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Hall Effects On Unsteady Magnetohydrodynamics Three Dimensional Flow through a Porous Channel in an Inclined Magnetic Field with No Slip Condition

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ABSTRACT

The hall effects on unsteady magnetohydrodynamic three dimensional flow of an incompressible viscous fluid through a porous channel in an inclined magnetic field with no slip condition was investigated. The fluid is bounded by a medium under the influence of a uniform magnetic field of strength H_0 inclined at an angle of inclination α. The governing equations of the flow were transformed to ordinary differential equation by a regular perturbation method and the expression for the solution of velocity was obtained. The behavior of the flow was computationally discussed with reference to the various governing parameters. It was found that the resultant velocity q and the magnitude of shear stress (at both lower and upper plate) increase with increase in hall parameter m and rotation parameter K. While, both decrease with increase in Hartmann number M and inverse Darcy parameter D^{-1}

Keywords: Hall effects, Unsteady, MHD, Inclined Magnetic field

1. Introduction

The hall effect on unsteady MHD flow in three dimension through a porous channel in an inclined magnetic field with no slip condition is a classical problem that has important application in magneto hydro dynamic (MHD) power generators and pumps, accelerators, aerodynamic, heating, electrostatic precipitation polymer technology, petroleum industry, purification of crude oil and fluid droplets, sprays, designing cooling systems with liquid metal, centrifugal separation of matter from fluid and flow meters. The flows of fluid through porous medium are very important particularly in the fields of agricultural engineering for irrigation processes; in petroleum technology to study petroleum transport; in chemical engineering for filtration and purification processes.

Sulochana (2014) [1] investigated the unsteady flow of incompressible viscous fluid between two rigid non-conducting rotating parallel plates bounded by a porous medium under the influence of a uniform magnetic field of strength H_0 inclined at an angle of inclination α with normal to the boundaries taking hall current into account. The perturbations are created by a constant pressure gradient along the plates in addition to the non-torsional oscillations of the upper plate while the lower plate is at rest. The flow in the porous medium is governed by the Brinkman's equations. The exact solution of the velocity in the porous medium consist of steady state and transient state. The time required for the transient state to decay is evaluated in detail and the ultimate quasi-steady state solution has been derived analytically. Its behaviour is computationally discussed with reference to the various governing parameters. The shear stresses on the boundaries are also obtained analytically and their behaviour is computationally discussed. Singh and Rastogi (2012) [4] studied the unsteady MHD flow of viscous incompressible and electrically conducting fluid through a porous medium adjacent to an accelerated impermeable plate in a rotating system. Heat transfer is also determined. In the analysis hall current is considered, and boundary wall slip flow and temperature slip conditions are assumed, Governing equations of velocity and temperature fields are solved numerically by Crank-Nicolson implicit finite difference scheme. It is found that interplay of Coriolis force and hydro magnetic force in the presence of boundary slip and hall current plays an important role in characterizing the flow behaviour. Effects of permeability, slip parameters, hall parameter, rotation parameter, magnetic field and prandtl number are determined on velocity and temperature fields, depicted graphically and discussed.

Farhad *et al* (2012) [7] investigated the slip effect on hydro magnetic rotating flow of a viscous fluid through a porous space where the fluid is an electrically conducting fluid with the consideration of Hall Current. The entire system rotates about the axis normal to a porous plate of uniform suction or injection with uniform angular velocity. The closed form solution is obtained using laplace transform technique. The analytical expression for skin friction is evaluated. The graphical results are displayed to see the effects of various embedded flow parameters such as magnetic parameter M , permeability parameter K , Hall parameter m , rotation parameter η , suction or injection parameter S , slip parameter γ and dimensionless time τ . It was found that the magnetic field and slip parameter decrease the velocity magnitude whereas permeability and Hall parameter increased it. The slip and magnetic field play an important role in retarding the growth of both the primary and secondary flows, whereas Hall parameter enhances the flow.

Swarnalathamma and Krishna (2016) [2] studied the heat transfer on MHD convective flow of viscous electrically conducting heat generating/absorbing fluid through porous medium in a rotating channel under uniform transverse magnetic field normal to the channel and taking Hall current. The flow is governed by the Brinkman's model. The diagnostic solutions for the velocity and temperature are obtained by perturbation technique and computationally discussed with respect to flow parameters through the graphs. The skin friction and Nusselt number are also evaluated and computationally discussed with reference to pertinent parameters in detail. Pandit *et al* (2016) [5] investigated the effects of Hall current and rotation on an unsteady Magneto-Hydrodynamic (MHD) free convection heat and mass transfer of an electrically conducting, viscous, incompressible and heat absorbing fluid flow past a vertical infinite flat plate embedded in non-Darcy porous medium. The flow is induced by a general time-dependent movement of the vertical plate, and the cases of ramped temperature and isothermal plates are studied. Exact solution of the governing time-dependent boundary layer equations for the momentum, energy and concentration were obtained in closed form by using Laplace transform technique. Expressions for skin friction due to primary and secondary flows and Nusselt number are derived for both ramped temperature and isothermal plates. Expression for Sherwood number is also derived. Some applications of practical interest for different types of plates. Veera and Chand (2016) [3] studied the effects of radiation and hall current on MHD free convection three dimensional flow in a vertical channel through a porous medium. The study involved the incompressible viscous and electrically conducting incompressible viscous fluid in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform pressure gradient parallel to the channel plate and the entire flow field is subjected to a uniform inclined magnetic field of strength H_0 inclined at an angle of inclination α with normal to the boundaries in the transverse xy plane. The temperature of one of the plates varies periodically and the temperature difference of the plates is high enough to induce radiative heat transfer. The effects of various parameters on the velocity profiles, the skin friction, temperature field, rate of heat transfer in terms of their amplitude and phase angles are shown graphically. Pattnaik (2017) [6] analysed an unsteady hydro magnetic flow past an infinite vertical porous plate showing the effect of an additional cross transport phenomenom, i.e. heat flux caused by concentration gradient in addition to the heat flux caused by temperature gradient. The effect of magnetic field on the fluid temperature and the heat transfer between fluid and wall is of considerable importance affecting the flow. Further, hall current, an additional electric current density so generated perpendicular to both applied electric field and magnetic has been taken into consideration in the study. Moreover, the Dufor effect has been considered in energy equation leaving the equation of thermal diffusion and mass diffusion coupled. The coupled non-linear equations are solved by applying a special function $Hh_n\left(x\right)$. The effects of flow parameters are shown with the help of graphs and tables. A phenomenal observation, i.e. a radical change is marked near the plate in respect of Dufor number in the presence of suction. Further, it is to note that suction induces backflow in conjunction with opposing buoyancy forces. Hall current contributes to greater skin friction at the bounding surface.

In this present paper, the hall effects on unsteady magnetohydrodynamic three dimensional flow of an incompressible viscous fluid through a porous channel in an inclined magnetic field with no slip condition was investigated. The fluid is bounded by a medium under the influence of a uniform magnetic field of strength H_0 inclined at an angle of inclination α. The governing equations of the flow were transformed to ordinary differential equation by a regular perturbation method and the expression for the solution of velocity was obtained. The behavior of the flow was computationally discussed with reference to the various governing parameters.

2. Formulation of the Problem

We consider the unsteady flow of an incompressible electrically conducting viscous fluid bounded by porous medium with two non-conducting rotating parallel plates. A uniform transverse magnetic field is applied to the *z*-axis. In the presence of a strong magnetic field a current is inclined in a direction normal to both the electric and magnetic field viz. Magnetic field of strength *Ho* inclined at an angle of inclination α to the normal of the boundaries in the transverse *xz* plane. The inclined magnetic field gives rise to a secondary flow transverse to the channel. In the equation of motion along x-direction, the component current density — $\mu_e J_Z H_o \sin\alpha$ and the *z*-component current density $\mu_e J_x H_o \sin\alpha$. We choose a Cartesian system 0 (*x, y, z*) such that the boundary walls are at z = 0 and z = 1. Under the Brinkman's equations of flow through porous medium, the unsteady hydro magnetic flow under the influence of a transverse magnetic field with reference to a rotating frame is governed by the following Cartesian equations;

Momentum Equations

$$\frac{\partial u}{\partial t}+2\Omega w=-\frac{I}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial z^2}-\frac{\mu_e J_z H_o \sin\alpha}{\rho}-\frac{v}{k}u \qquad (1)$$

$$\frac{\partial w}{\partial t}-2\Omega u=v\frac{\partial^2 w}{\partial z^2}+\frac{\mu_e J_x H_o \sin\alpha}{\rho}-\frac{v}{k}u \qquad (2)$$

Where , (u,w) is the velocity component along $O(x,z)$ directions respectively. ρ is the density of the fluid. μ_e is the magnetic permeability, v is the coefficient of kinematic viscosity , k is the ρ permeability of the medium, H_o is the applied magnetic field. When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current, so that

$$J+\frac{\omega_e\tau_e}{H_O}J\times H=\sigma(E+\mu_e q\times H) \qquad (3)$$

Where, q is the velocity vector, H is the magnetic field intensity vector, E is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time, σ is the fluid conductivity and, μ_e is the magnetic permeability . In Equation (3) the electron pressure gradient, the ion-slip and thermoelectric effects are neglected. We also assume that the electric field $E=0$ under assumptions, it then reduces to

$$J_x-mJ_z\sin\alpha=-\sigma\mu_e H_o\sin\alpha w \qquad (4)$$

$$J_z+mJ_x\sin\alpha=\sigma\mu_e H_o\sin\alpha u \qquad (5)$$

Where $m=\omega_e\tau_e$ is the Hall parameter.

We shall now assume that the equation of motion with reference to a rotating frame is given by;

$$\frac{\partial q}{\partial t}-2iK^2q=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 q}{\partial z^2}-\frac{\sigma\mu_e^2H_o^2\sin^2\alpha}{\rho(1+im\sin\alpha)}q-\frac{v}{k}q. \qquad (6)$$

The boundary and initial conditions are

$$q = 0,\ t \le 0,\ z = 0 \tag{7}$$

$$q = 0,\ t > 0,\ z = l \tag{8}$$

We now introduce the following non dimensional variables

$$z = z^{*}l,\quad q = \frac{q^{*}v}{l},\ t = \frac{t^{*}l^{2}}{v},\ \ \omega = \frac{\omega^{*}v}{l^{2}},\quad \xi = \xi^{*}l,\quad p = \frac{p^{*}v^{2}}{l} \tag{9}$$

Dropping asterisks the non-dimensional variables governing the equations becomes,

$$\frac{\partial q}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^{2} q}{\partial z^{2}} - \frac{M^{2}\sin^{2}\alpha}{(1 + im\sin\alpha)}q - D^{-1}q \tag{10}$$

Where,

$M^{2} = \dfrac{\sigma\mu_{e}^{2}H_{o}^{2}l^{2}}{\rho v}$ Is the Hartmann number;

$D^{-1} = \dfrac{l^{2}}{k}$ Is the inverse Darcy parameter

$K^{2} = \dfrac{\Omega^{2}l^{2}}{v}$ Is the rotating parameter;

$D^{-1} = \dfrac{l^{2}}{k}$ Is the inverse Darcy parameter

$m = \omega_{e}\tau_{e}$ Is the Hall parameter.

And the corresponding boundary conditions are

$$q = 0,\ t \le 0,\ z = 0 \tag{11}$$

$$q = 0,\ t > 0,\ z = 0 \tag{12}$$

3. Method/Solution of the Problem

In this Section, we used the Perturbation method to solve equation (10) subjects to the boundary conditions in equations (11) and (12) as follows

$q(z,t) = q_0 + \varepsilon q_1 e^{iwt} + 0(\varepsilon^2)$ and we choose $-\frac{\partial p}{\partial x} = \partial e^{iwt}$ as the prescribed pressure gradient and the amplitude of oscillation as $\varepsilon << 1$.

The periodic and non – periodic terms of equation (10) are

$$\frac{\partial^2 q_0}{\partial z^2} - L_2 q_0 = -\lambda e^{iwt} \quad (13)$$

$$\frac{\partial^2 q_1}{\partial z^2} - L_3 q_1 = 0 \quad (14)$$

Where, $\left(L_1 = \frac{M^2 \sin^2 \alpha}{1 + im\sin\alpha}\right)$, $\left(L_2 = L_1 + \frac{1}{D}\right)$, $(L_3 = L_2 + iw)$

And the corresponding boundary conditions become

$$q_0 = q_1 = 0,\ t \le 0,\ z = 0 \quad (15)$$

$$q_0 = q_1 = 0,\ t > 0,\ z = 1 \quad (16)$$

Solving equations (13) and (14) subject to the boundary conditions (15) and (16), we obtained the solution for the velocity as follows;

$$q(z,t) = C_1 e^{m_1 z} + C_2 e^{m_2 z} + \frac{\lambda e^{i\omega t}}{L_2} + (C_3 e^{m_3 z} + C_4 e^{m_4 z}) e^{i\omega t} \quad (17)$$

Where, $\Rightarrow m_1 = \sqrt{L_2}$, $m_2 = -\sqrt{L_2}$, $L_4 = -\frac{\lambda e^{iwt}}{L_2}$, $C_1 e^{m_1} = L_4 - C_2 e^{m_2}$, $C_2 = \frac{L_4 e^{m_1} - L_4}{e^{m_1 - m_2}}$, ,

$\Rightarrow m_3 = \sqrt{L_3}, m_4 = -\sqrt{L_3}$, $C_3 = \sin m\pi$, $C_4 = -C_3$

The Shear Stress at the lower and upper plate are respectively;

$$\tau_L = \frac{\partial q}{\partial z}\Big|_{z=0} \text{ and } \tau_L = \frac{\partial q}{\partial z}\Big|_{z=1} \quad (18)$$

4. Results and Discussion

The hall effects on unsteady magnetohydrodynamic three dimensional flow of an incompressible viscous fluid through a porous channel in an inclined magnetic field with no slip condition has been studied. The flow is governed by non-dimensional parameters M the Hartman number, D^{-1} the inverse Darcy parameter, K is the rotation parameter and m is the Hall parameter. The velocity field in the porous region is evaluated analytically its behaviour with reference to variations in the governing parameters has been computationally analysed. The profile for k have been plotted in the entire flow field in the porous medium using MATLAB.

We now discuss the quasi steady solution for the velocity for different sets of governing parameters namely viz. M the Hartman number and D^{-1} the inverse Darcy parameter, K the rotation parameter, m is the Hall parameter, the frequency oscillations ω , a and b the constants related to non-torsional oscillations of the boundary, for computational analysis purpose we are fixing the axial pressure gradient as well as a and b and $\omega = \pi/4$, $\omega_1 = \pi/4$, $\alpha = \pi/3$. Figures 1-4 corresponding to the velocity component q along the prescribed pressure gradient for different sets of governing parameters when the upper boundary plates executes non-torsional oscillations. Figure 1 depicts that the magnitude of the velocity q decreases as the Hartmann number M increases. To this effect the magnetic field suppresses the turbulence of the flow.

Figure 2 demonstrates that the magnitude of the velocity q increases with increasing the inverse Darcy parameter D^{-1} . The magnitude of the velocity q reduces in on the upper part of the fluid region, while it increases on the lower part with increasing the Hall parameter m in Figure 3. Finally we notice that, from (figure 4) the magnitude of the velocity component enhances and reduces with increase in the rotation parameter K .

The shear stress at the lower and upper plates (τ_L and τ_U) have been calculated for the different variations in the governing parameters and are tabulated in the Table 1 and Table 2 respectively. On the upper plate we notice that the magnitudes of t_u enhances the inverse Darcy parameter D^{-1} the hall parameter m , rotation parameter K decreases with increase in the Hartmann number M . The similar behaviour is observed on the lower plate (Table 1). The lower plate was also noticed to have a smaller magnitude of the shear stress compare to its upper plate values.

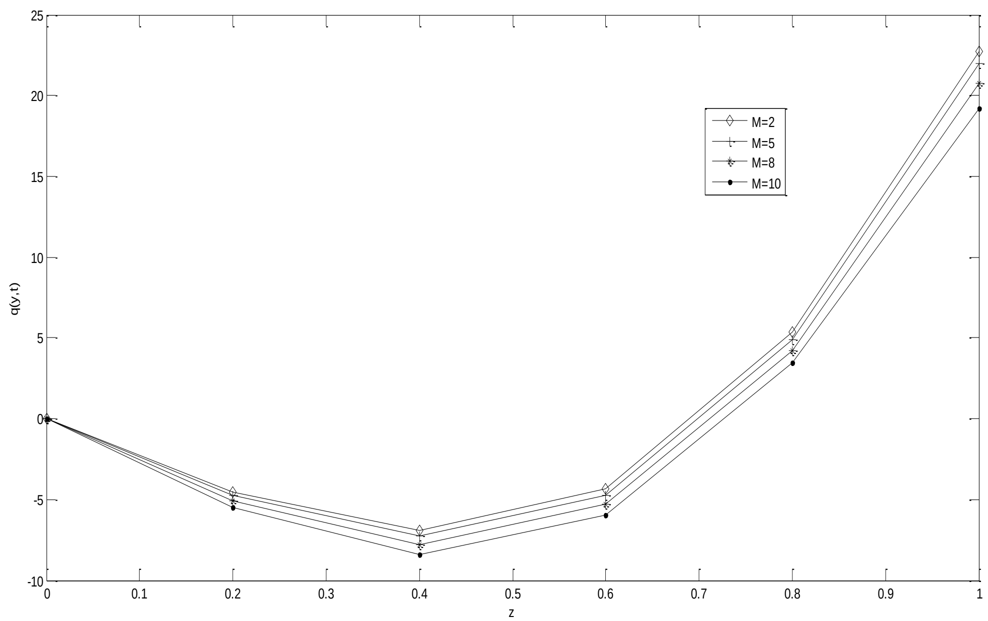

Figure 1: The velocity profile for q with M

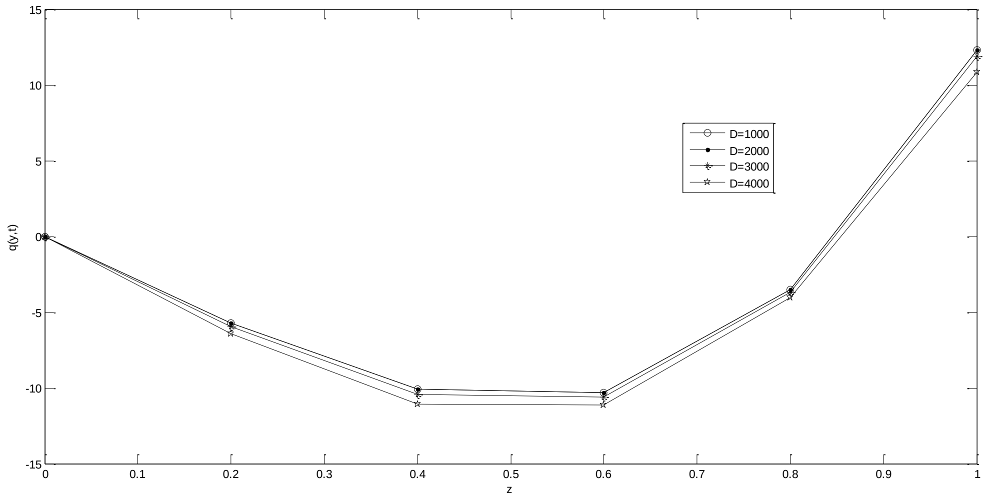

Figure 2: The velocity profile for q with D

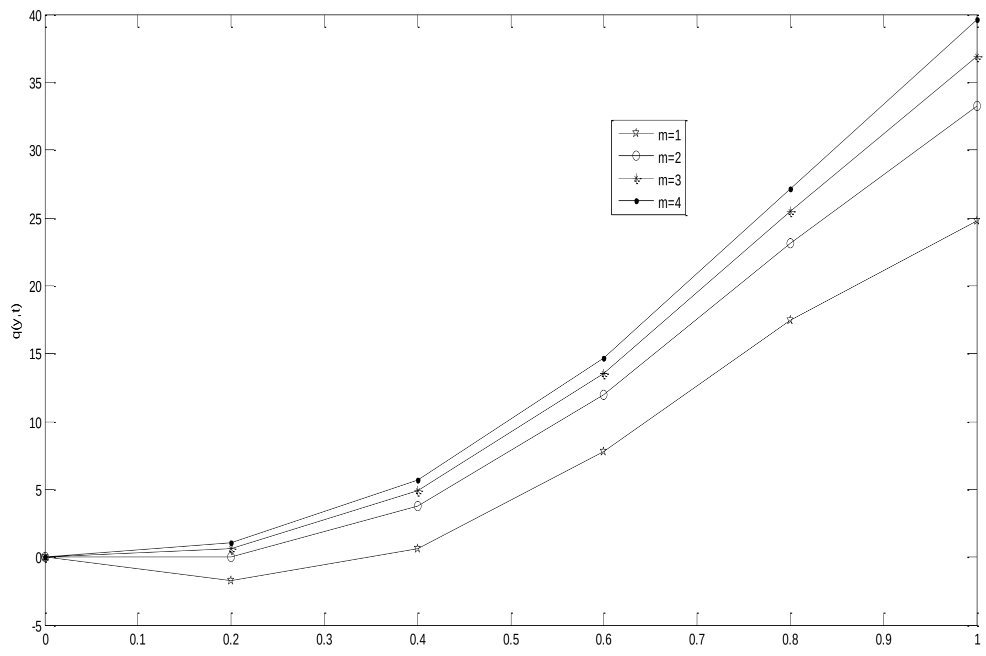

Figure 3: The velocity profile for q with m

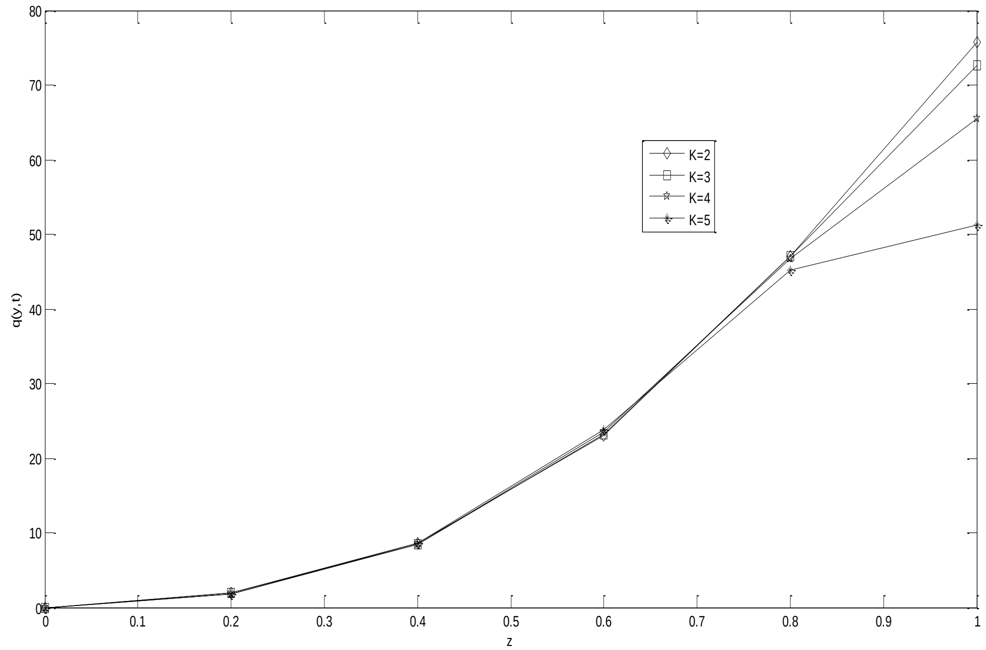

Figure 4: The velocity profile for q with K

Table 2. The shear stress τ_U on the upper plate

M	I	II	III	IV	V	VI	VII
2	0.002272	0.0022714	0.0022784	0.0032186	0.0043209	0.00329	0.09458
5	0.0062196	0.0062149	0.005319	0.005209	0.063129	0.0064329	0.09237
D^{-1}	1000	2000	3000	1000	1000	1000	1000
m	1	1	1	2	3	1	1
K	2	2	2	2	2	3	4

Table 1. The shear stress τ_L on the lower plate

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5. Summary and Conclusion

The hall effects on unsteady magnetohydrodynamic three dimensional flow of an incompressible viscous fluid through a porous channel in an inclined magnetic field with no slip condition was investigated. The fluid is bounded by a medium under the influence of a uniform magnetic field of strength H_0 inclined at an angle of inclination α. The governing equations of the flow were transformed to ordinary differential equation by a regular perturbation method and the expression for the solution of velocity was obtained. The behavior of the flow was computationally discussed with reference to the various governing parameters. It was found that the resultant velocity q and the magnitude of shear stress (at both lower and upper plate) increase with increase in hall parameter m and rotation parameter K. While, both decrease with increase in Hartmann number M and inverse Darcy parameter D^{-1}.

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M	I	II	III	IV	V	VI	VII
2	-12.6216	-12.5237	-12.4327	-12.3172	-12.3079	-12.2197	-12.10196
5	-11.6488	-11.6487	-11.5387	-11.52639	-11.51712	-11.41328	-11.31109
D^{-1}	1000	2000	3000	1000	1000	1000	1000
m	1	1	1	2	3	1	1
K	2	2	2	2	2	3	4

rotating vertical plate fluctuating with Heat Source/Sink and Chemical reaction. *IOSR Journal of Mathematics (IOSR-JM)* 2278-5728.

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