for minimizing the errors. So  $\lambda$  is more meaningful when the signal is sufficiently smooth or the length of the signal is close to infinity. Natural image, however, is usually neither sufficiently smooth nor composed of infinite number of pixels. In fact, if we suppose that an optimal threshold which minimize MSE (or maximized PSNR),  $\lambda$  is  $\alpha \lambda$ ,  $\alpha$  is always much less than 1.0 for natural image. Especially we got very similar  $\alpha$  value for different kinds and size of images when we applied soft thresholding rule.

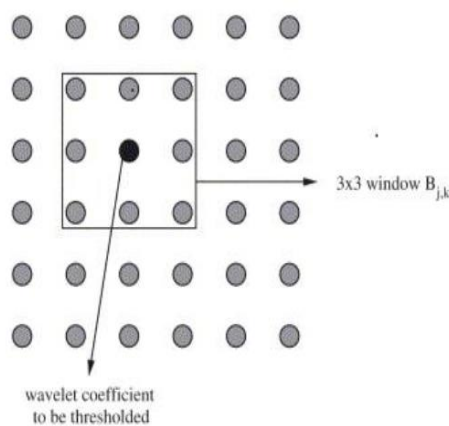


Fig.1. An illustration of the neighboring window of size  $3 \times 3$  centered at the wavelet coefficient to be shrunk. The shrinkage function for Neigh Shrink of any arbitrary  $3 \times 3$  window centered at  $(i,j)$  is expressed as:

$$\Gamma_{i,j} = \left[ 1 - \frac{T_U^2}{S_{ij}^2} \right] +$$

Where  $T_U$  is the universal threshold and  $S_{ij}^2$  is the squared sum of all wavelet coefficients in the given window.

Here very important consideration is “+” sign at the end of the formula it means keep the positive values while setting it to zero when it is negative. The estimated center wavelet coefficient is then calculated from its noisy counterpart  $Y_{ij}$  as:

$$\widehat{F}_{ij} = \Gamma_{ij} \cdot Y_{ij}$$

### 3.4. Block Shrink

Block Shrink is a data-driven block thresholding approach. It use the pertinence of the neighbor wavelet coefficients by using the block thresholding. It can decide the optimal block size and threshold for every wavelet subband by minimizing Stein’s unbiased risk estimate (SURE). The block thresholding simultaneously keeps or kills all the coefficients in groups rather than individually. The block thresholding increases the estimation precision by utilizing the information about the neighbor wavelet coefficients. Unfortunately, the block size and threshold level play important roles in the performance of a block thresholding estimator. The local block thresholding methods mentioned above all have the fixed block size and threshold and same thresholding rule is applied to all resolution levels regardless of the distribution of the wavelet coefficients.

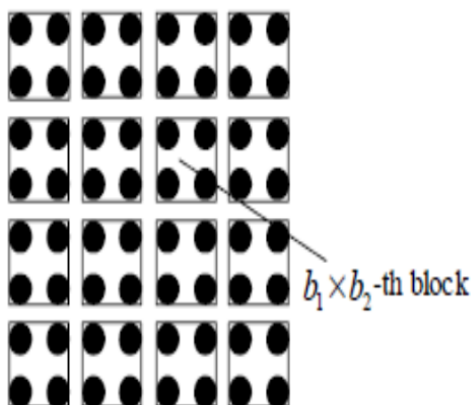


Figure 2:  $2 \times 2$  Block partition for a Wavelet subband As shown in Figure 2, there are a number of subbands produced when we perform wavelet decomposition on an image. For every subband, we need to divide it into a lot of square blocks.

Block Shrink can select the optimal block size and threshold for the given subband by minimizing Stein's unbiased risk estimate.

### Results and analysis

For the above mentioned three methods, image de-noising is performed using wavelets from the second level to fourth level decomposition and the results are shown in figures and table if formulated for second level decomposition for different noise variance as follows. It was found that three level decomposition and fourth level decomposition gave optimum results. However, third and fourth level decomposition resulted in more blurring. The experiments were done using a window size of  $3 \times 3$ ,  $5 \times 5$  and  $7 \times 7$ . The neighborhood window of  $3 \times 3$  and  $5 \times 5$  are good choices

Original Image



Fig 1: Input image

Noisy Image



Fig 2: Add noise to the input image

SURE Shrink image



Fig 3: Sure Shrink image



**Bivariate Shrinkage image**



**Fig 4 : Bivariate shrink image**

**Block Shrink Image**



**Fig 5: Block shrink image**

**NeighShrink Image**



**Fig 6: Neigh shrink image**

**Table for PSNR &MSE**

Method	PSNR	MSE
SURE Shrink image	23.6370	0.0043

Bivariate Shrinkage image	68.6184	1.3745e-07
Block Shrink Image	26.9144	0.0020
NeighShrink Image	26.1817	0.0024

From above table we observed that quality and noise ration of image in different methods

### Conclusion

In this paper, the image de-noising using discrete wavelet transform is analyzed. The experiments were conducted to study the suitability of different wavelet bases and also different window sizes. Among all discrete wavelet bases, coiflet performs well in image de-noising. Experimental results show that Bivariate Shrink Method gives better result than Sure Shrink, Bayes Shrink and Neigh Shrink methods when applied on series of images A lot of combinations have been applied in order to find the best method that can be followed for denoising intensity images. From the PSNR and MSE values as shown in tables, it is clear that Bivariate Shrinkage giving better results under different noise variance conditions for all of the images.

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