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Construction of Different Neighbour Balanced Incomplete Block Designs using Co-primes to Prime number (s) and v = s

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Abstract:- In field experiment of agriculture it is assumed that treatments are independent to each other but it is not true in all cases. In certain situations treatments are affected by their neighbors and special concern arises when treatment effect is small and neighbour effect is high which leads to inflate the error term. Usual experimental designs may not be applied here and hence neighbour designs are recommended for such situations. In neighbour designs, treatments are allotted in such a way that every treatment may occur equal number of times with every other treatment as neighbour such designs are said to be Neighbour Balanced Designs (NBD). Our aim is to construct neighbour design which may be balanced or incomplete balanced.

Key words:- Neighbor balanced incomplete block design, partially neighbour balanced complete block design, neighbor effect, primitive elements, Galois field.

1. **Introduction:-** Since the effect of treatment having no neighbour differs from the effect of the treatment with neighbour. it is considered here to have designs with border plots e. g. in agricultural experiments the output of any plot may be affected by the pesticide applied to its neighboring plots with itself. The concept of neighbor design was first of all introduced by Rees (1967) in serology and gave some designs, of use in serology, as collection of circular blocks in which any two distinct treatments appear as neighbors equally in virus research. Das and Saha (1976) gave some methods of construction of neighbor balanced designs. Misra et al. (1991) gave some methods of construction of neighbor designs of equal and unequal block sizes. Azais et al. gave a catalogue of efficient neighbor designs with border plots that are balanced and had also given a series of partially neighbor balanced designs. Meitei (1996) gave a method of construction of incomplete block neighbor designs. Bailey (2003) gave designs for one sided neighbor effects. Tomar et al. (2005) gave the totally balanced block designs for competition effects. Jaggi et al. (2006) gave the method of construction of complete and incomplete block designs partially balanced for neighbor competition effects. Pateria et al. (2007) constructed incomplete non circular block designs for competition effects by putting N – 1 MOLS with N treatments. Kedia and Misra (2008) constructed generalized neighbor design of use in serology. Laxmi and Rani (2009) constructed incomplete block designs for two sided (right and left) neighbor effects for OS1 series using MOLS. Laxmi and Parmita (2010) studied

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left neighbors in incomplete block designs for OS2 series. Laxmi and Parmita (2011) studied right neighbors in incomplete block designs for the above series. Laxmi and Parmita (2013) studied two sided (left and right) neighbors in incomplete block designs for same series. Neighbor effect may depend upon situation like direction, shed, wind ect. This may be distinguished as left and right neighbors.

1. If s is a prime number and $F_s = \{0,1,2,3,\ldots\ldots\ldots\ldots s-1\}$ then the system $f_s = (F_s, +_s, ._s)$ is Galois field and is denoted by GF(S). In fact f_s is the simplest example of GF(S). If x is any element of GF(s) and m is the least positive integer such that $x^m = 1$ then m is called the order of the element x. when for certain x = x', m has maximum value s – 1, x' is said to be a primitive element of GF(s). There exists a primitive element in every GF(s). If x is primitive element, all the non-zero elements of GF(s) can be expressed as

$x^0 = 1,\ x,\ x^2,\ x^3,\ x^4,\ x^5, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ x^{s-2}$

This is called as power cycle of x which generates the treatments needed for specific block size for different number of treatments, v = s - 1, where s is a prime number.

2.1 Generalization of primitive elements for prime number (s) :

s :	**x^1**	**x^2**	**x^3**	**x^4**	**x^5**	**x^6…………………………………………x^{s-1}**						**mod(s)**
3 :	x^1	x^2										mod(3)
5 :	x^1	x^2	x^3	x^4								mod(5)
7 :	x^1	x^2	x^3	x^4	x^5	x^6						mod(7)
11 :	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}		mod(11)
. :	,,	,,	,,	,,	,,	,,	,,	,,	,,	,,	………………………	
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s :	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	………x^{s-1}	mod(s)

The integers which are co-prime to s (from 1 to s-1) from a group of primitive classes (mod s) where s is the prime number and x is a primitive element.

Using these different co-primes to the prime number which generates the required size of the block may be considered for the construction of the design. Here method of construction of a design for v = s – 1 for block size k = s – 1 is proposed, for different co-prime to the prime number.

2.2 Primitive elements for prime numbers s = 3:

For s = 3 then the treatment v = s-1 = 2.

Table for power of all co-primes 3 under modulo s is:

X	X	X^2
1	1	
2	2	1

The order of co-prime 1 is 1, and the order of co-prime 2 is 2 so 2 is primitive element mod 3 as all the treatments number with mod 3 occurs, while other is not a primitive element because of missing value 2.

2.3 Primitive elements for prime number s = 5 :

For s= 5 then the treatment number v = s-1 = 4.

Co-primes under mod5 is:

x	x	x^2	x^3	x^4
1	1			
2	2	4	3	1
3	3	4	2	1
4	4	1		

Here the order of co-prime 1 is 1, the order of 4 is 2, and the order of 2 & 3 are 4, and the primitive elements since 2 & 3 as every number occurs while other are not primitive elements because of missing several values. Again the smallest primitive element among 2 and 3 is 2.

2.4 Primitive elements for prime number s = 7 :

For s = 7 then the treatment number v = s-1 = 6.

Co-primes under mod7 is:

X	X	x^2	x^3	x^4	x^5	x^6
1	1					
2	2	4	1			
3	3	2	6	4	5	1
4	4	2	1			
5	5	4	6	2	3	1
6	6	1				

The order of co-prime 1 is 1, the order of 6 is 2, the order of 2 & 4, is 3 and the order of 3 & 5 is 6. The primitive elements are 3 & 5 because every number occurs while others are not primitive elements because of missing several values. The smallest primitive element among them is 2.

2.5 Primitive elements for prime numbers s = 11:

For s = 11 then the treatment number v = s-1 = 10.

Co-primes under mod 11 is:

x	X	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}
1	1									
2	2	4	8	5	10	9	7	3	6	1
3	3	9	5	4	1					
4	4	5	9	3	1					
5	5	3	4	9	1					

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6	6	3	7	9	10	5	8	4	2	1
7	7	5	2	3	10	4	6	9	8	1
8	8	9	6	4	10	3	2	5	1	
9	9	4	3	5	1					
10	10	1								

The order of co-prime 1 is 1, the order of 10 is 2, the order of 8 is 9, the order of 3,4,5 & 9 is 5 and the order of 2,6 & 7is 10. So 2,6 & 7 are the primitive elements because every number occurs while other are not primitive elements because of missing several values. The smallest primitive element among these is 2.
Similarly one can find out the primitive elements for any prime numbers s.

3 Construction of Different Neighbour Balanced Incomplete Block Designs:

For v = s and r = k = s – 1 (where is the prime number) The initial block can be obtained easily for the primitive element x.

$$\mathbf{I = \{ x^{s-2}, x^{s-3}, x^{s-4}, \ldots\ldots\ldots\ldots\ldots\ldots, x^{1}, x^{s-1}\}}$$

This initial block when used to develop a design with cyclic shift provide a design.

3.1 For s = 5 then v = s = 5, and the primitive element are x = 2, 3.
The initial block for primitive element x = 2, with mod (s) of block size 5-1 = 4 is.

I = (3, 4, 2, 1)

Now taking mod(v) of this initial base block (since the total number of treatment is v), shall be the initial block of the design.

I = (3, 4, 2, 1)

It is the exactly same initial block as obtained with mod(s), as in this case v = s. Hence to obtained design with parameters v = s and r = k = s - 1 there is no need for obtaining further initial block.

The remaining blocks can be obtained using cyclically shift method, through this initial block and an Incomplete Block Design can be obtained.

3	4	2	1
4	0	3	2
0	1	4	3
1	2	0	4
2	3	1	0

The Neighbour Design of this block design can be obtained by using the method of border plots.

1	3	4	2	1	3
2	4	0	3	2	4

3	0	1	4	3	0
4	1	2	0	4	1
0	2	3	1	0	2

The neighbour design thus obtained is Neighbour Balanced Incomplete Block Design with parameters v = b = 5, r = k = 4 and λ = 2.

For v = s = 5 considering primitive element as x = 3 the initial block of size 4 is.

I = (2, 4, 3, 1)

The remaining blocks can be obtained using cyclically shift method through this initial block and an Incomplete Block Design can be obtained. Then using border plot the neighbour design obtained is Neighbour Balanced Incomplete Block Designs. Here initial block obtained from primitive elements x = 2 and x = 3 are same as both contains the identical treatments. Hence the obtained designs from these primitive elements x = 2 and x = 3 are also same.

3.2 For s = 11, then v = s = 11, r = k = 10, the primitive elements of s = 11 are x = 2, 6, 7.

The initial block for primitive element x = 2, with block size 10 is.

I = (6, 3, 7, 9, 10, 5, 8, 4, 2, 1)

The remaining blocks can be obtained using cyclically shift method through this initial block and an Incomplete Block Design obtained is:

6	3	7	9	10	5	8	4	2	1
7	4	8	10	0	6	9	5	3	2
8	5	9	0	1	7	10	6	4	3
9	6	10	1	2	8	0	7	5	4
10	7	0	2	3	9	1	8	6	5
0	8	1	3	4	10	2	9	7	6
1	9	2	4	5	0	3	10	8	7
2	10	3	5	6	1	4	0	9	8
3	0	4	6	7	2	5	1	10	9
4	1	5	7	8	3	6	2	0	10
5	2	6	8	9	4	7	3	1	0

The Neighbor Design is obtained by using the method of border plots.

1	6	3	7	9	10	5	8	4	2	1	6
2	7	4	8	10	0	6	9	5	3	2	7
3	8	5	9	0	1	7	10	6	4	3	8
4	9	6	10	1	2	8	0	7	5	4	9
5	10	7	0	2	3	9	1	8	6	5	10
6	0	8	1	3	4	10	2	9	7	6	0

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7	1	9	2	4	5	0	3	10	8	7	1
8	2	10	3	5	6	1	4	0	9	8	2
9	3	0	4	6	7	2	5	1	10	9	3
10	4	1	5	7	8	3	6	2	0	10	4
0	5	2	6	8	9	4	7	3	1	0	5

The neighbour design thus obtained is Neighbour Balanced Incomplete Block Designs with parameters v = b = 11, r = k = 10 and λ = 2.

For v = s = 11, considering primitive element x = 6 the initial block of size 10 is.

I = (2, 4, 8, 5, 10, 9, 7, 3, 6, 1)

The remaining blocks can be obtained using cyclically shift method through this initial block and an Incomplete Block Design can be obtained. Then using border plot the neighbour design obtained is Neighbour Balanced Incomplete Block Designs with parameters v = b = 11, r = k = 10 and λ = 2.

For v = s = 11, considering primitive elements x = 7 the initial block of size 10 is.

I = (8, 9, 6, 4, 10, 3, 2, 5, 7, 1)

The remaining blocks can be obtained using cyclically shift method, through this initial block and an Incomplete Block Design can be obtained. Then using border plot the neighbour design obtained is Neighbour Balanced Incomplete Block Designs with parameters v = b = 11, r = k = 10 and λ = 2.
Here designs for v = s = 11, and for primitive elements x = 2 and 6 are same but it is different from the neighbour design obtained for the primitive element as x = 7.

Conclusion Neighbour Balanced Incomplete Block Designs with different primitive elements of the prime number, s = 3, 5, 7, 11, 13, 17, 19, 23 and so on, may be constructed. It is to be noted that different primitive elements of the same prime number may provide different Neighbour Balanced Incomplete Block Designs for the same parameters v = b = s, r = k = s - 1 and λ = 2. Initial Blocks of the designs with parameters v = b = s, r = k = s-1 and λ = 2 for different primitive elements of s where s can be any prime number, are listed below:

Initial Block of the designs with parameters v = b = s, r = k = s-1 and λ = 2

s	**x**	**Initial block**
3	2	(2, 1)
5	2, 3	(3, 4, 2, 1)
7	3, 5	(5, 4, 6, 2, 3, 1)
11	2, 6	(6, 3, 7, 9, 10, 5, 8, 4, 2, 1)
11	7	(8, 9, 6, 4, 10, 3, 2, 5, 7, 1)

13	2, 7	(7, 10, 5, 9, 11, 12, 6, 3, 8, 4, 2, 1)
13	6, 11	(11, 4, 5, 3, 7, 12, 2, 9, 8, 10, 6, 1)
17	3, 6	(6, 2, 12, 4, 7, 8, 14, 16, 11, 15, 5, 13, 10, 9, 3, 1)
17	5, 7	(7, 15, 3, 4, 11, 9, 12, 16, 10, 2, 14, 13, 6, 8, 5, 1)
17	10	(12, 7, 11, 13, 3, 2, 7, 16, 5, 9, 6, 4, 14, 15, 10, 1)
17	11, 14	(14, 9, 7, 13, 12, 15, 6, 16, 3, 8, 10, 4, 5, 2, 11, 1)
19	2, 10	(10, 5, 12, 6, 3, 11, 15, 17, 18, 9, 14, 7, 13, 16, 8, 4, 2, 1)
19	3, 13	(3, 9, 8, 5, 15, 7, 2, 6, 18, 16, 10, 11, 14, 4, 12, 17, 13, 1)
19	14, 15	(15, 16, 12, 9, 2, 11, 13, 5, 18, 4, 3, 7, 10, 17, 8, 6, 14, 1)
23	5, 14	(14, 12, 7, 6, 15, 3, 19, 13, 21, 18, 22, 9, 11, 16, 17, 8, 20, 4, 10, 2, 5, 1)
23	7, 10	(10, 8, 11, 18, 19, 6, 14, 2, 20, 16, 22, 13, 15, 12, 5, 4, 17, 9, 21, 3, 7, 1)
23	11, 21	(21, 4, 15, 16, 14, 18, 10, 3, 17, 12, 22, 2, 19, 8, 7, 9, 5, 13, 20, 6, 11, 1)
23	15, 20	(20, 9, 19, 12, 10, 16, 21, 6, 5, 8, 22, 3, 14, 4, 11, 13, 7, 2, 17, 18, 15, 1)
23	19	(17, 13, 14, 8, 21, 12, 20, 18, 7, 4, 22, 6, 10, 9, 15, 2, 11, 3, 5, 16, 19, 1)
27	.	. . . .
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