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Bianchi Type I (Kasner form) with wet dark fluid in *f(R, T)* gravity

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Abstract: In this paper, Bianchi type I space time in Kasner form with wet dark fluid (WDF), which is a candidate for dark energy (DE), in the framework of *f(R, T)* gravity, *R* and *T* denote the Ricci scalar and the trace of the energy–momentum tensor, respectively (Harko et al. Phys. Rev. D, 84, 024020 (2011)) has been investigated. Equation of state in the form of WDF for the DE component of the universe is considered.It is modeled on the equation of state $p=\omega(\rho-\rho^*)$. The exact solutions to the corresponding field equations are obtained for power-law and exponential volumetric expansion. The geometrical and physical parameters for both the models are studied.

Keywords: Bianchi type I (Kasner form) , WDF , f(R,T) gravity .
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1. Introduction

The results of cosmological experiments: SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4], have provided the main evidences that the current universe is in accelerating expansion. A large majority of cosmological models explain the acceleration of the universe in terms of a component with the negative pressure, the so called dark energy. In view of the late time acceleration of the universe and the existence of the dark matter and dark energy, very recently, modified theories of gravity have been developed.

Different modified theories of gravitation area unit projected $f(R)$ gravity [5-8] and Gauss–Bonnet gravity or $f(G)$ gravity [9-12]. Another approach to modified gravity is so-called $f(T)$ gravity [13-15], where T is the scalar torsion. Harko et al. [16] developed modified gravity called $f(R,T)$ gravity. During this theory, the gravitative Lagrangian is given by associate degree whimsical operate of the Ricci scalar R and therefore the trace T of the strain energy tensor. It is to be observed that the dependence from T may be induced by exotic imperfect fluid or quantum effects. Several authors [17-39] studied different cosmological models in $f(R,T)$ theory of gravity .

The present paper is organized as follows. In Sect. 1, a brief introduction is given. A brief discussion about wet dark fluid (WDF) is given in Sect. 2. The field equations in metric version of f(R, T) gravity are given in Sect. 3. In Sect. 4, gravitational field equation in f(R, T) gravity is established with the aid of the Bianchi Type I space time in Kasner form in the presence of

WDF. In Sect. 5, the general discussion on the isotropization and solutions are given. Sections 6 and 7 deal with the cosmological model for the power law and exponential law of the volumetric expansion, respectively . Finally, conclusions are summarized in Sect. 8. Here we have used the natural system of units with G = c = 1.

2. Wet Dark Fluid (WDF).

WDF is a new candidate for DE in the script of generalized Chaplygin gas, where a physically motivated equation of state is offered with the properties relevant for a DE problem. The equation of state for a WDF is

$$\frac{p_{WDF}}{\omega} + \rho^* = \rho_{WDF}. \qquad (1)$$

Equation (1) is good approximation for many fluids, including water. The parameter ω and ρ^* are taken to be positive and we restrict ourselves to $0 \le \omega \le 1$. Note that if c_s denotes the adiabetic sound speed in WDF, then c_s^2 [40]. To find the WDF energy density, we use the energy conservation equation

$$\rho^*_{WDF} + 3H\left(p_{WDF} + \rho_{WDF}\right) = 0. \qquad (2)$$

From equation of state (1) and using $3H = \frac{\dot{V}}{V}$ in (2) equation, we get

$$p_{WDF} = \left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{V^{(1+\omega)}} \quad , \qquad (3)$$

where c is the constant of integration and V is the volume expansion. WDF naturally includes these components, a piece that behave as a cosmological constant as well as a standard fluid with an equation of state $p = \omega\rho$. We can show that if we take $c > 0$, this fluid will not violate the strong energy condition $p + \rho \ge 0$. Thus, we get

$$p_{WDF} + \rho_{WDF} = (1+\omega)\rho_{WDF} - \omega\rho^* = (1+\omega)\left(\frac{c}{V^{(1+\omega)}}\right) \ge 0 \qquad (4)$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW model by Holman and Naidu[41] . Many Relativists [42-56] studied cosmological models with WDF in General Relativity and theories of gravitations .

3. Gravitational field equations of $f(R,T)$ gravity

The $f(R,T)$ gravity is the generalization of General Relativity (GR). In this theory, the field equations are derived from a variation, Hilbert-Einstein type principle which is given as

$$S = \frac{1}{16\pi}\int \sqrt{-g}\, f(R,T)\, d^4x + \int \sqrt{-g}\, L_m d^4 x, \qquad (5)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T) of the matter T_{ij} $(T = g^{ij}T_{ij})$ and L_m is the matter Lagrangian density.

The stress energy tensor of matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}. \qquad (6)$$

Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} and not on its derivatives, in this case we obtain

$$T_{ij} = g_{ij}L_m - \frac{\delta(L_m)}{\delta g^{ij}}. \qquad (7)$$

The $f(R,T)$ gravity field equations are obtained by varying the action S with respect to the metric tensor components g_{ij} ,

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + f_R(R,T)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \qquad (8)$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\delta g^{ij}\partial g^{\alpha\beta}}. \qquad (9)$$

Here $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$

$\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ and ∇_i is the covariant derivative.

The contraction of equation (8) yields

$$f_R(R,T)R + 3\Pi f_R(R,T) - 2f(R,T) = (8\pi - f_T(R,T))T - f_T(R,T)\Theta$$

with $\Theta = g^{ij}\Theta_{ij}$. (10)

Equation (10) gives a relation between Ricci scalar and the trace of energy momentum tensor. Using matter Lagrangian L_m the stress energy tensor of the matter is given by

$$T_{ij} = (p_{WDF} + \rho_{WDF})u_i u_j - p_{WDF}\, g_{ij}, \qquad (11)$$

where $u^i = (0,0,0,1)$ denotes the four velocity vector in co-moving coordinates which satisfies the condition $u^i u_i = 1$. ρ_{WDF} and p_{WDF} is energy density and pressure of the fluid respectively.

The variation of stress energy of perfect fluid has the following expression

$$\Theta_{ij} = -2T_{ij} - pg_{ij}. \qquad (12)$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij}. Several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. [16] gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)\, f_3(T) \end{cases}. \qquad (13)$$

In this paper, it is focused to the first class $f(R,T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of tress energy tensor of the form $f(T) = \mu T$ where μ is constant. For this choice the gravitational field equations of $f(R,T)$ gravity becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2\dot f(T)T_{ij} - 2\dot f(T)\Theta_{ij} + f(T)g_{ij}, \qquad (14)$$

where the dot denotes differentiation with respect to the argument. If the matter source is a perfect fluid then the field equations (in view of Eq. (12)) becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2\dot f(T)T_{ij} + [2p_{WDF}\dot f(T) + f(T)]g_{ij}. \qquad (15)$$

3a. Field equations of Binachi Type I in Kasner form

Anisotropic [Bianchi type I] metric in Kasner form is given by

$$ds^2 = dt^2 - t^{2q_1}dx^2 - t^{2q_2}dy^2 - t^{2q_3}dz^2, \qquad (16)$$

where q_1, q_2, q_3 are three parameters that we shall require to be constant. Let $S = q_1 + q_2 + q_3$, $\theta = q_1^2 + q_2^2 + q_3^3$, we get $R = (S^2 - 2S + \theta)t^{-2}$.

With the choice of the function $f(T)$ of the trace of the stress–energy tensor of the matter so that

$$f(T) = \lambda T, \qquad (17)$$

where λ is a constant [16].

Using comoving coordinates and equation (11-12) and (17), the $f(R,T)$ gravity field equations, (15), for metric (16) can be written as

$$\left[q_1(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = (8\pi + 3\lambda)p_{WDF} - \rho_{WDF}\lambda, \qquad (18)$$

$$\left[q_2(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = (8\pi + 3\lambda)p_{WDF} - \rho_{WDF}\lambda, \qquad (19)$$

$$\left[q_3(s-1) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = (8\pi + 3\lambda)p_{WDF} - \rho_{WDF}\lambda, \qquad (20)$$

$$\left[(s-\theta) - \frac{1}{2}(s^2 - 2s + \theta)\right]t^{-2} = -(8\pi + 3\lambda)\rho_{WDF} + p_{WDF}\lambda. \qquad (21)$$

where a dot here in after denotes ordinary differentiation with respect to cosmic time "t" only.

4. Isotropization and the solution

Let us define $R=\left(t^{q_1}t^{q_2}t^{q_3}\right)^{\frac{1}{3}}$ as the average scale factor so that the Hubble parameter in our anisotropic models may be defined as

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\sum_{i=1}^{3}H_i \qquad (22)$$

where R is the mean scale factor and $H_i=\frac{\dot{R}_i}{R_i}$ are directional Hubble's factors in the direction of x^i respectively.

The anisotropy parameter of the expansion Δ is defined as

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \qquad (23)$$

in the x, y, z directions, respectively. In deciding whether the model is isotropic or anisotropic , the mean anisotropic parameter of the expansion Δ has a very crucial role . It is the measure of the deviation from isotropic expansion, the universe expands isotropically when $\Delta=0$.

Let us introduce the dynamical scalars, such as expansion parameter (θ) and the shear (σ^2) as usual

$$\theta=3H \qquad (24)$$

$$\sigma^2=\frac{3}{2}\Delta H^2 \qquad (25)$$

Using equations (18) and (19) , we get

$$(q_1-q_2)(s-1)t^{-2}=0. \qquad (26)$$

Equations (26) can be written as

$$\frac{d}{dt}\left(\frac{q_1}{t}-\frac{q_2}{t}\right)+\left(\frac{q_1}{t}-\frac{q_2}{t}\right)\frac{s}{t}=0. \qquad (27)$$

Let V be a function of t defined by

$$V=t^{(q_1+q_2+q_3)}=t^s. \qquad (28)$$

Then from equation (28), we obtain

$$\frac{d}{dt}\left(\frac{q_1}{t}-\frac{q_2}{t}\right)+\left(\frac{q_1}{t}-\frac{q_2}{t}\right)\frac{\dot{V}}{V}=0. \qquad (29)$$

Integrating the above equation, we get

$$\frac{q_1}{q_2}=d_1\exp\left(x_1\int\frac{1}{V}dt\right), \qquad (30)$$

where d_1 and x_2 are constants of integrations .

In view of $V=t^s$, we write $t^{q_1}, t^{q_2}, t^{q_3}$ in the explicit form

$$t^{q_1}=D_1V^{\frac{1}{3}}\exp\left(X_1\int\frac{1}{V}dt\right), \qquad (31)$$

$$t^{q_2}=D_2V^{\frac{1}{3}}\exp\left(X_2\int\frac{1}{V}dt\right), \qquad (32)$$

$$t^{q_3}=D_3V^{\frac{1}{3}}\exp\left(X_3\int\frac{1}{V}dt\right), \qquad (33)$$

where $D_i(i=1,2,3)$ and $X_i(i=1,2,3)$ satisfy the relation $D_1D_2D_3=1$ and $X_1+X_2+X_3=0$.

Since field equations (18)–(21) are four equations having five unknowns and are highly nonlinear, an extra condition is needed to solve the system completely. Here two different volumetric expansion laws are used ,i.e.

$$V = at^b \quad (34)$$

and

$$V = \alpha e^{\beta t}, \quad (35)$$

where a, b, α , β are constants. In this way, all possible expansion histories, the power law expansion, (34), and the exponential expansion, (35), have been covered.

5. Model for power law

Using (34) in (31) -(33), we obtain the scale factors as follows:

$$t^{q_1} = D_1 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{ \frac{X_1}{a(1-b)} t^{1-b} \right\}, \quad (38)$$

$$t^{q_2} = D_2 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{ \frac{X_2}{a(1-b)} t^{1-b} \right\}, \quad (39)$$

$$t^{q_3} = D_3 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{ \frac{X_3}{a(1-b)} t^{1-b} \right\}, \quad (40)$$

where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From (1) and (3) with the help of (34), the energy density (ρ_{WDF}) and pressure (p_{WDF}) of the WDF are obtained as

$$\rho_{WDF} = \left(\frac{\omega}{1+\omega} \right) \rho^* + \frac{c}{(at^b)^{1+\omega}} \quad (41)$$

and

$$p_{WDF} = \frac{c\omega}{(at^b)^{1+\omega}} - \left(\frac{\omega}{1+\omega} \right) \rho^* . \quad (42)$$

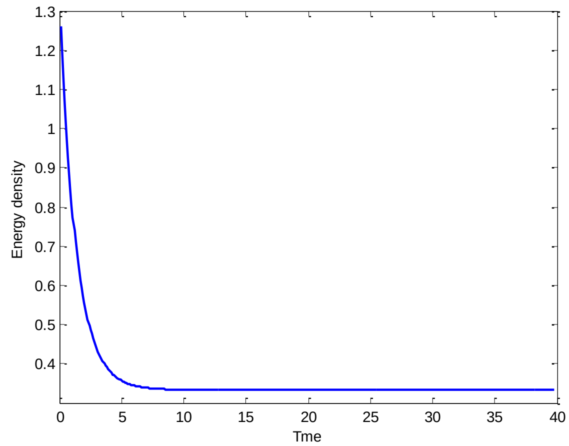

Figure No. 1 Energy Density vs time .

The mean Hubble's parameter, H, is given by

$$H = \frac{b}{3t}. \quad (43)$$

The anisotropic parameter is given by

$$\Delta = \frac{3X^2}{a^2 b^2 t^{2(b-1)}}. \quad (44)$$

The dynamical scalars are given by

$$\theta = \frac{b}{t}. \quad (45)$$

$$\sigma^2 = \frac{X^2}{2a^2 t^{2b}}, \quad (46)$$

where $X^2 = 2X_1^2 + X_2^2 = \text{constant}$.

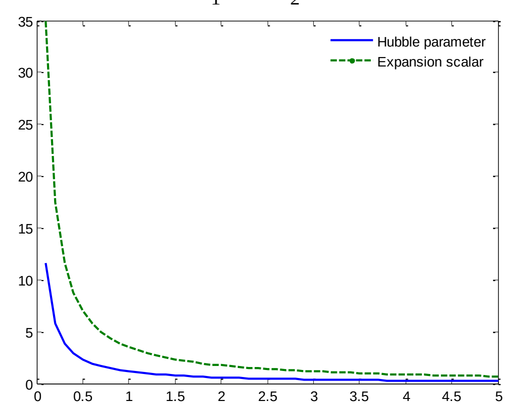

Figure No. 2 Hubble Parameter , Expansion vs time .

The deceleration parameter

$$q = \frac{3}{b} - 1. \qquad (47)$$

where $X^2 = 2X_1^2 + X_2^2 =$ constant .

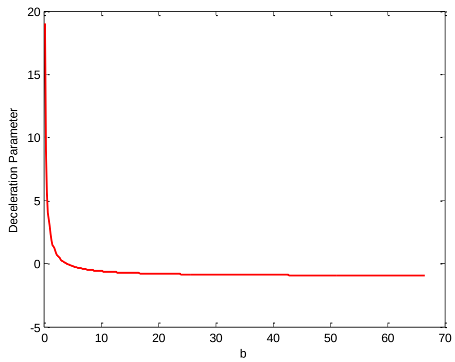

Figure No. 3. Deceleration Parameter vs *b*.

It reveals that near $t = 0$, the scale factor vanishes. Therefore, the model has an initial singularity. It is observed that the volume of the universe expands indefinitely for all positive values of b. Anisotropy of expansion is not promoted by the WDF. It behaves monotonically, decays to zero for $b > 1$, diverges for $b < 1$ as $t \to \infty$, and is constant for $b = 1$. The spatial volume is zero at t = 0. The matter pressure (p_{WDF}) and energy density (ρ_{WDF}) of WDF are infinity at t = 0. Thus the universe starts evolving with zero volume at t = 0 and expands with cosmic time. For $b > 3$ the deceleration parameter is negative. The model represents an accelerated universe.

6. Model for exponential law

Using (35) in (31) -(33), we obtain the scale factors as follows:

$$t^{q_1} = D_1 \alpha^{\frac{1}{3}} e^{\frac{\beta t}{3}} \exp\left\{\frac{-X_1}{\alpha\beta} e^{-\beta t}\right\}, \qquad (48)$$

$$t^{q_2} = D_2 \alpha^{\frac{1}{3}} e^{\frac{\beta t}{3}} \exp\left\{\frac{-X_2}{\alpha\beta} e^{-\beta t}\right\}, \qquad (49)$$

$$t^{q_3} = D_3 \alpha^{\frac{1}{3}} e^{\frac{\beta t}{3}} \exp\left\{\frac{-X_3}{\alpha\beta} e^{-\beta t}\right\}, \qquad (50)$$

where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From (1) and (3) with the help of (35), the energy density (ρ_{WDF}) and pressure (p_{WDF}) of the WDF are obtained as

$$\rho_{WDF} = \left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{\left(\alpha e^{\beta t}\right)^{1+\omega}} \qquad (51)$$

and

$$p_{WDF} = \frac{c\omega}{\left(\alpha e^{\beta t}\right)^{1+\omega}} - \left(\frac{\omega}{1+\omega}\right)\rho^* \qquad .(52)$$

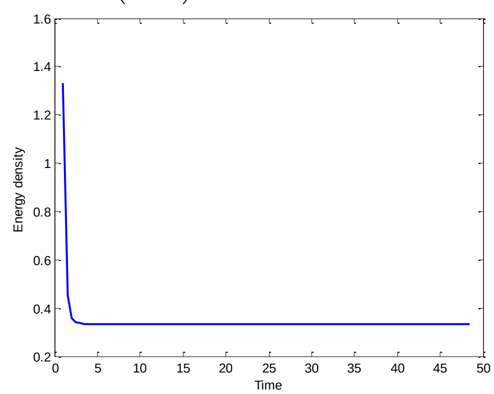

Figure No. 4 Energy Density vs time .

The mean Hubble's parameter, *H*, is given by

$$H = \frac{\beta}{3}. \qquad (53)$$

The anisotropy parameter of the expansion, Δ, is

$$\Delta = \frac{3X^2 e^{-2\beta t}}{\alpha^2 \beta^2}. \qquad (54)$$

The expansion scalar, θ, is found as

$$\theta = \beta . \tag{55}$$

The shear scalar, σ^2, is found as

$$\sigma^2 = \frac{X^2 e^{-2\beta t}}{2\alpha^2} . \tag{56}$$

The deceleration parameter

$$q = -1, \tag{57}$$

where $X^2 = 2X_1^2 + X_2^2 = \text{constant}$.

For large t, the model tends to be isotropic.

It is observed that the deceleration parameter $q = -1$ and $\frac{dH}{dt} = 0$ for this model. Hence, it gives the greatest values of the Hubble parameter and the fastest rate of expansion of the universe. The model may represent the inflationary era in the early universe and the very late time of the universe. The spatial volume is finite at $t = 0$. It expands exponentially as t increases and becomes infinitely large as $t \to \infty$. As t increases, the anisotropy of the expansion decreases exponentially to null. Thus the space approaches isotropy in this model. The expansion scalar is constant throughout the evolution of the universe. The shear scalar is also finite at $t \to 0$ and becomes zero as $t \to \infty$.

7. Conclusions:

Evolution of Bianchi type I space time in Kasner form cosmological model is studied in the presence of dark energy (DE) from a wet dark fluid (WDF) in *f(R,T)* theory of gravity [16]. The exact solutions of the field equations have been obtained by assuming two different volumetric expansion laws in a way to cover all possible expansion: namely, exponential expansion and power-law expansion. In the power law solutions , as scale factors diverge to infinity at large time there will be Big rip at least as far in the future. For $b > 3$ the deceleration parameter is negative. The model represents an accelerated universe, hence the model is consistent with the cosmological observations. In the Exponential volumetric expansion, the scale factors attain constant values at initial time. This is consistent with big bang scenario which resembles with Katore and Shaikh [57]. The value of the anisotropy parameter shows that at time tends to infinity the anisotropy parameter tends to zero that is the universe tends to isotropy. The deceleration parameter for this model is $q = -1$ and it predicts an accelerated expansion which resembles with Sahoo and Mishra [58].

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