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Practical Approach of Fixed Point Theorem In Metric Spaces for Mapping

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Abstract

Fixed Point Theorem 's being a constant study of many researchers , we want to utilize them for contractive mapping of metric spaces. In this paper, we presented an efficient way of using Fixed Point Theorems to prove its one of the kind in the world of mathematics , which can be used for marvelous applications . We have studied and taken few researchers previous results and tried to bring improvements in the results . We put our efforts in bringing the best results in Fixed Point for metric spaces and presented in the research work.

Keywords: Fixed point; coupled fixed point; generalized weak contractive mapping, metric spaces.

1. Introduction and Preliminaries

In1977, Alberetal. [1] Generalized Banach's contraction principle by introducing the concept of weak contraction mappings in Hilbert spaces . Weak contraction principle states that every weak contraction mapping on a complete Hilbert space has a unique fixed point. Rhoades [2] extended weak contraction principle in Hilbert spaces to metric spaces. Since then, many authors (for example , [3–14]) obtained generalizations and extensions of the weak contraction principle . Khanetal . [15] Obtained fixed point theorems in metric spaces by introducing the concept of altering distance functions . In particular , Choudhury et al. [8] obtained a generalization of the weak contraction principle in metric spaces by using altering distance functions as follows.

There are a lot of fixed point theorems for different type contractions in the literature . The fundamental linear contractive conditions are given by Banach [7], Kannan [8], Chatterjea [11], Reich [18], Zamfirescu [15], Hardy and Rogers [14] and the fundamental nonlinear contractive conditions are given by Boyd and Wong [17] and Matkowski [10]. In addition to the above , some fixed point theorems are proved using the ϕ-weak contraction , which is called weak contraction in some papers , in the sense of Alber and Guerre -Delabriere [3,18] and using the (ψ,ϕ)-weak contraction [12, 17]. On the other hand Berinde [8,10,11] defined weak contraction (or (δ, L)-weak contraction) mappings in a metric space as follow:

Theorem 1.1 ([8]) Suppose that a mapping g : X → X, where X is a metric space with metric d, satis -fies the following condition:

$$\psi\left(d(gx,gy)\right) \le \psi\left(\max\left\{d(x,y), d(x,gx), d(y,gy), \frac{1}{2}\{d(x,gy)+d(y,gx)\}\right\}\right) - \phi\left(\max\{d(x,y), d(y,gy)\}\right) \quad - \qquad (1.1)$$

for all $x, y \in X$, *where* $\phi : [0,\infty) \to [0,\infty)$ *is a continuous function, and* $\psi : [0,\infty) \to [0,\infty)$ *is an altering distance function, that is,* ψ *is a nondecreasing and continuous function, and* $\psi(t) = 0$ *if and only if* $t = 0$.

Then T *has a unique fixed point.*

Matthews [16] introduced the notion of partial metric spaces, and extended Banach's contraction principle to partial metric spaces, and then a lot of authors gave fixed point results in partial metric spaces (see [5, 17–30]). Also, Aydi et al. [12] extended Ekeland ' s variational principle to partial metric spaces, and Aydi et al. [12] extended Caristi ' s fi xed point theorem to partial metric spaces.

In particular, Abdeljawad [3] extended the result of Choudhury et al. [8] to partial metric spaces.

Samet et al. [13]gave a generalization of Banach's contraction principle and an application tofixed point results in partial metric spaces.

In this paper, motivated and inspired by Samet et al. [13], we introduce the notion of generalized weakly contractive mappings in metric spaces and prove a fixed point theorem for generalized weakly contractive mappings de fined on complete metric spaces ,which is a generalization of the results of[8–10,13]. Also , we obtain a coupled fi xed point theorem in metric spaces by applying our main result,and we give applications to fixed point and coupled fi xed point theorems in partial metric spaces .

A function $f : X \to [0,\infty)$, where X is a metric space, is called *lower semicontinuous* if, for all $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x$, we have

$$f(x) \leq \lim_{n\to\infty} \inf f(x_n).$$

Let

$$\Psi = \left\{\psi : [0,\infty) \to [0,\infty) \mid \psi \text{ is continuous and } \psi(t) = 0 \Leftrightarrow t = 0\right\}.$$

Also, we denote

$$\Phi = \left\{\phi : [0,\infty) \to [0,\infty) \mid \phi \text{ is lower semicontinuous and } \phi(t) = 0 \Leftrightarrow t = 0\right\}.$$

Lemma 1.1 ([34]) *If a sequence* $\{x_n\}$ *in* X *is not Cauchy, then there exist* $\epsilon > 0$ *and two subsequences* $\{x_{m(k)}\}$ *and* $\{x_{n(k)}\}$ *of* $\{x_n\}$ *such that* $m(k)$ *is the smallest index for which* $m(k) > n(k) > k$,

$$d(x_{m(k)}, x_{n(k)}) \geq \epsilon, \tag{1.2}$$

and

$$d(x_{m(k)-1}, x_{n(k)}) < \epsilon. \tag{1.3}$$

Moreover, suppose that $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$.

Then we have:

(1) $\lim_{n\to\infty} d(x_{m(k)}, x_{n(k)}) = \epsilon$;

(2) $\lim_{n\to\infty} d(x_{m(k)-1}, x_{n(k)-1}) = \epsilon$;
(3) $\lim_{n\to\infty} d(x_{m(k)}, x_{n(k)-1}) = \epsilon$;
(4) $\lim_{n\to\infty} d(x_{m(k)-1}, x_{n(k)}) = \epsilon$.

2. Fixed Point Results

Let X be a metric space with metric d, let $T : X \to X$, and let $\varphi : X \to [0,\infty)$ be a lower semicontinuous function.

Then T is called a *generalized weakly contractive mapping* if it satisfies the following condition:

$$\psi\big(d(Tx,Ty)+\varphi(Tx)+\varphi(Ty)\big) \leq \psi\big(m(x,y,d,T,\varphi)\big) - \phi\big(l(x,y,d,T,\varphi)\big) \quad \forall x,y \in X, \tag{2.1}$$

where $\psi \in \Psi, \phi \in \Phi$, and

$$m(x,y,d,T,\varphi) = \max\Big\{d(x,y)+\varphi(x)+\varphi(y), d(x,Tx)+\varphi(x)+\varphi(Tx), d(y,Ty)+\varphi(y)+\varphi(Ty), \frac{1}{2}\{d(x,Ty)+\varphi(x)+\varphi(Ty) + d(y,Tx)+\varphi(y)+\varphi(Tx)\}\Big\}, \tag{2.2}$$

and

$$l(x,y,d,T,\varphi) = \max\{d(x,y)+\varphi(x)+\varphi(y), d(y,Ty)+\varphi(y)+\varphi(Ty)\}. \tag{2.3}$$

Let X be a metric space with metric d, let $T : X \to X$, and let $\varphi : X \to [0,\infty)$ be a lower semicontinuous function.

Theorem 2.1 *Let X be complete. If T is a generalized weakly contractive mapping, then there exists a unique $z \in X$ such that $z = Tz$ and $\varphi(z) = 0$.*

Proof Let $x_0 \in X$ be a fixed point, and define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n = 0,1,2,\ldots$.

If $x_n = x_{n+1}$ for some n, then $x_n = x_{n+1} = Tx_n$. So x_n is a fixed point of T, and the proof is finished.

From now on, assume that $x_n \neq x_{n+1}$ for all $n = 0,1,2,\ldots$.

From (2.2) with $x = x_{n-1}$ and $y = x_n$ we have

$$\begin{aligned} m(x_{n-1},x_n,d,T,\varphi) &= \max\Big\{d(x_{n-1},x_n)+\varphi(x_{n-1})+\varphi(x_n), \\ &d(x_{n-1},Tx_{n-1})+\varphi(x_{n-1})+\varphi(Tx_{n-1}), d(x_n,Tx_n)+\varphi(x_n)+\varphi(Tx_n), \\ &\frac{1}{2}\{d(x_{n-1},Tx_n)+\varphi(x_{n-1})+\varphi(Tx_n)+d(x_n,Tx_{n-1})+\varphi(x_n)+\varphi(Tx_{n-1})\}\Big\}. \end{aligned}$$

Since

$$\frac{1}{2}\left\{d(x_{n-1},Tx_n)+\varphi(x_{n-1})+\varphi(Tx_n)+d(x_n,Tx_{n-1})+\varphi(x_n)+\varphi(Tx_{n-1})\right\}$$
$$=\frac{1}{2}\left\{d(x_{n-1},x_{n+1})+\varphi(x_{n-1})+\varphi(x_{n+1})+d(x_n,x_n)+\varphi(x_n)+\varphi(x_n)\right\}$$
$$\le\frac{1}{2}\left\{d(x_{n-1},x_n)+\varphi(x_{n-1})+\varphi(x_n)+d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right\}$$
$$\le\max\left\{d(x_{n-1},x_n)+\varphi(x_{n-1})+\varphi(x_n),d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right\},$$

we obtain

$$m(x_{n-1},x_n,d,T,\varphi)$$
$$=\max\left\{d(x_{n-1},x_n)+\varphi(x_{n-1})+\varphi(x_n),d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right\}. \tag{2.4}$$

Also, we have

$$l(x_{n-1},x_n,d,T,\varphi)$$
$$=\max\left\{d(x_{n-1},x_n)+\varphi(x_{n-1})+\varphi(x_n),d(x_n,Tx_n)+\varphi(x_n)+\varphi(Tx_n)\right\}$$
$$=\max\left\{d(x_{n-1},x_n)+\varphi(x_{n-1})+\varphi(x_n),d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right\}.$$

It follows from (2.1) that

$$\psi\left(d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right)$$
$$=\psi\left(d(Tx_{n-1},Tx_n)+\varphi(Tx_{n-1})+\varphi(Tx_n)\right)$$
$$\le\psi\left(m(x_{n-1},x_n,d,T,\varphi)\right)-\phi\left(l(x_{n-1},x_n,d,T,\varphi)\right). \tag{2.5}$$

If

$$d(x_{n-1},x_n)+\varphi(x_{n-1})+\varphi(x_n)<d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})$$

for some positive integer n, then from (2.5) we obtain

$$\psi\left(d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right)$$
$$\le\psi\left(d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right)-\phi\left(d(x_n,x_{n+1})+\varphi(x_n)+\varphi(x_{n+1})\right),$$

which implies

$$\phi\left(d(x_{n+1},x_n)+\varphi(x_{n+1})+\varphi(x_n)\right)=0,$$

and so

$$d(x_{n+1},x_n)+\varphi(x_{n+1})+\varphi(x_n)=0.$$

Hence

$$x_{n+1} = x_n \quad \text{and} \quad \varphi(x_{n+1}) = \varphi(x_n) = 0,$$

which is a contradiction.

Thus we have

$$d(x_n, x_{n+1}) + \varphi(x_n) + \varphi(x_{n+1}) \leq d(x_{n-1}, x_n) + \varphi(x_{n-1}) + \varphi(x_n) \tag{2.6}$$

for all $n = 1, 2, 3, \ldots$, and so

$$m(x_{n-1}, x_n, d, T, \varphi) = d(x_{n-1}, x_n) + \varphi(x_{n-1}) + \varphi(x_n)$$

and

$$l(x_{n-1}, x_n, d, T, \varphi) = d(x_{n-1}, x_n) + \varphi(x_{n-1}) + \varphi(x_n)$$

for all $n = 1, 2, 3, \ldots$.

It follows from (2.5) that

$$\begin{aligned} &\psi\left(d(x_n, x_{n+1}) + \varphi(x_n) + \varphi(x_{n+1})\right) \\ &\quad \leq \psi\left(d(x_{n-1}, x_n) + \varphi(x_{n-1}) + \varphi(x_n)\right) \\ &\qquad - \phi\left(d(x_{n-1}, x_n) + \varphi(x_{n-1}) + \varphi(x_n)\right). \end{aligned} \tag{2.7}$$

It follows from (2.6) that the sequence $\{d(x_n, x_{n+1}) + \varphi(x_n) + \varphi(x_{n+1})\}$ is nonincreasing. Thus we have

$$d(x_n, x_{n+1}) + \varphi(x_n) + \varphi(x_{n+1}) \to r \quad \text{as } n \to \infty$$

for some $r \geq 0$.

Assume that $r > 0$.

Letting $n \to \infty$ in (2.7), by the continuity of ψ and the lower semicontinuity of ϕ it follows that

$$\begin{aligned} \psi(r) &\leq \psi(r) - \lim_{n\to\infty} \inf \phi\left(d(x_{n-1}, x_n) + \varphi(x_{n-1}) + \varphi(x_n)\right) \\ &\leq \psi(r) - \phi(r). \end{aligned}$$

Since $r > 0$, $\phi(r) > 0$. Hence

$$\psi(r) \leq \psi(r) - \phi(r) < \psi(r),$$

a contradiction.

Hence

$$\lim_{n\to\infty} \left\{d(x_n, x_{n+1}) + \varphi(x_n) + \varphi(x_{n+1})\right\} = 0,$$

which implies

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0 \tag{2.8}$$

and

$$\lim_{n\to\infty} \varphi(x_n) = 0. \tag{2.9}$$

Now, we prove that the sequence $\{x_n\}$ is Cauchy.

If $\{x_n\}$ is not Cauchy, then by Lemma 1.1 there exist $\epsilon > 0$and subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ such that (1.2) and (1.3) hold.

From (2.2) we have

$$\begin{aligned} & m(x_{n(k)}, x_{m(k)}, d, T, \varphi) \\ &= \max\Big\{ d(x_{n(k)}, x_{m(k)}) + \varphi(x_{n(k)}) + \varphi(x_{m(k)}), \\ &\quad d(x_{n(k)}, Tx_{n(k)}) + \varphi(x_{n(k)}) + \varphi(Tx_{n(k)}), d(x_{m(k)}, Tx_{m(k)}) + \varphi(x_{m(k)}) + \varphi(Tx_{m(k)}), \\ &\quad \frac{1}{2}\left\{ d(x_{n(k)}, Tx_{m(k)}) + \varphi(x_{n(k)}) + \varphi(Tx_{m(k)}) + d(x_{m(k)}, Tx_{n(k)}) + \varphi(x_{m(k)}) + \varphi(Tx_{n(k)}) \right\} \Big\} \\ &= \max\Big\{ d(x_{n(k)}, x_{m(k)}) + \varphi(x_{n(k)}) + \varphi(x_{m(k)}), d(x_{n(k)}, x_{n(k)+1}) + \varphi(x_{n(k)}) + \varphi(x_{n(k)+1}), \\ &\quad d(x_{m(k)}, x_{m(k)+1}) + \varphi(x_{m(k)}) + \varphi(x_{m(k)+1}), \frac{1}{2}\left\{ d(x_{n(k)}, x_{m(k)+1}) + \varphi(x_{n(k)}) + \varphi(x_{m(k)+1}) \right. \\ &\quad \left. + d(x_{m(k)}, x_{n(k)+1}) + \varphi(x_{m(k)}) + \varphi(x_{n(k)+1}) \right\} \Big\}. \end{aligned} \tag{2.10}$$

Letting $k \to \infty$ in (2.10) and applying Lemma 1.1, (2.8), and (2.9), it follows that

$$\lim_{k\to\infty} m(x_{n(k)}, x_{m(k)}, d, T, \varphi) = \epsilon. \tag{2.11}$$

Also, it follows from (2.3) that

$$\begin{aligned} & l(x_{n(k)}, x_{m(k)}, d, T, \varphi) \\ &= \max\left\{ d(x_{n(k)}, x_{m(k)}) + \varphi(x_{n(k)}) + \varphi(x_{m(k)}), d(x_{m(k)}, Tx_{m(k)}) + \varphi(x_{m(k)}) + \varphi(Tx_{m(k)}) \right\} \\ &= \max\left\{ d(x_{n(k)}, x_{m(k)}) + \varphi(x_{n(k)}) + \varphi(x_{m(k)}), d(x_{m(k)}, x_{m(k)+1}) + \varphi(x_{m(k)}) + \varphi(x_{m(k)+1}) \right\}. \end{aligned}$$

Hence

$$\lim_{n\to\infty} l(x_{n(k)}, x_{m(k)}, d, T, \varphi) = \epsilon. \tag{2.12}$$

From (2.1) we have

$$\begin{aligned} & \psi\left(d(x_{n(k)+1}, x_{m(k)+1}) + \varphi(x_{n(k)+1}) + \varphi(x_{m(k)+1}) \right) \\ &\quad \le \psi\left(m(x_{n(k)}, x_{m(k)}, d, T, \varphi) \right) - \phi\left(l(x_{n(k)}, x_{m(k)}, d, T, \varphi) \right). \end{aligned}$$

Letting $k \to \infty$ in this inequality, by Lemma 1.1, the continuity of ψ, the lower semicontinuity of ϕ, and by (2.9), (2.11), and (2.12) we have

$$\psi(\epsilon) \le \psi(\epsilon) - \phi(\epsilon),$$

which is a contradiction because $\phi(\epsilon) > 0$.

Hence the sequence $\{x_n\}$ is Cauchy, and hence

$$\lim_{n\to\infty} x_n = z \in X \text{ exists}$$

because X is complete. Since φ is lower semicontinuous,

$$\varphi(z) \le \lim_{n\to\infty} \inf \varphi(x_n) \le \lim_{n\to\infty} \varphi(x_n) = 0,$$

which implies

$$\varphi(z) = 0. \tag{2.13}$$

It follows from (2.2) that

$$\begin{aligned} m(x_n, z, d, T, \varphi) = \max\Big\{ & d(x_n, z) + \varphi(x_n) + \varphi(z), \\ & d(x_n, Tx_n) + \varphi(x_n) + \varphi(Tx_n), d(z, Tz) + \varphi(z) + \varphi(Tz), \\ & \frac{1}{2}\left\{d(x_n, Tz) + \varphi(x_n) + \varphi(Tz) + d(z, Tx_n) + \varphi(z) + \varphi(Tx_n)\right\}\Big\} \\ = \max\Big\{ & d(x_n, z) + \varphi(x_n) + \varphi(z), \\ & d(x_n, x_{n+1}) + \varphi(x_n) + \varphi(x_{n+1}), d(z, Tz) + \varphi(z) + \varphi(Tz), \\ & \frac{1}{2}\left\{d(x_n, Tz) + \varphi(x_n) + \varphi(Tz) + d(z, x_{n+1}) + \varphi(z) + \varphi(x_{n+1})\right\}\Big\}. \end{aligned}$$

So we have

$$\lim_{n\to\infty} m(x_n, z, d, T, \varphi) = d(z, Tz) + \varphi(z) + \varphi(Tz) = d(z, Tz) + \varphi(Tz). \tag{2.14}$$

Also, we have

$$\begin{aligned} \lim_{n\to\infty} l(x_n, z, d, T, \varphi) &= \lim_{n\to\infty} \max\left\{d(x_n, z) + \varphi(x_n) + \varphi(z), d(z, Tz) + \varphi(z) + \varphi(Tz)\right\} \\ &= d(z, Tz) + \varphi(z) + \varphi(Tz) = d(z, Tz) + \varphi(Tz). \end{aligned} \tag{2.15}$$

It follows from (2.1) that

$$\begin{aligned} \psi\left(d(x_{n+1}, Tz) + \varphi(x_{n+1}) + \varphi(Tz)\right) &= \psi\left(d(Tx_n, Tz) + \varphi(Tx_n) + \varphi(Tz)\right) \\ &\le \psi\left(m(x_n, z, d, T, \varphi)\right) - \phi\left(l(x_n, z, d, T, \varphi)\right). \end{aligned} \tag{2.16}$$

By taking the limit as $n \to \infty$ in (2.16) and by applying the continuity of ψ, the lower semicontinuity of ϕ, (2.14), and (2.15) we have

$$\psi\left(d(z, Tz) + \varphi(Tz)\right) \leq \psi\left(d(z, Tz) + \varphi(Tz)\right) - \phi\left(d(z, Tz) + \varphi(Tz)\right).$$

Hence $d(z, Tz) + \varphi(Tz) = 0$, and hence $z = Tz$ and $\varphi(Tz) = 0$.

Suppose that u is another fixed point of T.

Then

$$u = Tu \quad \text{and} \quad \varphi(u) = 0.$$

By applying (2.1) with $x = z$ and $y = u$ we have

$$\begin{aligned} \psi\left(d(z,u)\right) &= \psi\left(d(Tz, Tu)\right) \\ &= \psi\left(d(Tz, Tu) + \varphi(Tz) + \varphi(Tu)\right) \\ &\leq \psi\left(m(z,u,d,T,\varphi)\right) - \phi\left(l(z,u,d,T,\varphi)\right) \\ &= \psi\left(d(z,u)\right) - \phi\left(d(z,u)\right), \end{aligned}$$

which implies $z = u$. □

The following example illustrates Theorem 2.1 and shows that it is a real generalization of Theorem 3.1 in [8].

Example 2.1 Let $X = [0, \infty)$ and $d(x, y) = |x - y|$ for $x, y \in X$, let $\psi(t) = \frac{3}{2}t$ for $t \geq 0$, and let

$$\varphi(t) = \begin{cases} \frac{1}{2}t & (0 \leq t \leq 1), \\ \frac{1}{2}t + \frac{1}{2} & (1 < t \leq 2), \\ t & (t > 2). \end{cases}$$

Then $\psi \in \Psi$, φ is lower semicontinuous, and $\frac{1}{2}t \leq \varphi(t) \leq t$, $t \geq 0$.

Define the map $T : X \to X$ by

$$Tx = \frac{x^2}{2(1+x)}.$$

Assume that a function $\phi : [0, \infty) \to [0, \infty)$ is defined by

$$\phi(t) = \frac{3t}{4 + 2t}.$$

Then $\phi \in \Phi$.

We now show that (2.1) holds.

Without loss of generality, suppose that $x \geq y$.

Then we have

$$\begin{aligned}
&\frac{1}{2}\{d(x,Ty)+\varphi(x)+\varphi(Ty)+d(y,Tx)+\varphi(y)+\varphi(Tx)\}\\
&\geq \frac{1}{2}\left\{d(x,Ty)+\frac{1}{2}x+\frac{1}{2}Ty+d(y,Tx)+\frac{1}{2}y+\frac{1}{2}Tx\right\}\\
&\geq \frac{1}{2}\left\{\frac{1}{2}\{d(x,Ty)+x+Ty+d(y,Tx)+y+Tx\}\right\}\\
&= \begin{cases} \frac{1}{2}\{x+\frac{x^2}{1+x}\} & (y\leq \frac{x^2}{2(1+x)}),\\ \frac{1}{2}\{x+y\} & \text{otherwise}\end{cases}\\
&> \frac{1}{2}x.
\end{aligned}$$

Thus we have

$$\begin{aligned}
&m(x,y,d,T,\varphi)\\
&= \max\Big\{d(x,y)+\varphi(x)+\varphi(y), d(x,Tx)+\varphi(x)+\varphi(Tx),\\
&\quad d(y,Ty)+\varphi(y)+\varphi(Ty), \frac{1}{2}\{d(x,Ty)+\varphi(x)+\varphi(Ty)+d(y,Tx)+\varphi(y)+\varphi(Tx)\}\Big\}\\
&\geq \frac{1}{2}\max\Big\{d(x,y)+x+y, d(x,Tx)+x+Tx,\\
&\quad d(y,Ty)+y+Ty, \frac{1}{2}\{d(x,Ty)+x+Ty+d(y,Tx)+y+Tx\}\Big\}\\
&= \frac{1}{2}\max\left\{2x,2x,2y,\frac{1}{2}x\right\}\\
&= x
\end{aligned}$$

and

$$\begin{aligned}
l(x,y,d,T,\varphi) &= \max\{d(x,y)+\varphi(x)+\varphi(y), d(y,Ty)+\varphi(y)+\varphi(Ty)\}\\
&\leq \max\{d(x,y)+x+y, d(y,Ty)+y+Ty\}\\
&= \max\{2x,2y\}\\
&= 2x.
\end{aligned}$$

Also, we have

$$\begin{aligned}
\psi\left(d(Tx,Ty)+\varphi(Tx)+\varphi(Ty)\right) &\leq \psi\left(d(Tx,Ty)+Tx+Ty\right)\\
&= \frac{3}{2}\left(\left|\frac{x^2}{2(1+x)}-\frac{y^2}{2(1+y)}\right|+\frac{x^2}{2(1+x)}+\frac{y^2}{2(1+y)}\right)\\
&= \frac{3}{2}\cdot\frac{2x^2}{2(1+x)}\\
&= \frac{3}{2}\cdot\frac{x^2}{1+x}.
\end{aligned}$$

Hence

$$\begin{aligned}\psi\left(m(x,y,d,T,\varphi)\right)-\phi\left(l(x,y,d,T,\varphi)\right) &\geq \frac{3}{2}x-\frac{3x/2}{1+x}\\ &= \frac{3}{2}\cdot\frac{x^2}{1+x}\\ &\geq \psi\left(d(Tx,Ty)+\varphi(Tx)+\varphi(Ty)\right),\end{aligned}$$

where the equality is satisfied when $x = 0$.

Thus (2.1) is satisfied.

By Theorem 2.1, T has a unique fixed point $z = 0$, and $\varphi(z) = 0$.

However, (1.1) is not satisfied. In fact, let $x = 3$, $y = 1$, and $\varphi(t) = 0$, $t \geq 0$.

Then

$$\psi\left(m((x,y,d,T,\varphi))\right)=\frac{45}{2},$$

$$\phi\left(l((x,y,d,T,\varphi))\right)=\frac{3}{4},$$

$$\psi\left(d(Tx,Ty)\right)=\frac{51}{2},$$

and so

$$\psi\left(d(Tx,Ty)\right)=\frac{204}{8}>\frac{147}{8}=\psi\left(m((x,y,d,T,\varphi))\right)-\phi\left(l((x,y,d,T,\varphi))\right).$$

The proofs of the following Corollary 2.2 and Corollary 2.3 are similar to that of Theorem 2.1. So, here the proofs are omitted.

Corollary 2.2 *Let X be complete. Suppose that T satisfies the following condition:*

$$\begin{aligned}&\psi\left(d(Tx,Ty)+\varphi(Tx)+\varphi(Ty)\right)\\ &\quad\leq \psi\left(m(x,y,d,T,\varphi)\right)-\phi\left(m(x,y,d,T,\varphi)\right)\\ &\qquad \forall x,y\in X,\ \text{where } \psi\in\Psi \text{ and } \phi\in\Phi.\end{aligned}$$

Then there exists a unique $z \in X$ such that $z = Tz$ and $\varphi(z) = 0$.

Corollary 2.3 *Let (X,d) be complete. Suppose that T satisfies the following condition:*

$$\begin{aligned}&\psi\left(d(Tx,Ty)+\varphi(Tx)+\varphi(Ty)\right)\\ &\quad\leq \psi\left(d(x,y)+\varphi(x)+\varphi(y)\right)-\phi\left(d(x,y)+\varphi(x)+\varphi(y)\right)\\ &\qquad \forall x,y\in X,\ \text{where } \psi\in\Psi \text{ and } \phi\in\Phi.\end{aligned}$$

Then there exists a unique $z \in X$ such that $z = Tz$ and $\varphi(z) = 0$.

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Corollary 2.4 *Let X be complete. Suppose that T satisfies the following condition*:

$$\psi\left(d\left(T^k x, T^k y\right) + \varphi\left(T^k x\right) + \varphi\left(T^k y\right)\right)$$
$$\leq \psi\left(m\left(x, y, d, T^k, \varphi\right)\right) - \phi\left(l\left(x, y, d, T^k, \varphi\right)\right)$$
$$\forall x, y \in X,\ \textit{where}\ \psi \in \Psi, \phi \in \Phi,\ \textit{and } k \textit{ is a positive integer.}$$

Then there exists a unique $z \in X$ such that $z = Tz$ and $\varphi(z) = 0$.

Proof Let $S = T^k$. Then by Theorem 2.1 S has a unique fixed point, say z.
Then $T^k z = Sz = z$ and

$$\varphi(z) = \varphi(Sz) = \varphi\left(T^k z\right) = 0.$$

Since $T^{k+1} z = Tz$,

$$STz = T^k(Tz) = T^{k+1} z = Tz,$$

and so Tz is a fixed point of S. By the uniqueness of a fixed point of S, $Tz = z$. □

Remark 2.1 If we have $\varphi = 0$, then ψ is nondecreasing and continuous, and ϕ is continuous in Theorem 2.1 (resp. Corollary 2.3, Corollary 2.4), then we obtain Theorem 3.1 of [8] (resp. Theorem 2.1 of [10], Corollary 3.1 of [8]).

Remark 2.2 If $\varphi = 0$ and if ψ and ϕ are nondecreasing and continuous in Corollary 2.3, then we obtain Theorem 2.1 of [10].

Remark 2.3 If $\varphi = 0$ and ψ is nondecreasing and continuous in Corollary 2.2, then we obtain Theorem2.2of[9].

3. Discussion

In this section, we obtain a new coupled fixed point result from Theorem 2.1.
Let X be a nonempty set.
We say that $(x, y) \in X \times X$ is a *coupled fixed point* [35] of a mapping $G : X \times X \to X$ if

$$G(x, y) = x \quad \text{and} \quad G(y, x) = y.$$

Lemma 3.1 ([36]) *Let X be a nonempty set, $(x, y) \in X \times X$, and let $G : X \times X \to X$.*
Then the following are equivalent.
(1) *$G(x, y) = x$ and $G(y, x) = y$;*
(2) *$H(x, y) = (x, y)$, that is, H has a fixed point (x, y),*
where $H : X \times X \to X \times X$ is a mapping defined by

$$H(x, y) = \left(G(x, y), G(y, x)\right). \tag{3.1}$$

Lemma 3.2 ([37]) *Let (X, d) be a complete metric space (resp. complete partial metric space).*

Define $\rho : X \times X \to [0, \infty)$ *by*

$$\rho\big((x,y),(u,v)\big) = \max\big\{d(x,u), d(y,v)\big\}. \tag{3.2}$$

Then $(X \times X, \rho)$ *is a complete metric space* (*resp. complete partial metric space*).

Let (X, d) be a metric space (or partial metric space), $G : X \times X \to X$, and let $H : (X \times X, \rho) \to (X \times X, \rho)$ be a mapping defined as in (3.1).

Let

$$\begin{aligned} &M\big((x,y),(u,v),d,G,\varphi^*\big) \\ &\quad = \max\Big\{ \max\big\{d(x,u), d(y,v)\big\} + \varphi^*(x,y) + \varphi^*(u,v), \\ &\qquad \max\big\{d\big(x, G(x,y)\big), d\big(y, G(y,x)\big)\big\} + \varphi^*(x,y) + \varphi^*\big(G(x,y), G(y,x)\big), \\ &\qquad \max\big\{d\big(u, G(u,v)\big), d\big(v, G(v,u)\big)\big\} + \varphi^*(u,v) + \varphi^*\big(G(u,v), G(v,u)\big), \\ &\qquad \frac{1}{2}\big[\max\{d\big(x, G(u,v)\big), d\big(y, G(v,u)\big) + \varphi^*(x,y) + \varphi^*\big(G(u,v), G(v,u)\big) \\ &\qquad + \max\big\{d\big(u, G(x,y)\big), d\big(v, G(y,x)\big)\big\} + \varphi^*(u,v) + \varphi^*\big(G(x,y), G(y,x)\big)\big]\Big\}, \end{aligned}$$

and let

$$\begin{aligned} &L\big((x,y),(u,v),d,G,\varphi^*\big) \\ &\quad = \max\big\{\max\big\{d(x,u), d(y,v)\big\} + \varphi^*(x,y) + \varphi^*(u,v), \\ &\qquad \max\big\{d\big(u, G(x,y)\big), d\big(v, G(y,x)\big)\big\} + \varphi^*(u,v) + \varphi^*\big(G(x,y), G(y,x)\big)\big\}, \end{aligned}$$

where $\varphi^* : X \times X \to [0, \infty)$.

Then we have

$$\begin{aligned} &M\big((x,y),(u,v),d,G,\varphi^*\big) \\ &\quad = \max\Big\{\rho\big((x,y),(u,v)\big) + \varphi^*(x,y) + \varphi^*(u,v), \\ &\qquad \rho\big((x,y), H(x,y)\big) + \varphi^*(x,y) + \varphi^*\big(H(x,y)\big), \\ &\qquad \rho\big((u,v), H(u,v)\big) + \varphi^*(u,v) + \varphi^*\big(H(u,v)\big), \\ &\qquad \frac{1}{2}\big[\rho\big((x,y), H(u,v)\big) + \varphi^*(x,y) + \varphi^*\big(H(u,v)\big) \\ &\qquad + \rho\big((u,v), H(x,y)\big) + \varphi^*(u,v) + \varphi^*\big(H(x,y)\big)\big]\Big\} \\ &\quad = m\big((x,y),(u,v),\rho,H,\varphi^*\big). \end{aligned} \tag{3.3}$$

Also, we have

$$L\big((x,y),(u,v),d,G,\varphi^*\big) = l\big((x,y),(u,v),\rho,H,\varphi^*\big). \tag{3.4}$$

Theorem 3.1 *Let X be complete. Suppose that $G : X \times X \to X$ is a mapping such that*

$$\psi\left(d\left(G(x,y),G(u,v)\right)+\varphi\left(G(x,y),G(y,x)\right)+\varphi\left(G(u,v),G(v,u)\right)\right) \leq \psi\left(M\left((x,y),(u,v),d,G,\varphi^*\right)\right)-\phi\left(L\left((x,y),(u,v),d,G,\varphi^*\right)\right), \tag{3.5}$$

for all $(x,y),(u,v) \in X \times X$, where $\psi \in \Psi$, $\phi \in \Phi$, and $\varphi^* : X \times X \to [0,\infty)$ is lower semi-continuous.

Then G has a unique coupled fixed point $(x^*,y^*) \in X \times X$, and $\varphi^*(x^*,y^*) = 0$.

Proof Let ρ be the metric on $X \times X$ defined as (3.2), and let $H : (X \times X, \rho) \to (X \times X, \rho)$ be a mapping defined as in (3.1) for $(x,y),(u,v) \in X \times X$.

It follows from (3.3), (3.4), and (3.5) that

$$\rho\left(H(x,y),H(u,v)\right) \leq \psi\left(m\left((x,y),(u,v),\rho,H,\varphi^*\right)-\phi\left(l\left((x,y),(u,v),\rho,H,\varphi^*\right)\right)\right.$$

for $(x,y),(u,v) \in X \times X$.

By Theorem 2.1, H has a unique fixed point, and so by Lemma 3.1 G has a unique coupled fixed point. □

Corollary 3.2 *Let X be complete. Suppose that $G : X \times X \to X$ is a mapping such that*

$$\psi\left(d\left(G(x,y),G(u,v)\right)+\varphi\left(G(x,y),G(y,x)\right)+\varphi\left(G(u,v),G(v,u)\right)\right) \leq \psi\left(M\left((x,y),(u,v),d,G,\varphi^*\right)\right)-\phi\left(M\left((x,y),(u,v),d,G,\varphi^*\right)\right)$$

for all $(x,y),(u,v) \in X \times X$, where $\psi \in \Psi$, $\phi \in \Phi$, and $\varphi^* : X \times X \to [0,\infty)$ is lower semi-continuous.

Then G has a unique coupled fixed point $(x^*,y^*) \in X \times X$, and $\varphi^*(x^*,y^*) = 0$.

Corollary 3.3 *Let X be complete. Suppose that $G : X \times X \to X$ is a mapping such that*

$$\begin{aligned}\psi\left(d\left(G(x,y),G(u,v)\right)+\varphi\left(G(x,y),G(y,x)\right)+\varphi\left(G(u,v),G(v,u)\right)\right) &\leq \psi\left(\max\left\{d(x,u),d(y,v)\right\}+\varphi^*(x,y)+\varphi^*(u,v)\right) \\ &\quad -\phi\left(\max\left\{d(x,u),d(y,v)\right\}+\varphi^*(x,y)+\varphi^*(u,v)\right)\end{aligned}$$

for all $(x,y),(u,v) \in X \times X$, where $\psi \in \Psi$, $\phi \in \Phi$, and $\varphi^* : X \times X \to [0,\infty)$ is lower semi-continuous.

Then G has a unique coupled fixed point $(x^*,y^*) \in X \times X$, and $\varphi^*(x^*,y^*) = 0$.

Taking $\varphi^* = 0$ in Theorem 3.1, we have the following corollary.

Corollary 3.4 *Let X be complete. Suppose that $G : X \times X \to X$ is a mapping such that*

$$\psi\left(d\left(G(x,y),G(u,v)\right)\right) \leq \psi\left(M\left((x,y),(u,v),d,G,0\right)-\phi\left(L\left((x,y),(u,v),d,G,0\right)\right)\right.$$

for all $(x,y),(u,v) \in X \times X$, where $\psi \in \Psi$ and $\phi \in \Phi$.

Then G has a unique coupled fixed point.

Corollary 3.5 *Let X be complete. Suppose that $G : X \times X \to X$ is a mapping such that*

$$\psi\left(d\left(G(x,y),G(u,v)\right)\right) \leq \psi\left(M\left((x,y),(u,v),d,G,0\right) - \phi\left(M\left((x,y),(u,v),d,G,0\right)\right)\right)$$

for all $(x,y),(u,v) \in X \times X$, where $\psi \in \Psi$ and $\phi \in \Phi$.
Then G has a unique coupled fixed point.

Corollary 3.6 *Let X be complete. Suppose that $G : X \times X \to X$ is a mapping such that*

$$\psi\left(d\left(G(x,y),G(u,v)\right)\right) \leq \psi\left(\max\left\{d(x,u),d(y,v)\right\} - \phi\left(\max\left\{d(x,u),d(y,v)\right\}\right)\right)$$

for all $(x,y),(u,v) \in X \times X$, where $\psi \in \Psi$ and $\phi \in \Phi$.
Then G has a unique coupledfixed point.

4 Applications

In this section, we give application to fixed point theorems in partial metric spaces.

Recall some definitions and basic results in partial metric spaces. For more details, we refer to [16].

Let Z be a nonempty set. A function $p : Z \times Z \to [0,\infty)$ is called a *partial metric* on Z if, for all $x,y,z \in Z$, the following are satisfied:

(1) $p(x,x) = p(y,y) = p(x,y) \Leftrightarrow x = y$;
(2) $p(x,x) \leq p(x,y)$;
(3) $p(x,y) = p(y,x)$;
(4) $p(x,z) \leq p(x,y) + p(y,z) - p(y,y)$.

The pair (Z,p) is called a partial metric space.

Note that if $p(x,y) = 0$, then $x = y$.

An example of a partial metric defined on $[0,\infty)$ is $p(x,y) = \max\{x,y\}$, $x,y \geq 0$. For more examples of partial metrics, we refer to [16].

It is well known that each partial metric p on a nonempty set Z generates a T_0 topology on Z and that $\{B(x,\epsilon) : \epsilon > 0, x \in Z\}$ is a base for the topology, where $B(x,\epsilon) = \{y \in Z : p(x,y) < p(x,x) + \epsilon\}$ for all $x \in Z$ and $\epsilon > 0$.

Also, it is known that the function $p_s : Z \times Z \to [0,\infty)$ defined by

$$p_s(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{4.1}$$

is a metric on Z.

Let Z be a partial metric space with partial metric p, let $\{x_n\}(\subset Z)$ be a sequence, and let $x \in Z$. Then we say that

(1) $\{x_n\}$ is *convergent to* x if $\lim_{n\to\infty} p(x,x_n) = p(x,x)$;
(2) $\{x_n\}$ is called a *Cauchy sequence* if there exists $\lim_{n,m\to\infty} p(x_n,x_m)$ such that it is finite;
(3) Z is *complete* if every Cauchy sequence in Z is convergent to a point $z \in Z$ such that

$$\lim_{n,m\to\infty} p(x_n,x_m) = p(z,z).$$

Remark 4.1 A partial metric space Z is complete if and only if for every Cauchy sequence $\{x_n\}$ in Z, there exists $z \in Z$ such that

$$\lim_{n,m\to\infty} p(x_n, x_m) = \lim_{n,m\to\infty} p(x_n, z) = p(z, z).$$

Remark 4.2 Let $\{x_n\}(\subset Z)$ be a sequence, and let $x \in Z$. If the sequence $\{x_n\}$ is convergent to $x$in(Z,p_s),then it is convergent to$x$in(Z,p),and the converse is not true(see[16]).

4.1 Fixed Points on Partial Metric Spaces

Theorem4.1 *Let Z be complete with partial metric p.Suppose that $T:Z\to Z$ is a map - ping such that*

$$\psi\left(p(Tx,Ty)\right) \le \psi\left(\max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{1}{2}\{p(x,Ty)+p(y,Tx)\}\right\}\right)$$
$$-\phi\left(\max\left\{p(x,y), p(y,Ty)\right\}\right) \tag{4.2}$$

for all $x, y \in Z$, where $\psi \in \Psi$ and $\phi \in \Phi$.

Then there exists a unique $z \in Z$ such that $z = Tz$ and $p(z,z) = 0$.

Proof From (4.1) we have

$$p(x,y) = \frac{p_s(x,y) + p(x,x) + p(y,y)}{2}$$

for all $x, y \in Z$.

Let $d(x,y) = \frac{p_s(x,y)}{2}$ and $\varphi(x) = \frac{p(x,x)}{2}$ for all $x, y \in Z$.

Then Z is a complete metric space with metric d, and $\varphi : Z \to [0,\infty)$ is a lower semicontinuous function. Also, (4.2) reduces to (2.1). By Theorem 2.1 there exists a unique $z \in Z$ such that $z = Tz$ and $p(z,z) = 0$. □

Remark 4.3 Theorem 4.1 is a generalization of Theorem 8 of [3]. In fact, let ϕ and ψ be nondecreasing and continuous functions.

Then Theorem 4.1 reduces to Theorem 8 of [3].

Corollary 4.2 *Let Z be complete with partial metric p. Suppose that $T : Z \to Z$ is a mapping such that*

$$\psi\left(p(Tx,Ty)\right) \le \psi\left(\max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{1}{2}\{p(x,Ty)+p(y,Tx)\}\right\}\right)$$
$$-\phi\left(\max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{1}{2}\{p(x,Ty)+p(y,Tx)\}\right\}\right)$$

for all $x, y \in Z$, where $\psi \in \Psi$ and $\phi \in \Phi$.

Then there exists a unique $z \in Z$ such that $z = Tz$ and $p(z,z) = 0$.

Remark 4.4 If ϕ is continuous in Corollary 4.2, then we obtain Theorem 2.5 of [6].

Corollary 4.3 *Let Z be complete with partial metric p. Suppose that $T : Z \to Z$ is a mapping such that*

$$\psi\left(p(Tx, Ty)\right) \leq \psi\left(p(x,y)\right) - \phi\left(p(x,y)\right)$$

for all $x, y \in Z$, where $\psi \in \Psi$ and $\phi \in \Phi$.

Then there exists a unique $z \in Z$ such that $z = Tz$ and $p(z,z) = 0$.

4.2 Coupled fixed points on partial metric spaces

Theorem 4.4 *Let Z be complete with partial metric p. Suppose that $G : Z \times Z \to Z$ is a mapping such that*

$$\psi\left(p\left(G(x,y), G(u,v)\right)\right) \leq \psi\left(M\left((x,y),(u,v),p,G,0\right)\right) - \phi\left(L\left((x,y),(u,v),p,G,0\right)\right) \tag{4.3}$$

for all $(x,y),(u,v) \in Z \times Z$, where $\psi \in \Psi$ and $\phi \in \Phi$.

Then G has a unique coupled fixed point.

Proof Let ρ be the partial metric on $Z \times Z$ defined as in (3.2), and let $H : (Z \times Z, \rho) \to (Z \times Z, \rho)$ be a mapping defined as in (3.1).

It follows from (4.3), (3.3), and (3.4) with $\varphi^* = 0$ that

$$\rho\left(H(x,y), H(u,v)\right) \leq \psi\left(m\left((x,y),(u,v),\rho,H,0\right) - \phi\left(l\left((x,y),(u,v),\rho,H,0\right)\right)\right.$$

for all $(x,y),(u,v) \in Z \times Z$.

By Theorem 4.1, H has a unique fixed point, and so by Lemma 3.1 G has a unique coupled fixed point. □

Corollary 4.5 *Let Z be complete with partial metric p. Suppose that $G : Z \times Z \to Z$ is a mapping such that*

$$\psi\left(p\left(G(x,y), G(u,v)\right)\right) \leq \psi\left(M\left((x,y),(u,v),p,G,0\right)\right) - \phi\left(M\left((x,y),(u,v),p,G,0\right)\right)$$

for all $(x,y),(u,v) \in Z \times Z$, where $\psi \in \Psi$ and $\phi \in \Phi$.

Then G has a unique coupled fixed point.

Corollary 4.6 *Let Z be complete with partial metric p. Suppose that $G : Z \times Z \to Z$ is a mapping such that*

$$\psi\left(p\left(G(x,y), G(u,v)\right)\right) \leq \psi\left(\max\left\{p(x,y), p(u,v)\right\}\right) - \phi\left(\max\left\{p(x,y), p(u,v)\right\}\right)$$

for all $(x,y),(u,v) \in Z \times Z$, where $\psi \in \Psi$ and $\phi \in \Phi$.

Then G has a unique coupled fixed point.

5. Conclusion

Motivated by the result of some research scholars , we introduced the best results in contractive mappings and prove it is one of its kind in mathematics for such mappings.We give applications to the existence of fixed point in partial metric spaces .This investigation can be extended to a quasi -metric spaces with applications to studies of fixed points in quasi-partial metric spaces.

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