

# MIMO OFDM PAPR Reduction by DHT Based Residue Number System

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## ABSTRACT

*The peak-to-average power (PAPR) is one of the main challenges in multicarrier transmissions. Aiming at reducing the PAPR, we propose a residue number system (RNS)-based OFDM parallel transmission scheme. The key idea of the proposed scheme is to utilize the parallel property of RNS to convert the input signals into the parallel smaller residue signals while utilizing the characteristic of RNS modular operation to effectively limit the output in each residue subchannel after inverse fast Fourier transform, which is smaller than the corresponding modulus. The main contribution of the proposed scheme is to reduce the dynamic range of the transmitted signal without nonlinear distortion so as to reduce the PAPR during the transmission. A generalized performance of the proposed scheme is analyzed in this paper, including the PAPR reduction, the complexity, the transmission bandwidth, etc. Also, an approximate formula to calculate the transmission bandwidth of the proposed scheme is derived, which simplifies design procedure in practice and implies that a minor increase of the dynamic range of RNS will bring comparative improvement of the transmission bandwidth consumption. Theoretical analysis and simulation results demonstrate that the proposed scheme has the*

*ability to achieve desirable PAPR reduction and low computational complexity without nonlinear distortion. In the future work Discerte Hartley Transform (DHT) is one of the advanced technique which is used to improve the performance of the signal by reducing PAPR. DHT provides reduced PAPR and low complexity when compared to RNS.*

**Keywords:** RNS, PAPR, OFDM, wireless communication systems, DHT.

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM), known as a multicarrier transmission, divides high-rate serial data streams into a number of parallel lower rate data streams that are transmitted on different subcarriers. The main advantages of OFDM-based systems include robustness to frequency selective fading, high spectral efficiency, low-complexity equalization, etc. However, since the transmitted signal of multicarrier transmission is the sum of data on different subcarriers, the variation of OFDM signal amplitudes is very wide with high peak-to-average power ratio (PAPR). The system performance could be degraded due to high PAPR, which introduces signal distortion when the dynamic

range of transmitted signals is larger than the amplifier accommodation. As a consequence, PAPR becomes one of the bottlenecks for OFDM-based systems in practical applications.

These years, great interest has been focused on PAPR reduction. In general, these schemes can be classified into lossy and lossless techniques depending on whether the transmitted signals are distorted or not. Common lossy schemes include clipping, peak windowing, companding transform, etc. Among them, clipping which limits parts of the signals over the allowed region, is the simplest and most widely used. However, there are some limitations for these lossy schemes. For example, when the distortion caused by amplitude clipping is serious, it will lead to bit error rate (BER) performance degradation. Lossless schemes include coding and probabilistic scheme Coding scheme selects the codeword that reduces the PAPR for transmission and may address the problem of error control, but it is hard to be adapted to OFDM with a larger number of subcarriers.

Residue number system (RNS), a parallel number system, is based on Chinese remainder theorem (CRT), which divides a large integer into several independent and parallel smaller ones with a specific modulus set. Due to the carry-free and parallel properties, RNS further simplifies the computations by decomposing a problem into a set of parallel, independent residue computations. Thus, RNS has received wide attention in very large scale integration applications. The activities of RNS focus on RNS to-binary conversion, RNS parity check, and RNS scaling scheme.

When an RNS-based transmission scheme is employed in OFDM, one of the big advantages is that the dynamic range of the inverse fast Fourier transform (IFFT) output is limited by the corresponding modulus due to the characteristic of RNS modular operation. The main principle of the proposed scheme is to utilize the parallel property of RNS to divide the original frequency band into  $V$  equal portions and to convert the input signals into  $V$  smaller residues using the corresponding modulus set. Then, these  $V$  residue signals are preformed modulations (in particular, OFDM in this paper) in the corresponding  $V$  residue sub channels. Signals of each residue subchannel share the original frequency band through frequency division multiplexing (FDM). Specifically, the value of the corresponding modulus determines the dynamic range of the output in each residue subchannel. When the number of subcarriers is large, the proposed scheme is still able to limit the transmitted signals within a small dynamic range and reduce PAPR without nonlinear distortion. It is demonstrated that the PAPR performance has been improved by more than 5 dB compared with conventional OFDM.

## 2. BACKGROUND

Exchanging the information from one entity to another is called as communications and from the early ages communication is one of the predominant part of the science from ancient times. Communications has great importance in the human daily needs to high equipped applications and it replaces the traditional telegram, letters etc. The wireless communications discovery has revolutionized the communication scenario by introducing the innovative applications

which are once imagination of the 19<sup>th</sup> century. The wireless communication has moved one step ahead by implementing the mobile communication introduction to the human daily needs and as time passes on the mobile communication has become integral part of modern society to communicate the different people around the globe.

### A) *History*

Wireless communication usage has predominant evidences from ancient times but along with the time wireless communications has changed its face. The pigeons and smoke are used as primary elements to communicate people at different locations with different signs and in modern world in place of pigeons and smoke usage of mobiles, radars and satellites are witnessed.

The research on the communication is seriously started in the 16<sup>th</sup> century by the popular inventions like large mobile panels coding by the renowned mathematician and scientist Robert Hooke for alphabetical coding based on the early coding schemes and the research carried on by Robert Hooke paves way for the invention of the optical telegraph by the French physicist Claude Chappe in 17<sup>th</sup> century. The invention of the optical telegraph results in the long distance communication based on the transmission codes transmitted from the large signaling towers and along with the time the technology related to the communication changing frequently with rapid pace. With the immense development based on the past inventions results in the large network over major cities in France and surrounding countries which are considered as one of the most innovative invention by the historians. The

Electromagnetic spectrum invention and its usage in the communication applications have increased the speed and accuracy from ultra short range to ultra high range applications. Noted physicists like James Clerk Maxwell and Heinrich Hertz are important persons behind the foundation of the EM spectrum and its associated applications.

### B) *Partial Transmit Sequence (PTS)*

The partial transmit sequence (PTS) scheme is an efficient approach and a lossless scheme for PAPR reduction by optimally combining signal sub-blocks. Selective mapping (SLM) is also a good approach, in which some statistically independent sequences are generated from the same information and the sequence with the lowest PAPR is transmitted. Both schemes provide improved PAPR statistic at the cost of additional complexity and loss of the data rate, because they need to implement some extra IFFT and iterations of phase optimization and transmit the side information. In addition, SLM scheme leads to a higher computational complexity at the same level of PAPR reduction, because it operates on all carriers.

## 3. PROPOSED METHOD

### A) *RNS-based PAPR Reduction*

An RNS is defined by the relative prime modulus set  $m_v (v = 1, 2, \dots, V)$ . Any integer  $R$  can be represented in RNS by residue sequence  $\{r_1, r_2, \dots, r_v\}$

$$r_v = R(\text{mod} m_v) \quad (8)$$

The number  $r_v$  is said to be the residue of  $R$  with respect to  $m_v$ , and we shall usually denote this by

$r_v = \langle R \rangle_{m_v}$ . In this sense, a big integer can be converted into the small residues in RNS, and these residues are always smaller than the corresponding modulus. The integers in the range of  $[0, M_l)$  can be represented in this RNS uniquely and unambiguously, where  $M_l = \prod_{i=1}^V m_v$  is referred to as the information dynamic range, i.e., the legitimate range of the information symbol.

The information symbols can be uniquely recovered by residue sequence through CRT, which is one of the fundamental theorems of RNS. The relationship between the information symbols  $R$  and its residues is as follows

$$R = \left( \sum_v S_v \langle 1/S_v \rangle_{m_v} r_v \right) \text{mod } M_1 \quad (9)$$

Where,  $\langle 1/S_v \rangle_{m_v}$  called as multiplicative inverse of  $S_v$ ,  $S_v = M_l/m_v$  and  $(S_v \langle 1/S_v \rangle_{m_v}) \text{mod } M_v = 1$

The basic diagram of RNS-based PAPR reduction scheme in MIMO-OFDM is given in Fig.2. The number of modulus  $\{m_1, m_2, \dots, m_v\}$  is , and the input are converted into  $V$  residues by the corresponding modulus set, and the number of transmit antennas equals the number of residue sub-channels. These residue signals are preformed OFDM modulation in the corresponding residue channels. In the each of the  $V$  parallel residue sub-channels one IFFT of length  $N$  is employed.

The function of mapping module, if the input is positive, it can be sent into B/R (binary to residue) module directly; otherwise the input adds the legitimate MI before B/R. Through B/R conversion, according to (8), the serial data streams are divided

into  $V$  parallel residue sub-channels transmitting signals.

$$S_{m_v, k} = s(KT/N) = \sum_{i=0}^{N-1} r_{m_v, i} \exp\left(j \frac{2\pi i k}{N}\right), (0 \leq k \leq N-1, 0 \leq i \leq N-1) \quad (10)$$

### B) PAPR of RNS-based scheme

The real and imaginary parts of OFDM signals have asymptotically Gaussian distributions for a large number of subcarriers by the central limit theorem. Then the amplitude of the OFDM signals follows a Rayleigh distribution. The PAPR of RNS-based scheme in each sub-channel can be written as

$$PAPR_{n_t} = 10 \log \frac{\max\left\{\left|\sum_{i=0}^{N-1} r_{m_v, i} \exp\left(j \frac{2\pi i k}{N}\right)\right|^2\right\}}{E\left\{\left|\sum_{i=0}^{N-1} r_{m_v, i} \exp\left(j \frac{2\pi i k}{N}\right)\right|^2\right\}} \quad (11)$$

$$= 10 \log \frac{\max\left\{\left|\sum_{i=0}^{N-1} r_{m_v, i} \exp\left(j \frac{2\pi i k}{N}\right)\right|^2\right\}}{2\sigma^2} \quad (dB)$$

Where,  $\sigma$  is the variance of OFDM signals. In the MIMOOFDM sceneries, the PAPR performance is governed by the worst-case PAPR, it can be presented as

$$PAPR_{rns-mimo} = \max_{n_t=1,2,\dots,n_r} PAPR_{n_t}$$

$$= 10 \log \frac{\max\left\{\left|\sum_{i=0}^{N-1} r_{m_v, i} \exp\left(j \frac{2\pi i k}{N}\right)\right|^2\right\}}{2\sigma^2} \quad (dB) \quad (12)$$

According to (8), the residue is always smaller than the corresponding modulus, which may be chosen smaller than the original number. Then the residue is smaller than the original number. After multiplying a rotation factor and summing up all the  $N$  elements, it

is still smaller than the sum of original one. It can be seen that the proposed scheme has the potential to improve the PAPR reduction performance.

### C) *PAPR of Discrete Hartley Transform based scheme*

A discrete Hartley transform (DHT) is a Fourier-related transform of discrete, periodic data similar to the discrete Fourier transform (DFT), with analogous applications in signal processing and related fields. Its main distinction from the DFT is that it transforms real inputs to real outputs, with no intrinsic involvement of complex numbers.

Just as the DFT is the discrete analogue of the continuous Fourier transform, the DHT is the discrete analogue of the continuous Hartley transform, introduced by R. V. L. Hartley in 1942.

Because there are fast algorithms for the DHT analogous to the fast Fourier transform (FFT), the DHT was originally proposed by R. N. Bracewell in 1983 as a more efficient computational tool in the common case where the data are purely real. It was subsequently argued, however, that specialized FFT algorithms for real inputs or outputs can ordinarily be found with slightly fewer operations than any corresponding algorithm for the DHT.

Formally, the discrete Hartley transform is a linear, invertible function  $H : R^n \rightarrow R^n$  (where  $R$  denotes the set of real numbers).

The  $N$  real numbers  $x_0, \dots, x_{N-1}$  are transformed into the  $N$  real numbers  $H_0, \dots, H_{N-1}$  according to the formula.

$$H_k = \sum_{n=0}^{N-1} x_n \left[ \cos\left(\frac{2\pi}{N}nk\right) + \sin\left(\frac{2\pi}{N}nk\right) \right] \quad (13)$$

where,  $k = 0, 1, \dots, N - 1$

### D) *Complexity*

In RNS, the addition and multiplication are modular operations. In theoretical analysis, they can be designed for flexibility in which case the methodology allows the design of adders for any modulus. The basic adder for any modulo- $m$  is defined as (13)

$$\langle A + B \rangle_m = \begin{cases} A + B & \text{if } A + B < m \\ A + B - m & \text{otherwise} \end{cases} \quad (14)$$

In the most straightforward implementation, the most complex way, a basic modular requires 3 adders: one for the addition, one for the subtraction, and one for the comparison [13].

A modular multiplication of complex signals can be expressed as (14)

$$\langle A \times B \rangle_m = \langle \langle a_1 a_2 \rangle_m - \langle b_1 b_2 \rangle_m \rangle_m + i \langle \langle b_1 a_2 \rangle_m + \langle a_1 b_2 \rangle_m \rangle_m \quad (15)$$

The modular multiplication of complex signals needs more 6 modular operations than complex multiplier. In each modular operation, it needs 2 adders (one for addition and one for comparison), which is similar to the case of the modular adder. Based on the definition of RNS, the residue is smaller than its corresponding modulus. Regardless of the number of addition and multiplication, the sum of residue signals in each residue sub-channel is still smaller than its corresponding modulus. It can be seen that

PAPR (in db)	CCDF for RNS	CCDF for DHT
0	1	1
1	0.9688	0.9688
2	0.999	0.661
3	0.7432	0.088
4	0.35	0.008

this scheme effectively controls the dynamic range of the transmitted signals to improve the PAPR reduction performance.

#### 4. SIMULATION RESULTS:

In the simulation results we finally proved that DHT with RNS will provide more efficient results compared with state of art existing systems.

Table 1. Comparison of CCDF in RNS and DHT

The performance of PAPR reduction is evaluated by CCDF and from the above table we can conclude that DHT performance is better than the RNS as CCDF values for particular PAPR are less for DHT as compare to RNS.

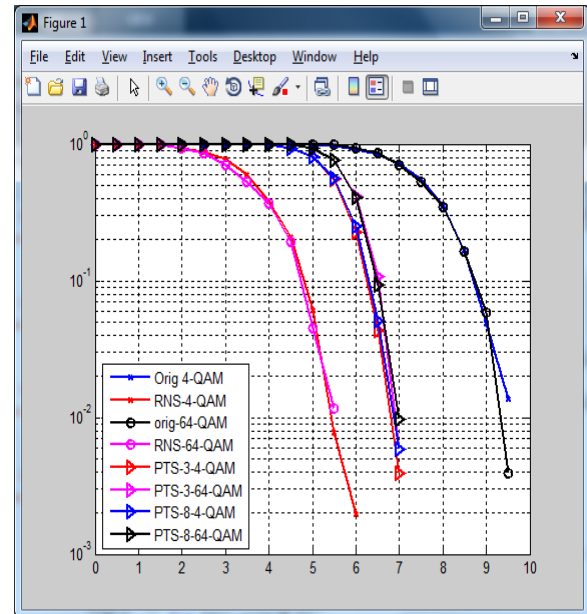


Figure1. PAPR reduction performance of the proposed scheme, PTS scheme and the conventional MIMO-OFDM.

#### Analysis.1:

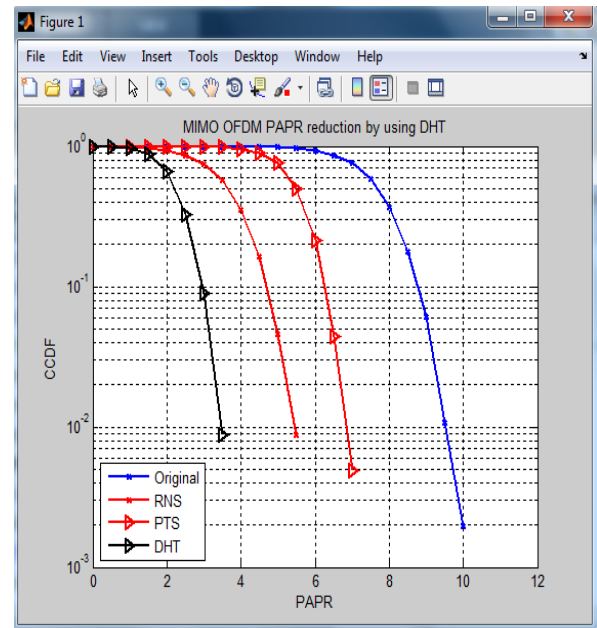


Figure2. PAPR reduction performance of extension scheme (DHT), RNS, PTS scheme of MIMO-OFDM.

## 5. CONCLUSION

In this paper, an RNS based PAPR scheme in MIMO-OFDM is proposed, which utilizes the properties and characteristics of RNS module to efficiently reduce the PAPR without any side information. Theoretical analysis and simulation results demonstrate the proposed scheme outperforms the PTS scheme in the PAPR reduction performance and the computational complexity. Again if performance of proposed system is compared with the DHT (Discrete Hartley transform) with the help of CCDF at particular PAPR, we get better performance for DHT than RNS.

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