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Construction of Different Partially Neighbour Balanced Complete Block Designs using Co-primes to Prime number (s) and v = s – 1

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Abstract

In the experiments of agriculture it is assumed that treatments are independent to each other, which is not true in all cases. In some situations treatments are affected by their neighbors, special concern arises when treatment effect is small and the neighbor effect is high which leads to inflate the error term. Our aim is to construct neighbour designs which may be balanced or partially balanced.

Key words:- Neighbor balanced complete block design, partially neighbour balanced complete block design, neighbor effect, primitive elements, Galois field.

1. Introduction

All the designs are assumed to be in linear blocks with neighbor effect only in the direction of the blocks (say, left neighbor effect and right neighbor effect). Since the effect of having no treatment differs from the neighbor effect of any treatment, it is considered here to have only designs with border plots that are one plot added at each end of each block. In agricultural experiments the output of any plot may be affected by the pesticide applied to its neighboring plots and itself. If a design consists each pair of treatments in the same number of blocks with block size equal to the number of treatments is balanced complete block design and a design each pair of treatments in the same number of blocks with block size is less than the number of treatment is balanced incomplete block design.

The concept of neighbor design was introduce by Rees (1967) in serology who gave some designs, of use in serology as collection of circular blocks in which any two distinct treatments appear as neighbors equally in virus research. Das and Saha (1976) proposed some methods of construction of neighbor balanced designs. Misra et al. (1991) suggested some methods of construction of neighbour designs of equal and unequal block sizes. Azais et al. (1993) gave a catalogue of efficient neighbor designs with border plots that are balanced and had also given a series of partially neighbor balanced designs. Meitei (1996) gave a method of construction of incomplete block neighbor designs. Bailey (2003) gave designs for one sided neighbor effects.

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Tomar et al. (2005) gave the totally balanced block designs for competition effects. Jaggi et al. (2006) gave the method of construction of complete and incomplete block designs partially balanced for neighbor competition effects. Pateria et al. (2007) constructed incomplete non circular block designs for competition effects by using N – 1 MOLS. Kedia and Misra (2008) constructed generalized neighbor designs. Laxmi and Rani (2009) constructed incomplete block designs for two sided (right and left) neighbor effects for OS1 series using MOLS. Laxmi and Parmita (2010) studied left neighbors in incomplete block designs for OS2 series obtained using MOLS. Laxmi and Parmita (2011) studied right neighbors in incomplete block designs for OS2series using MOLS. Laxmi and Parmita (2013) studied two sided (left and right) neighbors in incomplete block designs for OS2 series.

2. If s is a prime number and $F_s = \{0,1,2,3,\ldots\ldots\ldots\ldots s-1\}$ then the system $f_s = (F_s, +_s, ._s)$ is Galois field and is denoted by GF(S). In fact f_s is the simplest example of GF(S). If x is any element of GF(s) and m is the least positive integer such that $x^m = 1$ then m is called the order of the element x. when for certain x = x', m has maximum value s – 1, x' is said to be a primitive element of GF(s). There exists a primitive element in every GF(s). If x is primitive element, all the non-zero elements of GF(s) can be expressed as

$x^0 = 1,\ x,\ x^2,\ x^3,\ x^4,\ x^5, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ x^{s-2}$

This is called as power cycle of x which generates the treatments needed for specific block size for different number of treatments, v = s - 1, where s is a prime number.

2.1 Generalization of primitive elements for prime number (s) :

s :	x^1	x^2	x^3	x^4	x^5	x^6……………					……x^{s-1}	**mod(s)**
3 :	x^1	x^2										mod(3)
5 :	x^1	x^2	x^3	x^4								mod(5)
7 :	x^1	x^2	x^3	x^4	x^5	x^6						mod(7)
11 :	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}		mod(11)
. :	,,	,,	,,	,,	,,	,,	,,	,,	,,	,,	………………………	
. :	,,	,,	,,	,,	,,	,,	,,	,,	,,	,,	………………………	
. :	,,	,,	,,	,,	,,	,,	,,	,,	,,	,,	………………………	
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. :	,,	,,	,,	,,	,,	,,	,,	,,	,,	,,	………………………	
s :	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	………x^{s-1}	mod(s)

The integers which are co-prime to s (from 1 to s-1) form a group of primitive classes (mod s) where s is the prime number and x is a primitive element.

Using these different co-primes to the prime number which generates the required size of the block may be considered for the construction of the design. Here a method of construction of a design for v = s – 1 for block size k = s – 1 is proposed, for different co-primes to the prime number.

2.2 Primitive elements for prime number s = 3:

For s = 3 then the treatment v = s-1 = 2.

Table for power of all co-primes 3 under modulo s is:

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X	X	X^2
1	1	
2	2	1

The order of co-prime 1 is 1, and the order of co-prime 2 is 2 so 2 is primitive element with mod 3 as all the treatments number with mod 3 occurs, while other is not a primitive element because of missing value 2.

2.3 Primitive elements for prime number s = 5 :

For s= 5 then the treatment number v = s-1 = 4.

Co-primes under mod5 is:

x	**x**	**x^2**	**x^3**	**x^4**
1	1			
2	2	4	3	1
3	3	4	2	1
4	4	1		

Here the order of co-prime 1 is 1, the order of 4 is 2, and the order of 2 & 3 are 4, and the primitive elements since 2 & 3 as every number occurs while other are not primitive elements because of missing several values.

2.4 Primitive elements for prime number s = 7 :

For s = 7 then the treatment number v = s-1 = 6.

Co-primes under mod7 is:

X	**X**	**x^2**	**x^3**	**x^4**	**x^5**	**x^6**
1	1					
2	2	4	1			
3	3	2	6	4	5	1
4	4	2	1			
5	5	4	6	2	3	1
6	6	1				

The order of co-prime 1 is 1, the order of 6 is 2, the order of 2 & 4, is 3 and the order of 3 & 5 is 6. The primitive elements are 3 & 5 because every number occurs while others are not primitive elements because of missing several values.

2.5 Primitive elements for prime number s = 11:

For s = 11 then the treatment number v = s-1 = 10.

Co-primes under mod 11 is:

x	**X**	**x^2**	**x^3**	**x^4**	**x^5**	**x^6**	**x^7**	**x^8**	**x^9**	**x^{10}**

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1	1									
2	2	4	8	5	10	9	7	3	6	1
3	3	9	5	4	1					
4	4	5	9	3	1					
5	5	3	4	9	1					
6	6	3	7	9	10	5	8	4	2	1
7	7	5	2	3	10	4	6	9	8	1
8	8	9	6	4	10	3	2	5	1	
9	9	4	3	5	1					
10	10	1								

The order of co-prime 1 is 1, the order of 10 is 2, the order of 8 is 9, the order of 3,4,5 & 9 is 5 and the order of 2,6 & 7is 10. So 2,6 & 7 are the primitive elements because every number occurs while other are not primitive elements because of missing several values. Similarly one can find out the primitive elements for any prime numbers s.

3. Construction of Partially Neighbour Balanced Complete Block Designs:

For v = s-1 (i. e., s = v + 1) treatments where s be the prime number and x be any of the primitive elements of s. The initial block consisting of following treatments can be obtained easily.

$$\mathbf{I = \{1, x^{s-2}, x^{s-3}, x^{s-4}, \ldots\ldots\ldots\ldots, x^{1}\}}$$

This initial block provides a design when used with cyclic shift.

3.1 For s = 5 then v = s - 1 = 4, for which the primitive elements are x = 2, 3.
The initial block with primitive element as x = 2, with mod(s) of block size 4 is.

I = (1, 3, 4, 2)

Now taking mod(v) of this initial base block (since the total number of treatment is v), shall be the initial block of the design.

I = (1, 3, 0, 2)

It is the requisite initial block of the design with v = s – 1 = b, k = s – 1 = r, which is obtained in the systematic pattern. The remaining blocks can be obtained cyclically shift method through this initial block and a Complete Block Design can be constructed.

1	3	0	2
2	0	1	3
3	1	2	0
0	2	3	1

The Neighbour Design of this block design can be obtained by using the method of border plots.

2	1	3	0	2	1
3	2	0	1	3	2

0	3	1	2	0	3
1	0	2	3	1	0

The neighbour design thus obtained is Partially Neighbour Balanced Complete Block Design with parameters v = b = 4, r = k = 4 and $\lambda_1 = 2$, $\lambda_2 = 4$.

3.2 The initial block for v = s – 1 = 4 with other primitive element as x = 3, with mod(s) of block size 4 is.

I = (1, 2, 4, 3)

Now taking mod(v) of this initial base block (since the total number of treatment is v), shall be the requisite initial block of the design.

I = (1, 2, 0, 3)

Here requisite initial blocks for primitive elements x = 2 and x = 3 are same as both contains the identical treatments. Hence the designs obtained for these primitive elements x = 2 and x = 3 are also same.

3.3 For s = 11 then v = s - 1 = 10, for which the primitive elements are x = 2, 6, 7.

The initial block with primitive element as x = 2, with mod(s) of block size 10 is.

I = (1, 6, 3, 7, 9, 10, 5, 8, 4, 2)

Now taking mod(v) of this initial base block (since the total number of treatment is v), shall be the initial block of the design.

I = (1, 6, 3, 7, 9, 0, 5, 8, 4, 2,)

It is the requisite initial block of the design with v = s – 1 = b, k = s – 1 = r, which is obtained in the systematic pattern. The remaining blocks can be obtained cyclically shift method through this requisite initial block and a Complete Block Design can be constructed.

1	6	3	7	9	0	5	8	4	2
2	7	4	8	0	1	6	9	5	3
3	8	5	9	1	2	7	0	6	4
4	9	6	0	2	3	8	1	7	5
5	0	7	1	3	4	9	2	8	6
6	1	8	2	4	5	0	3	9	7
7	2	9	3	5	6	1	4	0	8
8	3	0	4	6	7	2	5	1	9
9	4	1	5	7	8	3	6	2	0
0	5	2	6	8	9	4	7	3	1

The Neighbour Design of this block design can be obtained by using the method of border plots.

2	1	6	3	7	9	0	5	8	4	2	1
3	2	7	4	8	0	1	6	9	5	3	2
4	3	8	5	9	1	2	7	0	6	4	3
5	4	9	6	0	2	3	8	1	7	5	4
6	5	0	7	1	3	4	9	2	8	6	5

7	6	1	8	2	4	5	0	3	9	7	6
8	7	2	9	3	5	6	1	4	0	8	7
9	8	3	0	4	6	7	2	5	1	9	8
0	9	4	1	5	7	8	3	6	2	0	9
1	0	5	2	6	8	9	4	7	3	1	0

The neighbour design thus obtained is Partially Neighbour Balanced Complete Block Design with parameters v = b = 10, r = k = 10 and $\lambda_1 = 2$, $\lambda_2 = 4$.

3.4 For v = s-1 = 10, now considering the primitive element x = 6, the initial block of size 10 is.

I = (1, 2, 4, 8, 5, 10, 9, 7, 3, 6)

Now taking mod(v) of this base block we get requisite initial block as:

I = (1, 2, 4, 8, 5, 0, 9, 7, 3, 6)

The remaining blocks can be obtained cyclically under modulo 10 through this requisite initial block and a Partially Neighbour Balanced Complete Block Design is constructed with parameters v = b = 10, r = k = 10 and $\lambda_1 = 2$, $\lambda_2 = 4$. This neighbour design is same as the neighbour design obtained with primitive element as x = 2.

3.5 For v = s-1 = 10, now considering the primitive element x = 7 the initial block of size 10 is.

I = (1, 8, 9, 6, 4, 10, 3, 2, 5, 7,)

Now taking mod(v) of this base block we get requisite initial block as:

I = (1, 8, 9, 6, 4, 0, 3, 2, 5, 7,)

The remaining blocks can be obtained cyclically under modulo 10 through initial block and a Partially Neighbour Balanced Block Design can be constructed with parameters v = b = 10, r = k = 10 and $\lambda_1 = 2$, $\lambda_2 = 4$.

Here designs for v = s – 1 = 10 obtained for primitive elements x = 2 and x = 6 are same but this is different from the neighbour design obtained for the primitive element as x = 7.

Partially Neighbour Balanced Block Designs with different primitive elements of the prime number s can be constructed for any value of s.

4 Conclusion :- Partially Neighbour Balanced Block Designs with different primitive elements of the prime numbers s= 3, 5, 7, 11, 13, 17, 19, 23 and so on, can be constructed easily with this method. A list is given below as :

Initial Block of the designs with parameters v = b = s-1, r = k = s-1 and $\lambda_1 = 2$, $\lambda_2 = 4$.

s	x	Initial block with mod (v)
5	2, 3	(1, 3, 0, 2,)
7	3,5	(1, 5, 4, 0, 2, 3)
11	2,6	(1, 6, 3, 7, 9, 0, 5, 8, 4, 2)
11	7	(1, 8, 9, 6, 4, 0, 3, 2, 5, 7)
13	2,7	(1, 7, 10, 5, 9, 11, 0, 6, 3, 8, 4, 2)
13	6,11	(1, 11, 4, 5, 3, 7, 0, 2, 9, 8, 10, 6)

17	3,6	(1, 6, 2, 12, 4, 7, 8, 14, 0, 11, 15, 5, 13, 10, 9, 3)
17	5,7	(1, 7, 15, 3, 4, 11, 9, 12, 0, 10, 2, 14, 13, 6, 8, 5)
17	10	(1, 12, 7, 11, 13, 3, 2, 7, 0, 5, 9, 6, 4, 14, 15, 10)
17	11,14	(1, 14, 9, 7, 13, 12, 15, 6, 0, 3, 8, 10, 4, 5, 2, 11)
19	2, 10	(1, 10, 5, 12, 6, 3, 11, 15, 17, 0, 9, 14, 7, 13, 16, 8, 4, 2)
19	3, 13	(1, 3, 9, 8, 5, 15, 7, 2, 6, 0, 16, 10, 11, 14, 4, 12, 17, 13)
19	14, 15	(1, 15, 16, 12, 9, 2, 11, 13, 5, 0, 4, 3, 7, 10, 17, 8, 6, 14)
23	5, 14	(1, 14, 12, 7, 6, 15, 3, 19, 13, 21, 18, 0, 9, 11, 16, 17, 8, 20, 4, 10, 2, 5)
23	7, 10	(1, 10, 8, 11, 18, 19, 6, 14, 2, 20, 16, 0, 13, 15, 12, 5, 4, 17, 9, 21, 3, 7)
23	11, 21	(1, 21, 4, 15, 16, 14, 18, 10, 3, 17, 12, 0, 2, 19, 8, 7, 9, 5, 13, 20, 6, 11)
23	15, 20	(1, 20, 9, 19, 12, 10, 16, 21, 6, 5, 8, 0, 3, 14, 4, 11, 13, 7, 2, 17, 18, 15)
23	19	(1, 17, 13, 14, 8, 21, 12, 20, 18, 7, 4, 0, 6, 10, 9, 15, 2, 11, 3, 5, 16, 19)

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