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Bianchi Type I (Kasner form) Cosmological models with Perfect Fluid and Dark Energy in Modified Theory of Gravity

A.Y.Shaikh, K.S.Wankhade*
Department of Mathematics, Indira Gandhi Mahavidyalaya, Ralegaon-445402.(M.S.)India.

*Department of Mathematics, Y.C.Science College, Mangrulpir.(M.S.)India.

e-mail: shaikh_2324ay@yahoo.com

Abstract: A self consistent system of Bianchi type-I space-time in Kasner form gravitational field and a binary mixture of perfect fluid and dark energy in a modified theory of gravity are considered. The perfect fluid is taken to be the one obeying the usual equation of state, i.e., $p = \gamma\rho$, with $\gamma \in [0, 1]$ whereas, the dark energy is considered to be either the quintessence like equation of state or Chaplygin gas. The exact solutions to the corresponding field equations are obtained for power-law and exponential volumetric expansion. The geometrical and physical parameters for both the models are studied.

Keywords: Dark energy, Perfect fluid, modified gravity, Bianchi type-I space-time in Kasner form .

1. Introduction

Recent data from distant supernovae Ia (Perlmutter et.al.(1998,1999), Riess et.al.(1998,2004), Kowalski et al. (2008)) are explained as guide of overdue time acceleration in the enlargement average of our universe. According to these observations, nearly 70% of the total energy density of the universe has a large negative pressure which is apparently unclustered and is dubbed as Dark energy (DE), which will in fact cause the cosmic expansion to speed up. Enormous DE proposals including k-essence, Chaplygin gas, quintessence, quintom, phantom, cosmological constant etc. have been suggested by (Al-Rawaf and Taha (1996), Caldwell, R.R. et. al(1998)). Cosmological studies of dark energy are mostly on to extract properties of a dark energy that focus on the determination of its equation of state. Several modified theories of gravity have been developed and studied, in view of the late time acceleration of the Universe and the existence of dark energy and dark matter.

Modify gravity is of great importance because it can successfully explain the rotation curve of galaxies and the motion of galaxy clusters in the universe. There are various modify gravity namely $f(R)$, $f(G)$, $f(R,G)$, $f(T)$ and $f(R,T)$ theory of gravity. Recently, Harko et al. (2011) developed a $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor. The $f(R,T)$ gravity model depends on a supply term, representing the variation of the matter stress energy tensor with regard to the metric. A general expression for this supply term is obtained as a operate of the matter Lagrangian L_m in order that every selection of L_m would generate a particular set of field equations.

Point like Lagrangian's for $f(R,T)$ gravity had been presented by Myrzakulov (2011). The $f(R,T)$ gravity model that satisfies the local tests and transition of matter from dominated era to accelerated phase was considered by Houndjo (2011). Adhav (2012) has obtained LRS Bianchi type I cosmological model in $f(R,T)$ gravity. Bianchi type III cosmological model in

$f(R,T)$ gravity have been discussed by Reddy et al. (2012a). Reddy et al. (2012b) investigated a five dimensional Kaluza-Klein space-time in the presence of a perfect fluid source in $f(R,T)$ theory of gravitation with negative constant deceleration parameter. The new class of cosmological model of the early Universe is considered with $f(R,T)$ modified theories of gravity has been discussed by Chaubey and Shukla (2013). The Einstein-Rosen space-time filled with perfect fluid in the framework of $f(R,T)$ gravity has been studied by Rao and Neelima (2013). Shri Ram and Priyanka (2013) presented some new classes of five dimensional Kaluza-Klein cosmological models in the presence of a perfect fluid source in $f(R,T)$ gravity theory. The exact solutions of the field equations in respect of Kantowski-Sachs universe filled with perfect fluid in the framework of $f(R,T)$ theory of gravity has been derived by Samanta (2013). Samanta and Dhal (2013) have studied higher dimensional spherical symmetric cosmological model in the presence of perfect fluid distribution in $f(R,T)$ theory of gravity. Yadav (2013) established the existence of Bianchi-V string cosmological model in $f(R,T)$ gravity. The dark energy model with EoS parameter are derived for Kantowaski-Sachs space-time filled with perfect fluid source in the frame work of $f(R,T)$ gravity by Katore and Shaikh (2012). Sahoo et al. (2014) and Ahmed and Pradhan (2014) studied the axially symmetric and Bianchi type V cosmological models in $f(R,T)$ gravity respectively. Rao et al.(2013) have studied perfect fluid cosmological models in general relativity and $f(R,T)$ gravity. Rao et al. (2013) have obtained cosmological models in a modified theory of gravity. The dark energy model with EoS parameter is derived by Shaikh and Wankhade (2015) for hypersurface-homogenous space-time filled with perfect fluid source in the frame work of $f(R,T)$ gravity.Shaikh and Bhoyar (2015) studied Plane symmetric cosmological models in $f(R,T)$ theory of gravity with a term Λ. A self consistent system of Plane Symmetric gravitational field and a binary mixture of perfect fluid and dark energy in a modified theory of gravity are considered by Shaikh (2016). The exact solutions of the field equations with respect to hypersurface-homogeneous Universe filled with perfect fluid in the framework of $f(R,T)$ theory of gravity is derived by Shaikh and Katore (2016).A spatially homogeneous and anisotropic Hypersurface-Homogenous cosmological model when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic string in $f(R,T)$ gravity, using Hybrid Expansion Law which exhibits a transition of the universe from decelerating phase to the present accelerating phase has been investigated by Shaikh (2016).Shaikh and Wankhade (2017) investigated Hypersurface-Homogeneous cosmological model in $f(R,T)$ theory of gravity with a term Λ.

A self consistent system of Bianchi type-I gravitational field and a binary mixture of perfect fluid and dark energy have been discussed by B. Shah (2006). Singh and Chaubey (2009) has considered Bianchi type-V model of the universe with a binary mixture of perfect fluid and dark energy. Katore et. al.(2013) studied Kaluza-Klein cosmological models with binary mixture of perfect fluid and dark energy . Katore et al. (2011a,2011b) have also considered cosmological models of the universe with a binary mixture of perfect fluid and dark energy. Adhav et al.(2011) studied the Bianchi type-V cosmological model with a binary mixture of perfect fluid and dark energy in higher dimensions. Tade and Sambhe (2012) studied Bianchi type-I cosmological models for binary mixture of perfect fluid and dark energy in general relativity. Kumar Suresh and Akarsu

Ozgur (2012) studied the spatially homogeneous but totally anisotropic and non-flat Bianchi type-II cosmological model in General Relativity in the presence of two minimally interacting fluids; a perfect fluid as the matter fluid and a hypothetical anisotropic fluid as the dark energy fluid. General Bianchi type-I cosmological models which containing a perfect fluid and dark energy with time varying G and Λ have been investigated by Fayaz, V. et al. (2012). G.C. Samanta and S.N. Dhala (2013) investigated a spatially homogeneous and anisotropic Bianchi type-V space time in $f(R,T)$ theory of gravity, where the universe is filled with perfect fluid and dark energy. Evolution of Hypersurface-Homogenous cosmological models is studied by Shaikh (2016) in the presence of dark energy (DE) from a wet dark fluid (WDF) in f(R,T) theory of gravity.

Motivating with on top of analysis work, in this paper Bianchi type I space time in Kasner form gravitational field and a binary mixture of perfect fluid and dark energy in $f(R,T)$ theory of gravity has been considered. The perfect fluid has equation of state $p=\gamma\rho$ with $\gamma \in [0, 1]$ and also the dark energy is given by either a quintessence or a Chaplygin gas. Precise solutions are obtained for the cases of power-law and exponential expansion.

2. Gravitational field equations of $f(R,T)$ gravity

The $f(R,T)$ theory of gravity is the modification of General Relativity (GR). The field equations are derived from a variational Hilbert-Einstein type, principle.

The action principle of modified gravity is given by

$$S=\frac{1}{16\pi G}\int\sqrt{-g}\; f(R,T)\,d^4x+\int\sqrt{-g}\;L_m d^4x, \quad (1)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T) of the matter T_{ij} $(T=g^{ij}T_{ij})$ and L_m is the matter Lagrangian density.

The stress energy tensor of matter is defined as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}. \quad (2)$$

Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} and not on its derivatives, in this case, it yields

$$T_{ij}=g_{ij}L_m-2\frac{\delta(L_m)}{\delta g^{ij}}. \quad (3)$$

By varying the action S with respect to the metric tensor components g_{ij}, the field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+f_R(R,T)\left(g_{ij}\nabla^i\nabla_i-\nabla_i\nabla_j\right)=8\pi T_{ij}-f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij},$$
(4)

where

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\alpha\beta}\frac{\partial^2 L_m}{\delta g^{ij}\partial g^{\alpha\beta}}. \quad (5)$$

Here $f_R=\frac{\delta\, f(R,T)}{\delta R}$, $f_T=\frac{\delta\, f(R,T)}{\delta T}$, $\Theta_{ij}=g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ and ∇_i is the covariant derivative and from the Lagrangian L_m, the standard matter energy momentum tensor T_{ij} is derived. One should note that when $f(R,T)=f(R)$ then (4) reduces to the field equations of $f(R)$ gravity.

Contracting Eq. (4), it gives relation between Ricci scalar R and T the trace of the energy momentum tensor as follow

$$f_R(R,T)R+3\Pi f_R(R,T)-2f(R,T)=(8\pi-f_T(R,T))T - f_T(R,T)\Theta$$

with $\Theta=g^{ij}\Theta_{ij}$. (6)

Using matter Lagrangian L_m the stress energy tensor of the matter is given by

$$T_{ij}=(p+\rho)u_i u_j-pg_{ij}, \quad (7)$$

where $u^i = (0,0,0,1)$ denotes the four velocity vector in co-moving coordinates which satisfies the condition $u^i u_i = 1$. The problem of the proper fluids delineate by energy density ρ , pressure p and matter Lagrangian may be taken as $L_m = -p$, since there's no distinctive definition of the matter Lagrangian.
The variation of stress energy of perfect fluid has the following expression

$$\Theta_{ij} = -2T_{ij} - pg_{ij} . \quad (8)$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij}. Several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. (2011) gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} . \quad (9)$$

In this paper, it is focused to the first class $f(R,T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of tress energy tensor of the form $f(T) = \lambda T$ where λ is constant.
For this choice the gravitational field equations of $f(R,T)$ gravity becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} + f(T)g_{ij}, \quad (10)$$

where the prime denotes differentiation with respect to the argument. The field equations (in view of Eq. (8)) in presence of matter as perfect fluid becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} . \quad (11)$$

3. Field equations of Binachi Type I in Kasner form

Consider anisotropic [Bianchi type I] metric in Kasner form

$$ds^2 = dt^2 - t^{2q_1}dx^2 - t^{2q_2}dy^2 - t^{2q_3}dz^2, (12)$$

where q_1, q_2, q_3 are three parameters that require to be constant . Let $S = q_1 + q_2 + q_3$, $\theta = q_1^2 + q_2^2 + q_3^3$, we get $R = (S^2 - 2S + \theta)t^{-2}$.

With the choice of the function $f(T)$ of the trace of the stress–energy tensor of the matter so that

$$f(T) = \lambda T , \quad (13)$$

where λ is a constant (Harko et. al. (2011)).
Using comoving coordinates and equations (7)–(8) and (13), the $f(R,T)$ gravity field equations, (11), for metric (12) can be written as

$$\left[q_1(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = (8\pi + 3\lambda)p - \rho\lambda, \quad (14)$$

$$\left[q_2(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = (8\pi + 3\lambda)p - \rho\lambda, \quad (15)$$

$$\left[q_3(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = (8\pi + 3\lambda)p - \rho\lambda, (16)$$

$$\left[(s-\theta) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = -(8\pi + 3\lambda)\rho + p\lambda, \quad (17)$$

where a dot here in after denotes ordinary differentiation with respect to cosmic time "t" only.

4. Isotropization and the solution

Define $R = \left(t^{q_1}t^{q_2}t^{q_3}\right)^{\frac{1}{3}}$ as the average scale factor so that the Hubble parameter in our anisotropic model may be defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\sum_{i=1}^{3} H_i \quad (18)$$

where R is the mean scale factor and $H_i = \frac{\dot{R}_i}{R_i}$ are directional Hubble's factors in the direction of x^i respectively.
The anisotropy parameter of the expansion Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \qquad (19)$$

in the x, y, z directions, respectively. In deciding whether the model is isotropic or anisotropic , the mean anisotropic parameter of the expansion Δ has a very crucial role . It is the measure of the deviation from isotropic expansion, the universe expands isotropically when $\Delta = 0$.

Let us introduce the dynamical scalars, such as expansion parameter (θ) and the shear (σ^2) as usual

$$\theta = 3H \qquad (20)$$

$$\sigma^2 = \frac{3}{2}\Delta H^2 \qquad (21)$$

Using equations (14) and (15) , it yields

$$(q_1 - q_2)(s-1)t^{-2} = 0. \qquad (22)$$

Equations (22) can be written as

$$\frac{d}{dt}\left(\frac{q_1}{t} - \frac{q_2}{t}\right) + \left(\frac{q_1}{t} - \frac{q_2}{t}\right)\frac{s}{t} = 0. \qquad (23)$$

Let V be a function of t defined by

$$V = t^{(q_1+q_2+q_3} = t^s. \qquad (24)$$

Then from equation (24), equation (23) gives

$$\frac{d}{dt}\left(\frac{q_1}{t} - \frac{q_2}{t}\right) + \left(\frac{q_1}{t} - \frac{q_2}{t}\right)\frac{\dot{V}}{V} = 0. \qquad (25)$$

Integrating the above equation, we get

$$\frac{q_1}{q_2} = d_1 \exp\left(x_1 \int \frac{1}{V} dt\right), \qquad (26)$$

where d_1 and x_2 are constants of integrations .

In view of $V = t^s$, we write $t^{q_1}, t^{q_2}, t^{q_3}$ in the explicit form

$$t^{q_1} = D_1 V^{\frac{1}{3}} \exp\left(X_1 \int \frac{1}{V} dt\right), \qquad (27)$$

$$t^{q_2} = D_2 V^{\frac{1}{3}} \exp\left(X_2 \int \frac{1}{V} dt\right), \qquad (28)$$

$$t^{q_3} = D_3 V^{\frac{1}{3}} \exp\left(X_3 \int \frac{1}{V} dt\right), \qquad (29)$$

where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Since field equations (14)–(17) are four equations having five unknowns and are highly nonlinear, an extra condition is needed to solve the system completely. Here two different volumetric expansion laws are used ,i.e.

$$V = at^b \qquad (30)$$

and

$$V = \alpha e^{\beta t}, \qquad (31)$$

where a, b, α, β are constants. In this way, all possible expansion histories, the power law expansion, (30), and the exponential expansion, (31) have been covered.

5. Universe as a binary mixture of Perfect fluid and Dark Energy

The evolution of the Bianchi type I space time in Kasner form crammed with perfect fluid and Dark Energy has been accounted. Taking under consideration that the energy density ρ and pressure p within the case comprise those of perfect fluid and dark energy .

$$\rho = \rho_{PF} + \rho_{DE}, \quad p = p_{PF} + p_{DE}. \qquad (32)$$

The energy momentum tensor can be decomposed as

$$T_i^j = (\rho_{PF} + \rho_{DE} + p_{PF} + p_{DE})u^j u_i - (p_{DE} + p_{PF})\delta_i^j. \qquad (33)$$

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In the above equation ρ_{DE} is the dark energy density, p_{DE} its pressure. Conjointly the notations ρ_{PF} and p_{PF} is introduced to denote the energy density and the pressure of the perfect fluid respectively. Here the perfect fluid obeys the subsequent equation of state

$$p_{PF} = \gamma \rho_{PF}. \quad (34)$$

Here γ is a constant and lies in the interval $\gamma \in [0,1]$.

Depending on its numerical value γ describes the following type of Universe

$$\gamma = 0 \text{ (Dust Universe)}, \quad (35)$$

$$\gamma = \frac{1}{3} \text{ (Radiation Universe)}, \quad (36)$$

$$\gamma \in (\frac{1}{3}, 1) \text{ (Hard Universe)}, \quad (37)$$

$$\gamma = 1 \text{ (Zel'dovich Universe or Stiff matter)}. \quad (38)$$

In a co-moving frame the conservation law of energy momentum tensor results in the balance equation for the energy density

$$\dot{\rho}_{DE} + \dot{\rho}_{PF} = -\frac{\dot{V}}{V}(\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF}). \quad (39)$$

The dark energy is meant to act with itself solely and it's minimally coupled to the field. As a result the evolution equation for the energy density decouples from that of the right fluid and from equation (39), two balance equations are obtained.

$$\dot{\rho}_{DE} + \frac{\dot{V}}{V}(\rho_{DE} + p_{DE}) = 0, \quad (40)$$

$$\dot{\rho}_{PF} + \frac{\dot{V}}{V}(\rho_{PF} + p_{PF}) = 0. \quad (41)$$

Equations (34) and (41) yield

$$\rho_{PF} = \frac{\rho_0}{V^{(1+\gamma)}}, \quad p_{PF} = \frac{\rho_0 \gamma}{V^{(1+\gamma)}}, \quad (42)$$

where ρ_0 is an integration constant.

6. Models with a Quintessence and Chaplygin gas

6.1 Case with a Quintessence

Let us take into account the case once the dark energy is given by a Quintessence that obeys the equation of state

$$p_q = w_q \rho_q, \quad (43)$$

where the constant w_q varies between -1 and zero i.e. $w_q \in [-1,0]$.

From equations (40) and (43), it yields

$$\rho_q = \frac{\rho_{0_q}}{V^{1+w_q}}, \quad p_q = \frac{w_q \rho_{0_q}}{V^{1+w_q}}, \quad (44)$$

with ρ_{0_q} being an integration constant.

6.2. Case with Chaplygin gas

Let us currently take into account the case once the dark energy is diagrammatic by Chaplygin gas

$$p_c = -\frac{\alpha}{\rho_c}, \quad (45)$$

with α being a positive constant.

From equations (40) and (45), it yields

$$\rho_c = \sqrt{\frac{\rho_{0_c}}{V^2} + \alpha}, \quad p_c = \frac{-\alpha}{\sqrt{\frac{\rho_{0_C}}{V^2} + \alpha}} \quad (46)$$

with ρ_{0c} being an integration constant.

7. Model for power law

Using (30) in (27) -(29), the scale factors are obtained as follows:

$$t^{q_1} = D_1 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{\frac{X_1}{a(1-b)} t^{1-b}\right\}, \quad (47)$$

$$t^{q_2} = D_2 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{\frac{X_2}{a(1-b)} t^{1-b}\right\}, \quad (48)$$

$$t^{q_3} = D_3 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{\frac{X_3}{a(1-b)} t^{1-b}\right\}, \quad (49)$$

where $D_i(i=1,2,3)$ and $X_i(i=1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

It reveals that near $t=0$, the scale factor vanishes. Therefore, the model has an initial singularity.

The mean Hubble's parameter, *H*, is given by

$$H = \frac{b}{3t}. \tag{50}$$

The anisotropic parameter is given by

$$\Delta = \frac{3X^2}{a^2 b^2 t^{2(b-1)}}. \tag{51}$$

The dynamical scalars are given by

$$\theta = \frac{b}{t}. \tag{52}$$

$$\sigma^2 = \frac{X^2}{2a^2 t^{2b}}, \tag{53}$$

where $X^2 = 2X_1^2 + X_2^2 = \text{constant}$.

The deceleration parameter

$$q = \frac{3}{b} - 1. \tag{54}$$

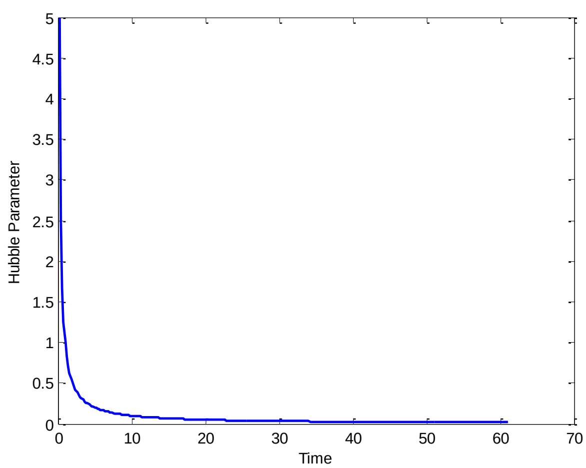

Figure No. 1. Hubble parameter vs Time.

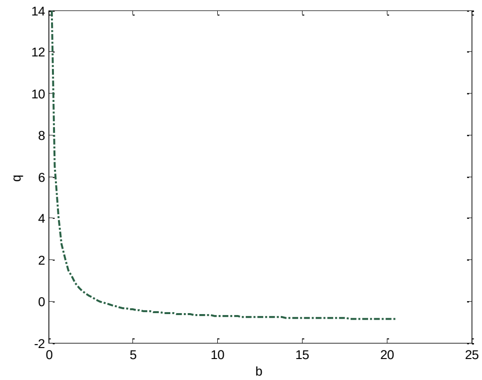

Figure No 2 Deceleration Parameter vs *b*.

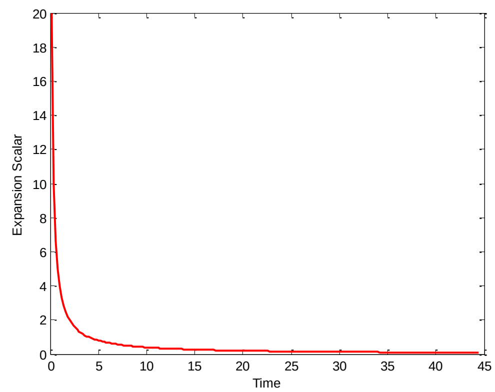

Figure No. 3. Expansion Scalar vs Time.

It is observed that the Hubble parameter, Expansion Scalar and Shear Scalar are very large at an initial epoch and finally tends to zero as $t \to \infty$. For all positive values of *b*, the volume of the universe expands indefinitely. The universe was anisotropic at early stage of evolution and approach to isotropy at large time is cleared from equation (51). The positive deceleration parameter corresponds to a decelerating model while the negative value provides inflation. For $b > 3$ the deceleration parameter is negative i.e. the universe is fast that is in agreement with current observations of SNe Ia and CMB as shown in figure 2.

Using equation (30) in (42), (44) and (46), it yields

$$\rho_{PF}=\frac{\rho_0}{(at^b)^{(1+\gamma)}}\ ,\quad p_{PF}=\frac{\rho_0\gamma}{(at^b)^{(1+\gamma)}}\ . \tag{55}$$

$$\rho_q=\frac{\rho_{0_q}}{(at^b)^{1+w_q}}\ ,\ p_q=\frac{w_q\rho_{0_q}}{(at^b)^{1+w_q}}\ . \tag{56}$$

$$\rho_c=\sqrt{\frac{\rho_{0_c}}{(at^b)^2}+\alpha}\ ,\ p_c=\frac{-\alpha}{\sqrt{\frac{\rho_{0_C}}{(at^b)^2}+\alpha}}\ . \tag{57}$$

8. Model for exponential law

Using (31) in (27) -(29), the scale factors are obtained as follows:

$$t^{q_1}=D_1\alpha^{\frac{1}{3}}e^{\frac{\beta t}{3}}\exp\left\{\frac{-X_1}{\alpha\beta}e^{-\beta t}\right\}, \tag{58}$$

$$t^{q_2}=D_2\alpha^{\frac{1}{3}}e^{\frac{\beta t}{3}}\exp\left\{\frac{-X_2}{\alpha\beta}e^{-\beta t}\right\}, \tag{59}$$

$$t^{q_3}=D_3\alpha^{\frac{1}{3}}e^{\frac{\beta t}{3}}\exp\left\{\frac{-X_3}{\alpha\beta}e^{-\beta t}\right\}, \tag{60}$$

where $D_i(i=1,2,3)$ and $X_i(i=1,2,3)$ satisfy the relation $D_1D_2D_3=1$ and $X_1+X_2+X_3=0$.

The scale factor are constant near $t=0$, afterwards start increasing with time and as $t\to\infty$, they diverges to infinity. Hence in this case, the volume of the universe is an exponential function which expands with increase in time from a constant to infinitely large. This is consistent with big bang scenario which resembles with Katore and Shaikh (2015).

The mean Hubble's parameter, *H*, is given by

$$H=\frac{\beta}{3}\ . \tag{61}$$

The anisotropy parameter of the expansion, Δ, is

$$\Delta=\frac{3X^2e^{-2\beta t}}{\alpha^2\beta^2}\ . \tag{62}$$

The expansion scalar, θ, is found as

$$\theta=\beta\ . \tag{63}$$

The shear scalar, σ^2, is found as

$$\sigma^2=\frac{X^2e^{-2\beta t}}{2\alpha^2}\ . \tag{64}$$

The deceleration parameter

$$q=-1, \tag{65}$$

where $X^2=2X_1^2+X_2^2=$ constant .

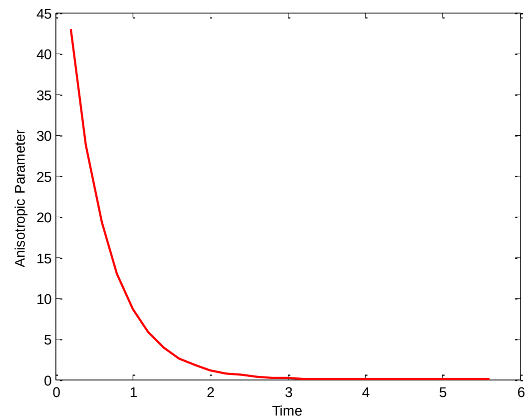

Figure No. 4. Anisotropic Parameter vs Time.

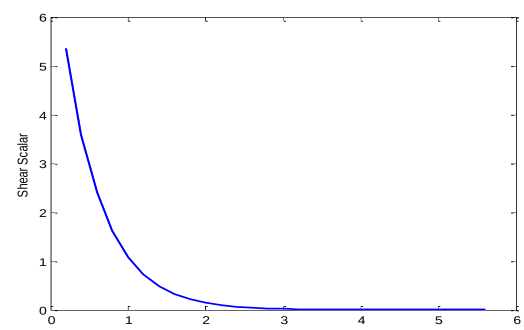

Figure No. 5. Shear Scalar vs Time.

The mean Hubble parameter is constant. The rate of expansion of the universe is constant for $\beta>0$. Thus, the universe evolves with constant rate of expansion. For large t , the model tends to be isotropic. At $t=0$, the anisotropy parameter is constant and decreases with time for $\beta_1>0$ as shown in figure 4. It means that the universe was anisotropic at early stage and approaching to isotropy as time increases. Here it is observed that this behaviour of anisotropic parameter is equivalent to the ones obtained for the model that correspond to the exponential expansion in Bianchi type –III cosmological model with anisotropic dark energy Akarsu and Kilinc (2010) .The Shear Scalar $\sigma\to 0$, as $t\to\infty$. Here from equation (65), it's ascertained that the deceleration parameter is negative. It is observed that the deceleration parameter $q=-1$ and $\frac{dH}{dt}=0$ for this model. Hence, it provides the best values of the Hubble parameter and also the quickest rate of growth of the universe. The model may represent the inflationary era in the early universe and the very late time of the universe.

Using equation (31) in (42), (44) and (46), it yields

$$\rho_{PF}=\frac{\rho_0}{(\alpha_1 e^{\beta_1 t})^{(1+\gamma)}},\ p_{PF}=\frac{\rho_0\gamma}{(\alpha_1 e^{\beta_1 t})^{(1+\gamma)}}.\quad(66)$$

$$\rho_q=\frac{\rho_{0_q}}{(\alpha_1 e^{\beta_1 t})^{1+w_q}},\ p_q=\frac{w_q\rho_{0_q}}{(\alpha_1 e^{\beta_1 t})^{1+w_q}}.\quad(67)$$

$$\rho_c=\sqrt{\frac{\rho_{0_c}}{(\alpha_1 e^{\beta_1 t})^2}+\alpha},\quad p_c=\frac{-\alpha}{\sqrt{\frac{\rho_{0_c}}{(\alpha_1 e^{\beta_1 t})^2}+\alpha}}.\quad(68)$$

9. Some Particular Cases:

Case i: For $\gamma=0, w_q=0$. From (55) , (56) ,(66) and (67) , it yields

$$\rho_{PF}=\frac{\rho_0}{(at^b)},\quad p_{PF}=0.\quad(69)$$

$$\rho_q=\frac{\rho_{0_q}}{(at^b)},\ p_q=0.\quad(70)$$

$$\rho_{PF}=\frac{\rho_0}{(\alpha_1 e^{\beta_1 t})},\quad p_{PF}=0.\quad(71)$$

$$\rho_q=\frac{\rho_{0_q}}{(\alpha_1 e^{\beta_1 t})},\ p_q=0.\quad(72)$$

Case ii: For $\gamma=1, w_q=0$. From (55) , (56) ,(66) and (67) , it yields

$$\rho_{PF}=\frac{\rho_0}{(at^b)^2}=p_{PF}.\quad(73)$$

$$\rho_q=\frac{\rho_{0_q}}{(at^b)},\ p_q=0.\quad(74)$$

$$\rho_{PF}=\frac{\rho_0}{(\alpha_1 e^{\beta_1 t})^2}=p_{PF}.\quad(75)$$

$$\rho_q=\frac{\rho_{0_q}}{(\alpha_1 e^{\beta_1 t})},\ p_q=0.\quad(76)$$

Case iii: For $\gamma=1, w_q=-1$. From (55) , (56) ,(66) and (67) , it yields

$$\rho_{PF}=\frac{\rho_0}{(at^b)^2}=p_{PF}.\quad(77)$$

$$\rho_q=\rho_{0_q},\ p_q=-\rho_{0_q}.\quad(78)$$

$$\rho_{PF}=\frac{\rho_0}{(\alpha_1 e^{\beta_1 t})^2}=p_{PF}.\quad(79)$$

10. Conclusions:

A self consistent system of Bianchi Type I space time in Kasner form with a binary mixture of perfect fluid and dark energy has been thought-about in modified theory of gravity. The precise solutions of the field equations are obtained by assuming two different volumetric expansion laws in a way to cover all possible expansion: specifically exponential and law expansion. Three fully completely different cases have put together been analyzed totally. The inclusion of dark energy in to the system provides rise to associate accelerated growth of the universe. As a result the initial property of the model quickly dies away. It's attention-grabbing to notice that obtained models correspond to the investigations of Singh and Chaubey (2009),Samanta and Dhal (2013) and Shaikh,A.Y.(2015).

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