International Journal of Research
Available at http://internationaljournalofresearch.org/

p-ISSN: 2348-6848
e-ISSN: 2348-795X

Volume 02 Issue 02
February 2015

Minimization of Error in Exponential Model Estimation via Jackknife Algorithm

Iheagwara Andrew .I.[*] & Opara Jude[**]

[*]E-mail Address: andyiheagwara@yahoo.com
Procurement Officer/Director Planning, Research & Statistics, Nigeria Erosion & Watershed Management Project (World Bank-Assisted), Ministry of Petroleum & Environment, Ploy 36, chief Executive Quarters, Area "B", New Owerri, Imo State Nigeria
GSM: +2348033285954

[**]E-mail Address: judend88@yahoo.com
Department of Statistics, Imo State University
PMB 2000, Owerri Nigeria
GSM: +2348064095411

ABSTRACT
This paper is on the Minimization of Error in exponential model Estimation via Jackknife Algorithm. The data were on 25 samples of percentage sugar and percentage of Nitrogen in tobacco leaf for organic and inorganic chemical constituents. The study concerning the use of jackknife methods in estimating the parameters of non linear regression models have been identified in this paper. An algorithm for the estimation of nonlinear regression parameters was stated. For estimating these parameters, computer programs were written in Stata for the implementation of these algorithms. In the estimation of the nonlinear regression parameters, the results obtained from numerical problems using the Jackknife based algorithm developed yielded a reduced error sum of squares than the analytic result. As the number of d observations deleted in each re-sampling stage increases, so does the error sum of squares reduces minimally. This reveals the appropriateness of the algorithms for the estimation of nonlinear regression parameters and in the reduction of the error terms in nonlinear regression estimation.

Key words: Jackknife algorithm, Bivariate non-linear regression, Delete – d, Gauss-Newton

Introduction

The jackknife or "leave one out" procedure is a cross-validation technique first developed by Quenouille to estimate the bias of an estimator. John Tukey then expanded the use of the jackknife to include variance estimation and tailored the name of jackknife because like a jackknife-a pocket knife akin to a Swiss army knife and typically

used by boy scouts-this technique can be used as a "quick and dirty" replacement tool for a lot of more sophisticated and specific tools. Curiously, despite its remarkable influence on the statistical community, the seminal work of Tukey is available only from an abstract (which does not even mention the name of jackknife) and from an almost impossible to find unpublished note (although some of this note found its way into Tukey's complete work).

The jackknife estimation of a parameter is an iterative process. First the parameter is estimated from the whole sample. Then each element is, in turn, dropped from the sample and the parameter of interest is estimated from this smaller sample. This estimation is called a partial estimate (or also a jackknife replication). A pseudo-value is then computed as the difference between the whole sample estimate and the partial estimate. These pseudo-values reduce the (linear) bias of the partial estimate (because the bias is eliminated by the subtraction between the two estimates). The pseudo-values are then used in lieu of the original values to estimate the parameter of interest and their standard deviation is used to estimate the parameter standard error which can then be used for null hypothesis testing and for computing confidence intervals. The jackknife is strongly related to the bootstrap (i.e., the jackknife is often a linear approximation of the bootstrap) which is currently the main technique for computational estimation of population parameters.

Related Literature Review

Batah et al (2008) carried out a research on the efficiency of modified Jackknife and ridge type regression estimators; a comparison. In their work, they proposed a new estimator known as, Modified Jackknife Ridge Regression Estimator (MJR). It was based on the criterion that combines the idea underlying both the Generalized Ridge Regression (GRR) and Jackknifed Ridge Regression (JRR) estimators. They investigated standard properties of this new estimator. From a simulation study, they found that the new estimator often outperforms the LASSO, and it was superior to both GRR and JRSS estimators, using the mean squared error criterion. The condition under which the MJR estimator was better than the other two competing estimators was also investigated.

Shao and Rao (1993) carried out a research paper on Jackknife inference for heteroscedastic linear regression models. Inference on the regression parameters in a heteroscedastic linear regression model with replication is considered, using either the ordinary least-squares (OLS) or the weighted least-squares (WLS) estimator. A delete-group jackknife method is shown to produce consistent variance estimators irrespective of within-group correlations, unlike the delete-one jackknife variance estimators or those based on the customary δ-method assuming within-group independence. Finite-sample properties of the delete-group variance estimators and associated confidence intervals are also studied through simulation.

International Journal of Research
Available at http://internationaljournalofresearch.org/
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 02 Issue 02
February 2015

Hongchang and Yuhe (2013) worked on Jackknifed Liu estimator in linear regression models. In their paper, they introduced a generalized Liu estimator and jackknifed Liu estimator in a linear regression model with correlated or heteroscedastic errors. Therefore, they extended the Liu estimator. Under the mean square error (MSE), the jackknifed estimator was superior to the Liu estimator and the jackknifed ridge estimator. They also gave a method to select the biasing parameter for *d*. Furthermore, a numerical example was given to illustrate these theoretical results.

Wu (1986) researched on Jackknife bootstrap and other re-sampling methods in regression analysis; motivated by a representation for the least squares estimator, they proposed a class of weighted jackknife variance estimators for the least squares estimator by deleting any fixed number of observations at a time. They are unbiased for homoscedastic errors and a special case, the delete-one jackknife, is almost unbiased for heteroscedastic errors. The method was extended to cover nonlinear parameters, regression M-estimators, non-linear regression and generalized linear models. Three bootstrap methods were considered. Two were shown to give biased variance estimators and one does not have the bias-robustness property enjoyed by the weighted delete-one jackknife. A general method for re-sampling residuals was proposed, and some simulation results were reported.

Ekezie et al (2014) researched on estimation of bivariate linear regression data via jackknife algorithm. The data used for the research were collected from Orji Town Primary School, Owerri North Imo State Nigeria. The data were on heights and weights of 20 randomly selected pupils in primary five and six. The jackknife algorithm was used to estimate the regression parameters, and the bias of the estimate. The result of their analysis revealed that the bias result was insignificant. For the regression, the jackknife parameters are linear functions of the standard estimates, which implies that the values of $\hat{Y}_n^*$ can be perfectly predicted from the values of $\hat{Y}_n$. The jackknife predicted values were calculated in their analysis.

Meterials and Method

Given a model of the form

$$Y = f(X_1, X_2, \ldots, X_k, b_1, b_2, \ldots b_j) + \varepsilon \quad \ldots \quad (1)$$

where the b's are the parameters, X's are the predictor variables and the error term $\varepsilon \sim N(0, \sigma^2)$ independently identically distributed and are uncorrelated.
Equation (1) is assumed to be intrinsically nonlinear.

Suppose we have a sample of n observations on the Y and X's, then, we can write equation.

$$Y = f(X_{i1}, X_{i2}, \ldots, X_{ik}, b_1, b_2, \ldots b_j) + \varepsilon_i \quad ; \quad i = 1, 2, \ldots, n \quad \ldots \quad (2)$$

The n-equation can be written in a matrix notation as

$$Y = f(X, b) + \varepsilon \quad \ldots (3)$$

where

International Journal of Research
Available at http://internationaljournalofresearch.org/
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 02 Issue 02
February 2015

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{k1} \\ X_{12} & X_{22} & \dots & X_{k2} \\ \vdots & \vdots & \dots & \vdots \\ X_{1n} & X_{2n} & \dots & X_{kn} \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_j \end{bmatrix}, \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \text{ and } E(\varepsilon) = 0$$

The error sum of squares for the nonlinear model is defined as

$$Q = S(\varepsilon) = \sum_{i=1}^{n} \{Y_i - f(X_i, b)\}^2 \qquad \dots \qquad (4)$$

Let the least square estimates of b by $\hat{b}$, these estimates minimize the S(ε). The least square estimates of b are obtained by differentiating (4) with respect to b, equate to zero and solve for $\hat{b}$, this results in J normal equations:

$$\frac{\partial Q}{\partial b_p} = \sum_{i=1}^{n} -2[Y_i - f(X_i, b)] \frac{\partial}{\partial b_p} (f(X_i, b)$$

$$-2\left[\sum_{i=1}^{n} Y_i \left[\frac{\partial}{\partial b_p} f(X_i, b)\right]_{b=\hat{b}} - \sum_{i=1}^{n} f(X_i, \hat{b}) \left[\frac{\partial}{\partial b_p} f(X_i, b)\right]\right]_{b=\hat{b}} = 0$$

$$\sum_{i=1}^{n} Y_i \left[\frac{\partial}{\partial b_p} f(X_i, b)\right]_{b=\hat{b}} - \sum_{i=1}^{n} f(X_i, \hat{b}) \left[\frac{\partial}{\partial b_p} f(X_i, b)\right]_{b=\hat{b}} = 0$$

$$\sum_{i=1}^{n} \{Y_i - f(X_i, \hat{b})\} \left[\frac{\partial}{\partial b_p} f(X_i, b)\right]_{b=\hat{b}} = 0 \text{; i =1, 2, …, n; p = 1, 2, …, J} \quad (5)$$

In estimating the parameters of nonlinear regression model, we use the Guass-Newton method based on Taylor's series to approximate equation (3). Now, considering the function f(X, b) which is the deterministic component of

$$Y_i = (X_i, b) + \varepsilon_i \qquad \text{i = 1, 2,…, n} \qquad \dots \qquad (6)$$

Let b^0 be the initial approximate value of b. Applying Taylor's series expansion of $f(X_i, b)$ about b^0., we have the linear approximation

$$f(X_i, b) = f(X_i, b^0) + (b - b^0) \frac{\partial}{\partial b} f(X_i, b)\Big|_{b=b^0} \qquad \dots \qquad (\dots)$$

Substituting expressions (7) in (6), we get

$$Y_i - f(X_i, b^0) + \sum_{i=1}^{J} \left[\frac{\partial}{\partial b_i} f(X_i, b)\right]_{b=b^0} \left(b_p - b_p^0\right) + \varepsilon_i \text{ i = 1, 2, …, n; p = 1, 2, …, J ..}$$

(8)

Equation (8) may be viewed as a linear approximation in a neighborhood of the starting value b^0. Let

$$f_i^0 = f(X_i, b^0)$$

International Journal of Research
Available at http://internationaljournalofresearch.org/
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 02 Issue 02
February 2015

$$\beta_p^0 = b_p - b_p^0$$

$$Z_{pi}^0 = \left[\frac{\partial}{\partial b_p} f(X_i, b)\right]_{b=b^0} \quad ; \text{ for i = 1, 2, …, n; and p = 1, 2, …, J}$$

Hence, equation (8) becomes

$$Y_i = f_i^0 + \sum_{p=1}^{J} Z_{pi}^0 \beta_p^0 + \varepsilon_i, \qquad \text{i = 1, 2, …, n} \qquad \ldots \qquad (9)$$

$$Y_i - f_i^0 = \sum_{p=1}^{J} Z_{pi}^0 \beta_p^0 + \varepsilon_i, \qquad \text{i = 1, 2, …, n} \qquad \ldots \qquad (10)$$

In a matrix form, we have

$$\begin{bmatrix} Y_1 - f_1^0 \\ Y_2 - f_2^0 \\ \vdots \\ Y_n - f_n^0 \end{bmatrix} = \begin{bmatrix} Z_{11}^0 & Z_{21}^0 & \ldots & Z_{J1}^0 \\ Z_{12}^0 & Z_{22}^0 & \ldots & Z_{J2}^0 \\ \vdots & \vdots & \ldots & \vdots \\ Z_{1n}^0 & Z_{2n}^0 & \ldots & Z_{Jn}^0 \end{bmatrix} \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \vdots \\ \beta_j^0 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \qquad \ldots \qquad (11)$$

In a Compactable form, equation (11) becomes

$$Y - f^0 = Z^0\beta^0 + \varepsilon \qquad \ldots \qquad (12)$$

where

$$Y - f^0 = [Y_1 - f_1^0, Y_2 - f_2^0 \ldots Y_n - f_n^0]' \; Z^0 = \begin{bmatrix} Z_{11}^0 & \ldots & Z_{J1}^0 \\ Z_{12}^0 & \ldots & Z_{J2}^0 \\ \vdots & \ldots & \vdots \\ Z_{1n}^0 & \ldots & Z_{Jn}^0 \end{bmatrix} \beta^0 = (\beta_1^0 \ldots \beta_J^0)' \; \varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)'$$

we obtain the sum of squares error (SSε)

$$SS\varepsilon = (\varepsilon'\varepsilon) = \left((Y - f^0) - Z^0\beta^0\right)' \left((Y - f^0) - Z^0\beta^0\right)$$

$$= (Y - f^0)'(Y - f^0) - 2(Y - f^0)'(Z^0\beta^0) + (Z^0\beta^0)'(Z^0\beta^0)$$

$$\frac{\partial SS\varepsilon}{\partial \beta^0} = -2(Y - f^0)'Z^0 + 2\left(Z^0\hat{\beta}^0\right)' Z^0 = 0$$

$$(Y - f^0)'Z^0 = Z^0\left(Z^0\hat{\beta}^0\right) \qquad \ldots \qquad (13)$$

Hence,

$$\hat{\beta}^0 = (Y - f)' Z^0 \left(Z^{0'} Z^0\right)^{-1} \qquad \ldots \qquad (14)$$

Therefore, the least square estimates of β^0 is

$$\hat{\beta}^0 = \left(Z^{0'} Z^0\right)^{-1} Z^{0'} (Y - f^0) \qquad \ldots \qquad (15)$$

Thus, $\hat{\beta}^0 = \left(\hat{\beta}_1^0, \hat{\beta}_2^0, \ldots, \hat{\beta}_J^0\right)'$ minimizes the error sum of squares

$$S^*(\varepsilon) = \sum_{i=1}^{n} \left(Y_i - f_i^0 - \sum_{p=1}^{J} Z_{pi}^0 \hat{\beta}_p^0\right)^2 \qquad \ldots \qquad (16)$$

Now, the estimates of parameters b_p of non-linear regression (1) are

$$b_p^1 = \hat{\beta}_p^0 + b_p^0; \; \text{p = 1, 2, …, J}$$

Iteratively, equation (17) reduces to

International Journal of Research
Available at http://internationaljournalofresearch.org/
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 02 Issue 02
February 2015

$$\begin{aligned} b^{1} &= \hat{\beta}^{0} + b^{0} \\ b^{2} &= \hat{\beta}^{1} + b^{1} \\ \vdots \quad &\quad \vdots \quad \quad \vdots \\ b^{r} &= \hat{\beta}^{r-1} + b^{r-1} \\ b^{r+1} &= \hat{\beta}^{r} + b^{r} \end{aligned}$$

Thus

$$b^{r+1} = b^{r} + \left(Z^{r'}Z^{r}\right)^{-1} Z^{r'}(Y - f^{r}) \qquad \ldots \qquad (18)$$

where $\hat{\beta}^{r} = \left(Z^{r'}Z^{r}\right)^{-1} Z^{r'}(Y - f^{r})$ are the least squares estimates of β obtained at the (r + 1)th iterations. The iterative process continues until $\left|\frac{\hat{b}_{p}^{(r+1)} - \hat{b}_{p}^{(r)}}{\hat{\theta}_{p}^{(r)}}\right| < \delta$ where $\delta = 10^{-5}$ is the error tolerance (Smith and Draper (1998); Nduka (1999)).

After each iteration, $S^{*}(\varepsilon)$ is evaluated to check if a reduction in its value has actually been achieved. At the end of the (r + 1)th iteration, we have

$$S^{*}(\varepsilon)^{r} = \sum_{i=1}^{n} \left(Y_i - f_i^{r} - \sum_{j=1}^{p} Z_{ji}^{r} \hat{\beta}_{p}^{r} \right)^{2} \qquad \ldots \qquad (19)$$

and iteration is stopped if convergence is achieved. The final estimates of the parameters at the end of the (r + 1)th iteration are: $\hat{b}_{1}^{(r+1)}, \hat{b}_{2}^{(r+1)}, \ldots, \hat{b}_{j}^{(r+1)}$.

Jackknife Delete-One Algorithm for the Estimation of Non-linear Regression Parameters

Let $W_i = \left(Y_i, Z_{ji}\right)'$ vector denotes the values associated with ith (w_1, w_2, …, w_n) observation sets. The steps of the delete-one jackknife regression are as follows.

Given randomly drawn sample of size n from a population and label the elements of the vector $W_i = (Y_i, Z_{ji})'$ as the vector $Y_i = (y_1, y_2, \ldots, y_n)'$ be the response variables, $Z_{ji} = (z_{j1}, z_{j2}, \ldots, z_{jn})'$ is the matrix of dimension n × k for the predictor variables, where j = 1, 2, …, k and i = 1, 2, …, n.

Step 1. Omit first row of the vector $W_i = (Y_i, Z_{ji})'$ and label remaining n – 1 observation sets $Y_i^{(J)} = \left(y_2^{(J)}, y_3^{(J)}, \ldots, y_n^{(J)}\right)'$ and $Z_{ji}^{(J)} = \left(z_{j2}^{(J)}, z_{j3}^{(J)}, \ldots, z_{jn}^{(J)}\right)'$ as the first delete-one Jackknife sample $\left(W_1^{(J)}\right)$.

Step 2. Calculate the least square estimates for nonlinear regression coefficient from the first jackknife sample; $\hat{\beta}^{0} = (Z'Z)^{-1}Z'(Y - f)$.

Step 3. Compute $\hat{b}^{1} = \hat{b}^{0} + \hat{\beta}^{0}$ using the Guass-Newton method, the $\hat{b}^{1}$ value is treated as the initial value in the first approximated linear model.

Step 4. We return to the second step and again compute $\hat{\beta}'s$. At each iteration, new $\hat{\beta}'s$ represent

International Journal of Research
Available at http://internationaljournalofresearch.org/
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 02 Issue 02
February 2015

increments that are added to the estimates from the previous iteration according to step 3 and eventually find $\hat{b}^2$, which is $\hat{b}^2 = \hat{b}^1 + \hat{\beta}^1$ up to $b^{r+1} = b^r + \hat{\beta}^r$.

Step 5. Stopping Rule; this iteration process continues until $\left|\frac{\hat{b}_p^{(r+1)} - \hat{b}_p^{(r)}}{\hat{b}_p^{(r)}}\right| < \delta$, where $\delta = 10^{-5}$, for the values of $\hat{b}_p^{(1)}, \hat{b}_p^{(2)}, \ldots, \hat{b}_p^{(r+1)}$ from the first delete-one Jackknife estimates $\hat{b}^{(J1)}$.

Step 6. Then omit second row of the vector $W_i = (Y_i, Z_{ji})'$ and label remaining n – 1 sized observation sets $Y_i^{(J)} = \left(y_1^{(J)}, y_3^{(J)}, \ldots, y_n^{(J)}\right)'$ $Z_{ji}^{(J)} = \left(z_{j1}^{(J)}, z_{j3}^{(J)}, \ldots, z_{jn}^{(J)}\right)'$ as $\left(W_2^{(J)}\right)$ and repeat steps 2 to 5 above for the estimate of regression coefficients $\hat{\theta}^{(J2)}$. Similarly, omit each one of the n observation sets and estimate the non linear regression coefficients as in the step 2 to 5 above for $\hat{b}^{(Ji)}$ alternatively, where $\hat{b}^{(Ji)}$ is Jackknife regression coefficient vector estimated after deleting of ith observation set from W_i..

Step 7. Obtain the probability distribution $F\left(\hat{b}^{(J)}\right), \hat{b}^{(J1)}, \hat{b}^{(J2)}, \ldots, \hat{b}^{(Jn)}$ of Jackknife estimates $\hat{b}^{(J1)}, \hat{b}^{(J2)}, \ldots, \hat{b}^{(Jn)}$.

Step 8. Calculate the jackknife regression coefficient estimate which is the mean of the $F\left(\hat{b}^{(J)}\right)$ distribution [Obiora-Ilonu et al (2012)] as;

$$\hat{b}^{(J)} = \frac{\sum_{r=1}^{n} \hat{b}^{(Ji)}}{n} = \bar{\hat{b}}^{(Ji)} \quad \ldots \quad (21)$$

Jackknife Delete-d Algorithm for Estimation of Non-Linear Regression

Let $W_i = \left(Y_i, Z_{ji}\right)'$ vector denotes the values associated with ith (w_1, w_2, …, w_n) observation sets. Draw a random sample of size n from the observation set (population) and label the elements of the vector $W_i(Y_i, Z_{ji})'$ as the vector $Y_i = (y_1, y_2, \ldots, y_n)'$ be the response variables, $Z_{ji} = (z_{j1}, z_{j2}, \ldots, z_{jn})'$ be the matrix of dimension n × k for the predictor variables, where j = 1, 2, …, k and i = 1, 2, …, n.

Stage 1: Divide the sample into "s" independent group of size d.

Stage 2: Omit first d observation set from full sample at a time and estimate the nonlinear regression parameter $\hat{b}^{(dd_1)}$ from (n – d) remaining observation set using the least square estimate for the nonlinear regression parameter from the first delete-d sample; $\hat{\beta}^0 = (Z'Z)^{-1}Z'(Y - f)$.

Stage 3: Compute $\hat{b}^1 = \hat{b}^0 + \hat{\beta}^0$ using the Guass-Newton method, the $\hat{b}^1$ value is assumed as the initial value in the first approximated.

International Journal of Research
Available at http://internationaljournalofresearch.org/
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 02 Issue 02
February 2015

Stage 4: Repeat the second step and again compute $\hat{\beta}'s$. At each iteration, new $\hat{\beta}'s$ represent increments that are added to the estimates $\hat{b}^1$ from the previous iteration according to stage 3 and eventually obtain $\hat{b}^2 = \hat{b}^1 + \hat{\beta}^1$ up to $\hat{b}^{r+1} = \hat{b}^r + \hat{\beta}^r$ and consequently $\hat{b}^{r+1} = \hat{b}^r + (Z^{r'}Z^r)^{-1}Z^{r'}(Y - f^r)$ ..

Stage 5: Stopping Rule; the iteration process continues until $\left|\frac{\hat{b}_p^{(r+1)} - \hat{b}_p^{(r)}}{\hat{b}_p^{(r)}}\right| < \delta$, where δ = 10^{-5}, is the tolerance magnitude and the parameters $\hat{b}_p^{(1)}, \hat{b}_p^{(2)}, \ldots, \hat{b}_p^{(r+1)}$ are computed from (n – d) delete-d samples $W^{(dd_1)}$.

Stage 6: Omit second d observation set from full sample at a time and estimate the nonlinear regression parameters $\hat{b}^{(dd_2)}$ from remaining (n – d) observation set based on the delete-d sample; and repeat stage 3 to stage 5 for the second delete-d sample.

Stage 7: Alternatively omit each d of the n observation set and estimate the parameters as $\hat{b}^{(dd_k)}$ where $\hat{b}^{(dd_k)}$ is the Jackknife regression parameters vector estimates after deletion of kth d observation set from full sample, for k = 1, 2, …, s; where $s = \binom{n}{d}$, and 1 < d < n – 1; where d is an integer.

Stage 8: Obtain the probability distribution $F\left(\hat{b}_p^{(dd)}\right)$ of nonlinear regression parameter estimates $\hat{b}_p^{(dd_1)}, \hat{b}_p^{(dd_2)}, \ldots, \hat{b}_p^{(dd_s)}$.

Stage 9: Calculate the nonlinear regression parameter estimates

$$\bar{\hat{b}}_p^{(dd)} = \frac{\sum_{k=1}^{n} \hat{b}_p^{(dd_k)}}{s}, \quad p = 1,2,\ldots, J \quad \ldots \quad (22)$$

Standard error for the nonlinear regression parameters $\hat{b}_p^{(dd)}$ is

$$Se\left(\hat{b}_p^{(dd)}\right) = \left\{ \frac{n-d}{\binom{n}{d}} \sum_{k=1}^{s} \left(\hat{b}_p^{(dd_k)} - \bar{\hat{b}}_p^{(dd)}\right)^2 \right\}^{\frac{1}{2}}$$

Where

$$\bar{\hat{b}}_p^{(dd)} = \frac{\sum_{k=1}^{s} \hat{b}_p^{(dd_k)}}{\binom{n}{d}}, \quad P = 1,2,\ldots, \quad J \quad \ldots \quad (23)$$

Data Analysis

The data used for this research was extracted from Neil H.T. (2002), Applied Multivariate Analysis, Exercises 4.3 page 216. A sample of 25 samples of tobacco leaf
for organic and inorganic chemical constituents was used for the study. The dependent and independent variables extracted are;

Y: Percentage sugar in the leaf
X: Percentage of Nitrogen

International Journal of Research
Available at http://internationaljournalofresearch.org/
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 02 Issue 02
February 2015

Table 1: Data on Percentage sugar in the leaf and Percentage of Nitrogen

13	14	15	16	17	18	19	20	21	22	23	24	25
14.66	17.31	14.32	15.05	15.47	16.85	17.42	18.55	18.74	14.79	18.86	15.62	18.56
2.50	1.72	2.53	1.90	2.18	2.16	2.14	1.98	1.89	2.07	2.08	2.21	2.00

	1	2	3	4	5	6	7	8	9	10	11	12
Y	20.05	12.58	18.56	18.56	14.02	15.64	14.52	18.52	17.84	13.38	17.55	17.97
X	2.02	2.62	2.08	2.20	2.38	2.03	2.87	1.88	1.93	2.57	1.95	2.03

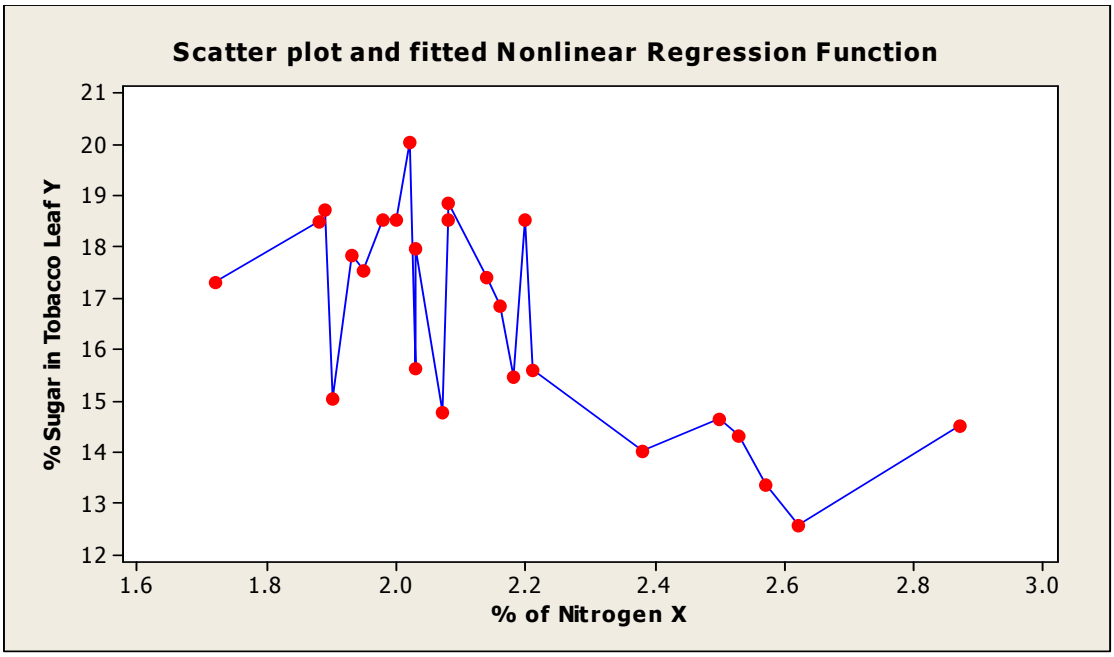

Figure1: Scatter Plot and Fitted Nonlinear Regression Function

Table 2: Analytical Result for the Residual Sum of Squares for Data in Table 1

Iteration	Residual Sum of Squares
0	51.60231
1	51.4928
2	51.4928

Table 3: **Summary Results of the Analytical, Jackknife Delete-1 and Jackknife Delete – d Techniques and their Least Squares Criterion Measure**

	Analytic	Delete-1	Delete-2	Delete-3	Delete-4	Delete-5	Delete-6
b_1	33.35535	32.37337	30.38542	30.11572	30.53931	29.61913	30.01736
b_2	0.72265	0.73045	0.75410	0.75564	0.74833	0.76067	0.75743
SSε	51.49279	43.57664	40.42682	37.51778	31.94526	30.09380	28.14058

Delete-7	Delete-8	Delete-9	Delete-10	Delete-11	Delete-12	Delete-13	Delete-14
32.75518	32.05469	31.78438	29.24974	29.10620	28.68749	27.73390	32.88183
0.72518	0.73195	0.73452	0.76596	0.76752	0.77121	0.78457	0.72515
26.64435	26.21737	26.12614	24.76304	24.73509	23.73708	23.58772	22.05071

Delete-15	Delete-16	Delete-17	Delete-18	Delete-19	Delete-20	Delete-21	Delete-22
29.62681	56.78670	50.30860	52.78934	61.35862	57.90773	79.55032	102.46470
0.76340	0.56263	0.59822	0.58370	0.54020	0.55440	0.47696	0.43203
21.88276	11.18568	10.48958	10.39654	9.66152	9.43320	9.21093	1.44551

Interpretation

The least squares criterion measure for the starting values has been reduced in the first iteration and also further reduced in the second iterations respectively. The second iteration led to no change in either the least squares criterion measure. Hence, convergence is obtained, and the iterations end. Table 3 displays the results of the analytical and the Jackknifes computation. The fitted regression functions for both analytical and Jackknifes delete -1 computation are:

$\hat{Y} = 33.35535(0.72265)^X$ and

$\hat{Y} = 32.37337(0.73045)^X$ respectively

.

The sums of squares error for the analytical and Jackknifes computation are also recorded in the Table 3. Also, as the number of *d* observations deleted in each re-sampling stage increases, the error sums of squares decreases minimally.

Conclusion

The Jackknife algorithm in estimation of the parameters of nonlinear regression model implementation in exponential regression model has been explained. The results obtained as shown in Tables 2 and 3 indicate that the Jackknife methods produced a minimum error sum of squares than the analytical method. We also observe that as the number of d observations deleted in each re-sampling stage increases, the error sum of squares decreases minimally. Hence, the Jackknife techniques yielded approximately the same inference as the analytical method with a better reduced error sum of squares.

References

Batah F.S.M., Thekke V. R., and Sharad D. G. (2008): The efficiency of modified jackknife and ridge type regression estimators: a comparison. Surveys in mathematics and its applications. ISSN 1842-6298. Volume 3 (2008), 111-122.

Ekezie D. D., Osuji G. A., Opara J. and Ogbonna C. J. (2014). Estimation of Bivariate Linear Regression Data Via Jackknife Algorithm. International Journal of Statistics and Mathematics. Vol. 1(1), pp. 009-015, April, 2014.

Hongchang, H., and Yuhe, X. (2013): Jackknifed Liu estimator in linear regression models. Wuhan University Journal of Natural Sciences. August 2013, Volume 18, Issue 4, pp 331-336.

Nduka, E. C., (1999): Principles of Applied Statistics 1, Regression and Correlation Analyses, Crystal Publishers, Imo State, Nigeria.

Neil, H.T.(2002): Applied Multivariate Analysis. Springer-Verlag, New York Berlin Heidelberg.

Obiora-Ilouno, H.O. and Mbegbu, J.I. (2012): Jackknife Algorithm for the Estimation of Logistic Regression Parameters, Journal of Natural Sciences Research, IISTE UK, Vol.2, No.4, 74-82, 2012.

Shao, J., and Rao J.N.K (1993): Jackknife inference for heteroscedastic linear regression models. The Canadian Journal of Statistics. Vol.21, No. 4, 1993, pages 377-395.

Smith, H. and Draper, N. R.(1998): Applied Regression Analysis, Third Edition, Wiley-Interscience, New York, 1998.

Wu, C.F.J. (1986): Jackknife bootstrap and other re-sampling methods in regression analysis. The Annals of Statistics 1986, vol. 14, No. 4, 1261-1295