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Study of Fourier series

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Abstract: The Fourier series, the founding principle behind the field of Fourier analysis, is an infinite expansion of a function in terms of sine's and cosines. In particular, the fields of electronics, quantum mechanics, and electrodynamics all make heavy use of the Fourier series.

Introduction

Recall that the mathematical expression

$$A_0 + \sum_{n=1}^{\infty} (A_n \cos(nx) + B_n \sin(nx)).$$

is called a **Fourier series**. Since this expression deals with convergence, we start by defining a similar expression when the sum is finite.

Definition. A **Fourier polynomial** is an expression of the form

$$F_n(x) = a_0 + \big(a_1 \cos(x) + b_1 \sin(x)\big) + \cdots + \big(a_n \cos(nx) + b_n \sin(nx)\big)$$

which may rewritten as

$$F_n(x) = a_0 + \sum_{k=1}^{k=n} \big(a_k \cos(kx) + b_k \sin(kx)\big).$$

The constants a_0, a_i and b_i, $i = 1, \cdots, n$, are called the **coefficients** of $F_n(x)$.

$$\sin(mx)\cos(nx) = \frac{1}{2}\Big[\sin((m+n)x) + \sin((m-n)x)\Big]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\Big[\cos((m+n)x) + \cos((m-n)x)\Big]$$

$$\sin(mx)\sin(nx) = \frac{1}{2}\Big[\cos((m-n)x) - \cos((m+n)x)\Big]$$

The Fourier polynomials are 2π-periodic functions. Using the trigonometric identities

we can easily prove the integral formulas

(1)

for $n \geq 0$, we have

$$\int_{-\pi}^{\pi} \sin(nx)dx = 0$$

for n>0 we have

$$\int_{-\pi}^{\pi} \cos(nx)dx = 0,$$

(2)

for m and n, we have

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)dx = 0,$$

(3)

for $n \neq m$, we have

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx = 0, \quad \text{and} \quad \int_{-\pi}^{\pi} \sin(mx)\sin(nx)dx = 0,$$

(4)

for $n \geq 1$, we have

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$$\int_{-\pi}^{\pi} \cos^2(nx)dx = \pi, \quad \text{and} \quad \int_{-\pi}^{\pi} \sin^2(nx)dx = \pi.$$

Using the above formulas, we can easily deduce the following result:

Theorem. Let

$$F_n(x) = a_0 + \sum_{k=1}^{k=n} \Big(a_k \cos(kx) + b_k \sin(kx)\Big).$$

We have

$$\begin{cases} a_0 &= \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} F_n(x)dx, \\ a_k &= \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} F_n(x)\cos(kx)dx, \quad 1 \leq k \leq n \\ b_k &= \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} F_n(x)\sin(kx)dx, \quad 1 \leq k \leq n. \end{cases}$$

This theorem helps associate a Fourier series to any 2π-periodic function.

Definition. Let $f(x)$ be a 2π-periodic function which is integrable on $[-\pi, \pi]$. Set

$$\begin{cases} a_0 &= \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} f(x)dx, \\ a_n = &= \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos(nx)dx, \quad 1 \leq n \\ b_n &= \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin(nx)dx, \quad 1 \leq n. \end{cases}$$

The trigonometric series

$$a_0 + \sum \left(a_n \cos(nx) + b_n \sin(nx)\right)$$

is called the **Fourier series** associated to the function $f(x)$. We will use the notation

CONCLUSION

The Fourier series is useful in many applications ranging from experimental instruments to rigorous mathematical analysis techniques. Thanks to modern developments in digital electronics, coupled with numerical algorithms such as the FFT, the Fourier Series has become one of the most widely used and useful mathematical tools available to any scientist.

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right).$$

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