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Control of Pollution in River Streams

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Abstract.

In industrial societies, rivers are used as dumps for sewage except that the sewage is treated at sewage stations prior to discharge into the stream. Untreated sewages discharged into the river contain high concentrations Biochemical Oxygen Demand (BOD) that has adverse effect on both human and aquatic organisms. The need of real-time monitoring and effective treatment of wastewater is urgent, and Dissolved Oxygen (DO) content in water quality monitoring is an important indicator. In this paper a methodology is presented to determine allowable concentrations of BOD in a wastewater discharge. A discrete dynamic model of first order difference equation is described for the dynamics of BOD and DO in a two reach river system. Kalman filtering technique is applied to the first order discrete dynamic model to estimate the concentrations of BOD in the effluent being discharged into the stream. This methodology was applied to Warri River in Delta state of Nigeria. It was observed that the maximum allowable concentration of BOD in the effluent discharged to the stream is 0.01 mg/l.

Key wards: River, DO, BOD, Pollution, Sewage, Kalman filter, Control.

1.0 **Introduction**

In recent years there has been much interest in regulating the levels of pollution in rivers. A good measure of the quality of a stream is given by the in-stream biochemical oxygen demand (B.O.D) and the dissolved oxygen (D.O) in the stream.

If the D.O falls below certain level or the B.O.D rises above certain level, fish die. In industrial societies, rivers are used as dumps for sewage except that the sewage is treated at sewage stations prior to discharge into the stream (fig.1). Sewage works in general operate a fixed level of treatment which could be determined by the level of B.O.D in the sewage which can be safely absorbed in the stream. However, the ecological balance of the river is often disturbed by unknown perturbations and it becomes necessary to vary the B.O.D content of the sewage (by increasing or decreasing the treatment levels) in order to bring the river quality back to the desired value. It is the problem of on-line regulation of sewage discharge B.O.D from multiple sewage works on polluted river that we treat in this paper.

The river pollution control problem is essentially how to control the BOD in the effluent discharge in order to maintain desired levels of DO and BOD in the river despite the stochastic effects (Singh and Titli,1978)

Fig.1: Treated domestic wastewater being discharged into a stream (Penn M.Retal, 1999)

2.0 **Definitions of Terms and Concepts**

(i)Sewage-they are used water and waste substances that are produced by human bodies and carried away from houses and factories through special pipes.

(ii)Effluent –is a treated wastewater, flowing from treatment plant.

(iii)Biochemical oxygen demand (BOD)-a parameter used to measure the rate of absorption of oxygen by decomposing organic matter. The unit is milligram per litre(mg/l)

(iv)Dissolve Oxygen (DO)-this is the amount of oxygen dissolved (and hence available to sustain marine life) in a body of water such as lake, river, or stream. The unit is in milligram/ litre (mg/l).

(v)A reach of river-is defined as a stretch of a river of some convenient length which receives one major effluent discharged from a sewage facility (fig. 2).

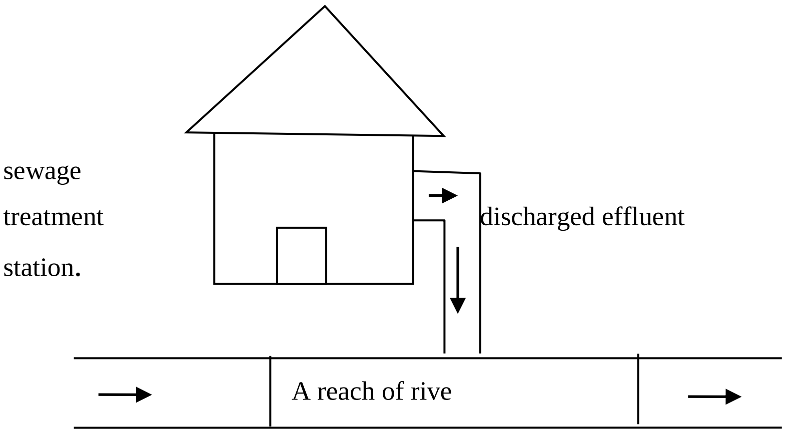

Fig.2**:** A reach of river

(vi)A two reach river system-is a stretch of a river of some convenient length divided into two sections such that each section receives one major effluent discharged from sewage facility (fig.3).

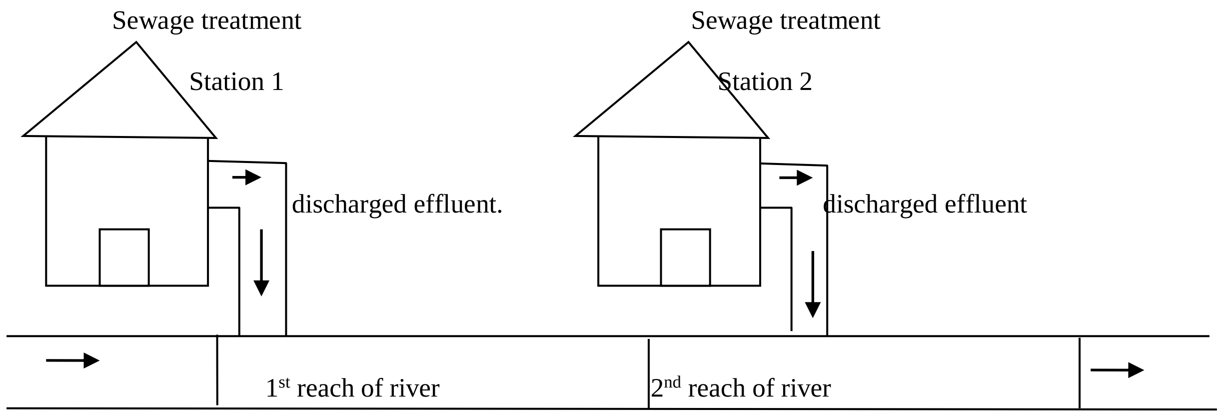

Fig.3: A two reach river system

Previously, a number of river-water quality models have been developed for supporting waste loading allocation, pollution control, and land-use planning in river basins (Guo etal, 2003).

The first water quality model was developed by Streeter and Phelps (1925). The basic principles behind this model include (i) DO is supplied by reaeration and photosynthesis and demanded by respiration and BOD, and, and (ii) BOD is due to emission from point and nonpoint sources and could be reduced by oxidation, sedimentation and absorption processes. After that a number of further studies were undertaken (Beck, 1975;

Beck and Young, 1976; Bowles and Grenney, 1978; DiCola et al, 1976; Eykhoff, 1974; Gnauk et al, 1976; Koivo and Philips, 1972; Renaldi et al,1979; Moore and Jones, 1978).

3.0 **The Kalman Filter**

Kalman filter is defined as a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the square error (Greg and Gary, 2006)

In order to use Kalman filter to estimate the internal state of a process, one must model the process in accordance with the framework of the Kalman filter. This means specifying the following matrices: Φ, the state-transition model, H, the observation model, Q, the covariance of the process noise, R, the covariance of the observation noise; for each time step, k.

The Kalman filter model assumes the true state at time k is evolved from the state at (k-1) as stated below.

$$X_k = \Phi X_{k-1} + \xi_{k-1} \qquad (1)$$

Where

Φ is the state transition model which is applied to the previous state X_{k-1};

ξ_k is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance Q_k, $\xi_k \sim N(0,Q_k)$

At time k an observation (or measurement) Y_k of the true state X_k is made according to

$$Y_k = HX_k + \eta_k \qquad (2)$$

Where H is the observation model which maps the true state space into the observed space and η_k is the observation noise which is assumed to be zero mean Gaussian white noise with covariance R_k, $\eta_k \sim N(0,R_k)$

The initial state, and the noise vectors at each step $\{X_o, \eta_1, ..., \eta_k, \xi, ... \xi_k\}$ are all assumed to be mutually independent.

The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state. No history of observations and /or estimates is required. In what follows, the notation $\hat{X}_{k|k-1}$ which is the 1 step prediction represents the estimates of X_k at time k given observations up to and including at time k-1.

$$\hat{X}_{k|k-1} = \Phi \hat{X}_{k-1|k-1} \qquad (3)$$

The covariance matrix for the one step prediction error is given by

$$p_{k|k-1} = \Phi p_{k-1|k-1} \Phi^T + Q \qquad (4)$$

$\hat{X}_{k|k-1}$ and $P_{k|k-1}$ are the Predicted(a priori) state estimate and Predicted(a priori) estimate of covariance respectively and represent the initial values for the Kalman filter.

The state of the filter is represented by two variables

$\hat{X}_{k|k}$, the updated (a posterior) state estimate at time k given observations up to and including at time k and given by

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k(Y_k - H \hat{X}_{k|k-1}) \qquad (5)$$

$P_{k|k}$, the updated(a posterior)error covariance matrix (variance of the estimation error) given by

$$P_{k|k} = (I - K_k H) P_{k|k-1} \qquad (6)$$

Where K_k is the Kalman (Filter) gain and given by

$$K_k = P_{k|k-1} H^T (H P_{k|k-1} H^T + R)^{-1} \qquad (7)$$

In Fig. 4, Kalman filter is summarized as Kalman Filter Loop(Robert and Patrick, 1992). Once the loop is entered it can be continued for any N (N≥ 1) iterations, k = 0,1,……..,N-1.

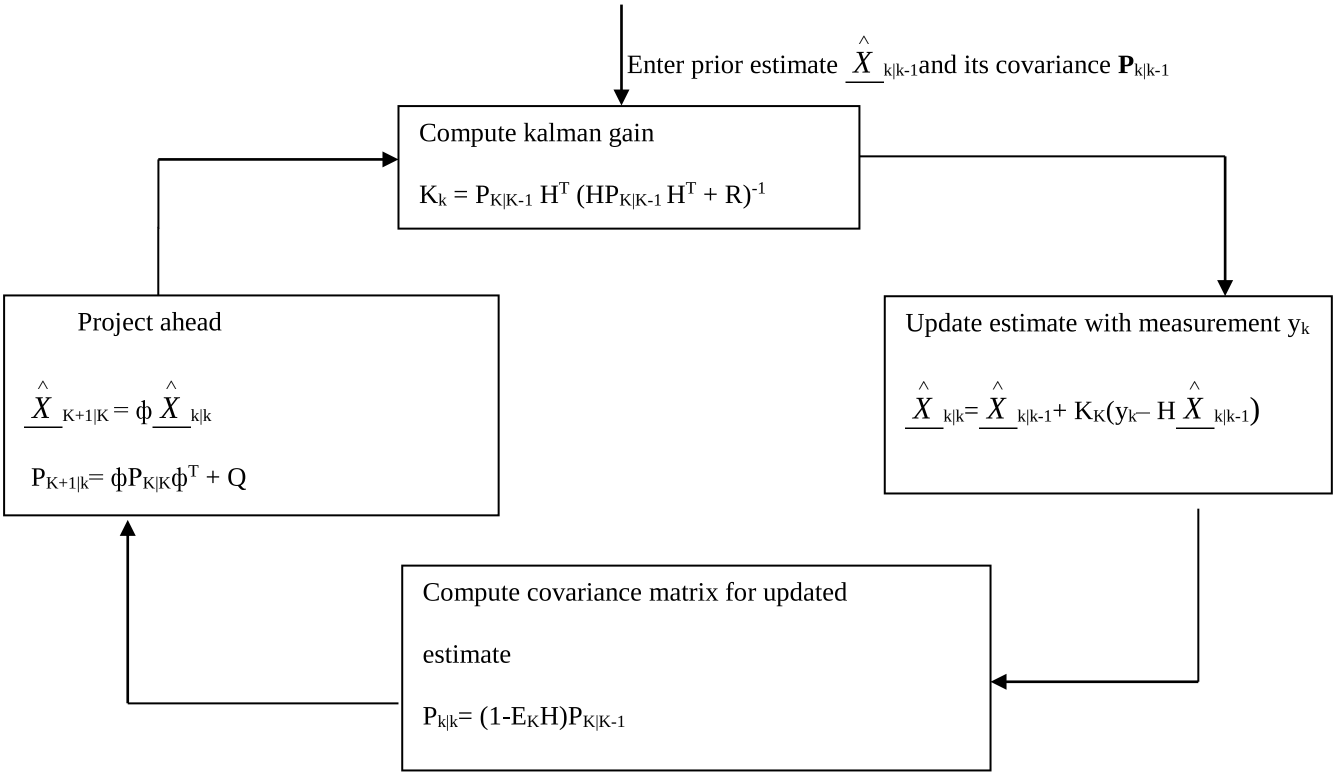

Fig.4: The Kalman Filter Loop (Robert and Patrick, 1992)

4.0 **Discrete-time Linear- Quadratic-Gaussian (LQG) Problem**

Consider the dynamic system,

$$\left.\begin{aligned} X_{k+1} &= \Phi X_k + BU_k + \xi_k \\ Y_k &= HX_k + \eta_k \end{aligned}\right\} \quad (8)$$

Where X_k is an n-vector , U_k is an m-vector ($m \leq n$) , Φ, B are n x n resp. n x m matrices whose elements are non-random functions of time. Given the observed values of $Y_0, \ldots, Y_k$, we are to find a sequence $U_1, U_2, \ldots, U_k$ of control vectors that minimizes the performance index

$$J = \sum_{k=0}^{N-1} X_k{}^T QX_k + U_k{}^T RU_k \quad (9)$$

Where Q , R are symmetric and positive definite matrices whose elements are non-random functions of time, N is the number of iterations.

The discrete-time controller is

$$U_k = -GX_k \quad (10)$$

5.0 **Theorem**: **Separation Principle**

In control theory, a separation theorem, more formally known as theorem of separation of estimation and control, states that under some assumptions the problem of designing an optimal feedback controller for a stochastic system can be solved by designing an optimal observer for the state of the system, which feeds into an optimal deterministic controller for the system (fig.5). Thus the problem can be broken into two separate parts, which facilitates the design (Wikipedia)

This result is the celebrated separation theorem of linear stochastic control and was proved independently by Joseph and Tou (1961) and Gunkel and Franklin (1963).

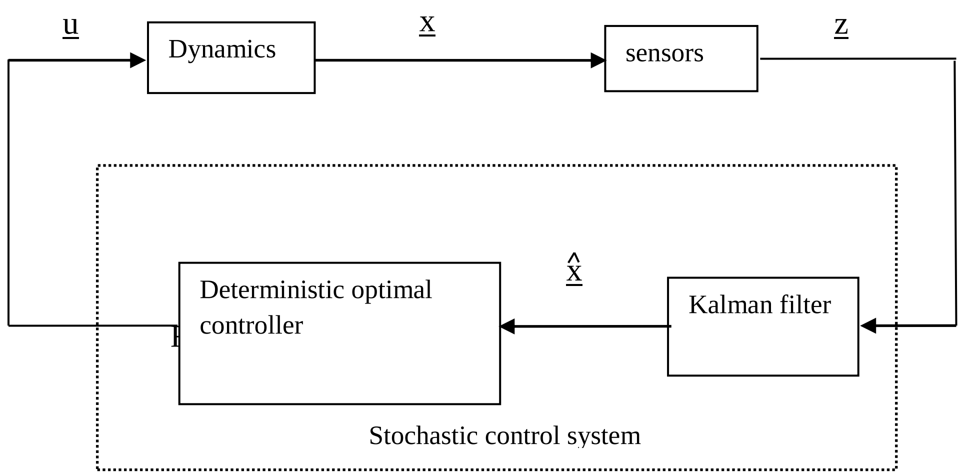

Fig.5: Observer-based controller(Grewal and Andrews, 2010)

6.0 **The Warri River**

The Warri river in Delta State of Nigeria is an example of inland water receiving sewage from several industries, factories and markets. These wastes often contain significant spectrum of organic and inorganic substances capable of producing adverse effects on the physical, chemical and biotic components of the environment either directly or indirectly on human health (Aghoghovwia, 2008). The rapid increase in human population, inadequate infrastructural facilities, lack of good/proper facilities for waste disposal as well as problem of refuse collection and disposal, have contributed to the environmental decay (Odiete, 1990). Warri River flows through the adjourning mangrove swamp forest area of the southern part of Nigeria, where the drainage and catchment areas are probably very rich in decaying organic matter and humus. Warri River stretches within latitude $5^0 21'$ to $6^0 00'$N and longitude $5^0 24'$to $6^0 2'$E. Its source is around Utagba Uno, covering a surface area of above 255 km^2 with a length of about 150 km. From its source, the river flows South-westerly direction passing between Oviorie and Ovu-inland and southwards at Odiete through Agbarho to Otokutu and Ugbolokposo (Egborge, 2001) It turns southwards to Effurun and forms a 'W' between Effurun and Warri. Important land marks in this River stretch are Enerhen, Igbudu, Ovwian and Aladja(steel town),Warri ports, main Warri market, NNPC Refinery, Globe star, etc.

Warri River was chosen as a case study for this research for the following reasons:

i) There are several industries, factories and markets within Warri as such Warri River is subject to large quantity of sewages flowing into it which causes a large quantity of pollution in the River.

ii) Many Researchers are interested in studying the amount of pollution in Warri River because of the above reason.

iii) This pollution produces adverse effects on the physical, chemical and biotic components of the environment either directly or indirectly on human health and so it needed to be controlled.

Many Researchers worked on Warri River to determine the quantity of the pollution, such as: Arimoro et al; Aghoghoviwia,2011; Aghoghovwia,2008; Olomukoro et al,2009; Edegbene et al,2012.

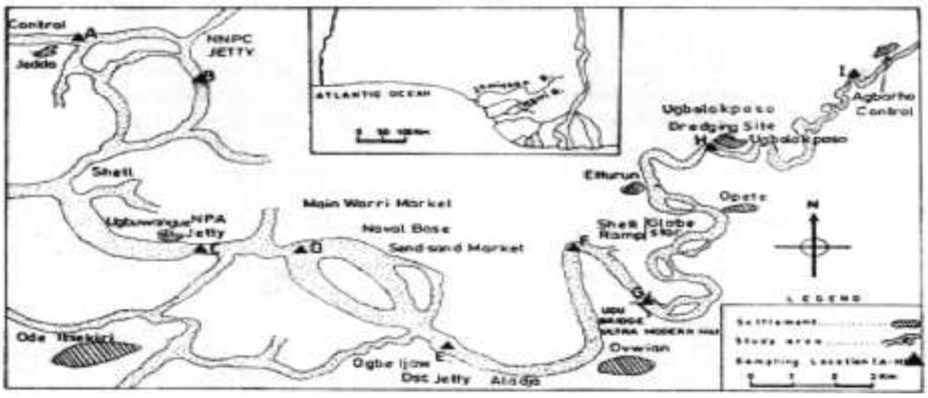

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Fig.6: Map showing Warri River (Aghoghovwia, 2008)

Reach 1 of the river covers the 500 metres length of the river along Naval base and Reach 2 which started from the end of Reach 1 covers the 500 metre length of the river along Main Warri Market.

7.0 **Formulation of the two reach River model.**

Beck (1978) presented a single reach model given by:

$$\left.\begin{aligned} \dot{z}_i &= -k_{1i} z_i + \frac{Q_{i-1}}{v_i} z_{i-1} - \frac{Q_i + Q_E}{v_i} z_i + \frac{\vartheta_i Q_E}{v_i} \\ \dot{q}_i &= k_{2i}(q_i^s - q_i) + \frac{Q_{i-1}}{v_i} q_{i-1} - \frac{Q_i + Q_E}{v_i} q_i - k_{1i} z_i - \frac{\mu_i}{v_i} \end{aligned}\right\} \qquad (11)$$

Where,

z_i , z_{i-1} are the concentrations of B.O.D. in reaches i and i-1 in mg/litre

q_i , q_{i-1} are the concentrations of D.O. in reaches i and i-1 in mg/litre

v_i is the volume of water in reach i in million gallons

Q_E is the flow rate of the effluent in reach i in million gallons/day

k_{li} is the oxygen consumption coefficient$(day)^{-1}$ in reach i

k_{2i} is the oxygen recovery coefficient$(day)^{-1}$ in reach i

Q_i , Q_{i-1} are the stream flow rates in reaches i and i-1 in million gallons/day

q_i^s is the D.O. saturation level for the i^{th} reach (mg/litre)

$\frac{\mu_i}{v_i}$ is the removal of D.O. due to bottom sludge requirements (mg/litre$(day)^{-1}$)

ϑ_i is the concentration of B.O.D. in the effluent in mg/litre

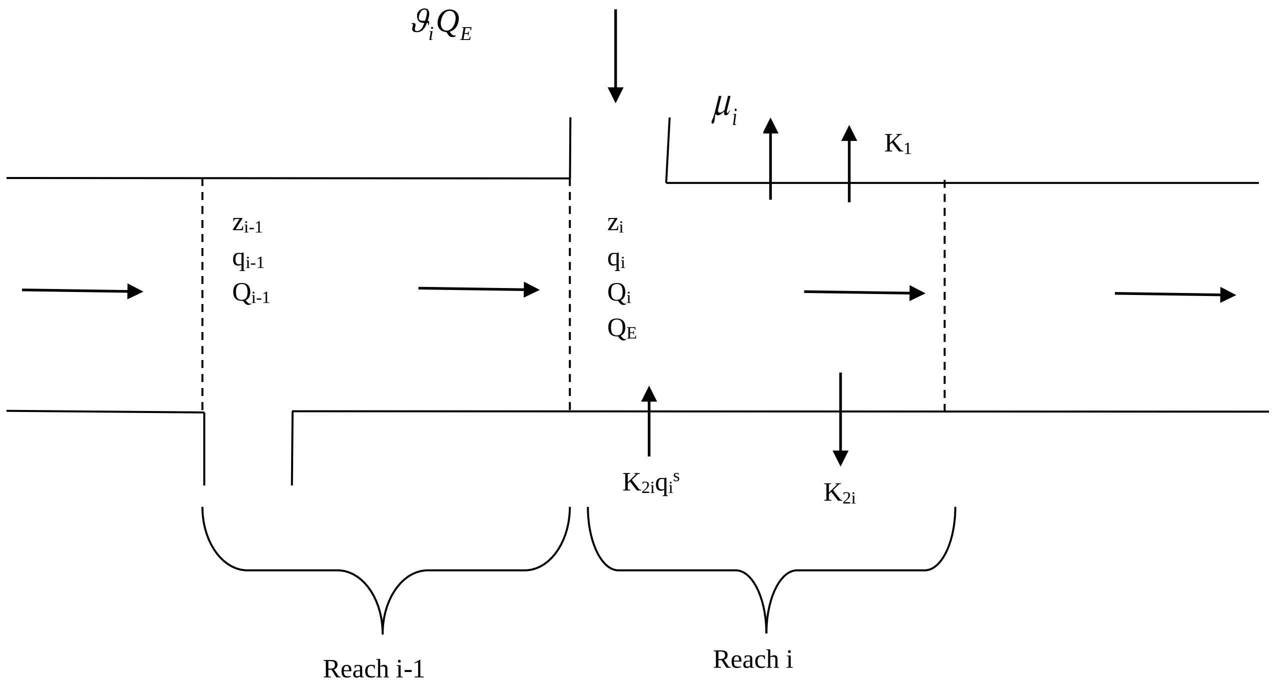

Fig.7: Flow diagram for Becks model.

A two reach river model is formulated as an extension of a single reach model. The stretch of the river is divided into two reaches (reach 1 and reach 2). Model equations for reach 1 and reach 2 were formulated independently and then merged together to form four order state space equation. Reach 1 is designated as i_1 and Reach 2 is designated as i_2.

7.1 **Model Equation for Reach 1**

$$\left.\begin{aligned}\frac{dz_{i_1}}{dt} &= -k_{1i_1} z_{i_1} + \frac{Q_{i_{1-1}}}{v_{i_1}} z_{i_{1-1}} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} z_{i_1} + \frac{\vartheta_{i_1} Q_{E_1}}{v_{i_1}} \\ \frac{dq_{i_1}}{dt} &= k_{2i_1} q_{i_1}{}^{s} - k_{2i_1} q_{i_1} + \frac{Q_{i_{1-1}}}{v_{i_1}} q_{i_{1-1}} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} q_{i_1} - k_{1i_1} z_{i_1} - \frac{\mu_{i_1}}{v_{i_1}}\end{aligned}\right\} \quad (12)$$

Where,

z_{i_1}, $z_{i_{1-1}}$ are the concentrations of B.O.D. in reaches i_1 and i_{1-1} in mg/litre

q_{i_1}, $q_{i_{1-1}}$ are the concentrations of D.O. in reaches i_1 and i_{1-1} in mg/litre

v_{i_1} is the volume of water in reach i_1 in million gallons

Q_{E_1} is the flow rate of the effluent in reach i_1 in million gallons/day

k_{1i_1} is the oxygen consumption coefficient(day)$^{-1}$ in reach i_1

k_{2i_1} is the oxygen recovery coefficient (day)$^{-1}$ in reach i_1

Q_{i_1}, $Q_{i_{1-1}}$ are the stream flow rates in reaches i_1 and i_{1-1} in million gallons/day

$q_{i_1}{}^{s}$ is the D.O. saturation level for the i_1^{th} reach (mg/litre)

$\frac{\mu_{i_1}}{v_{i_1}}$ is the removal of D.O. due to bottom sludge requirements (mg/litre(day)$^{-1}$)

ϑ_{i_1} is the concentration of B.O.D. in the effluent in mg/litre

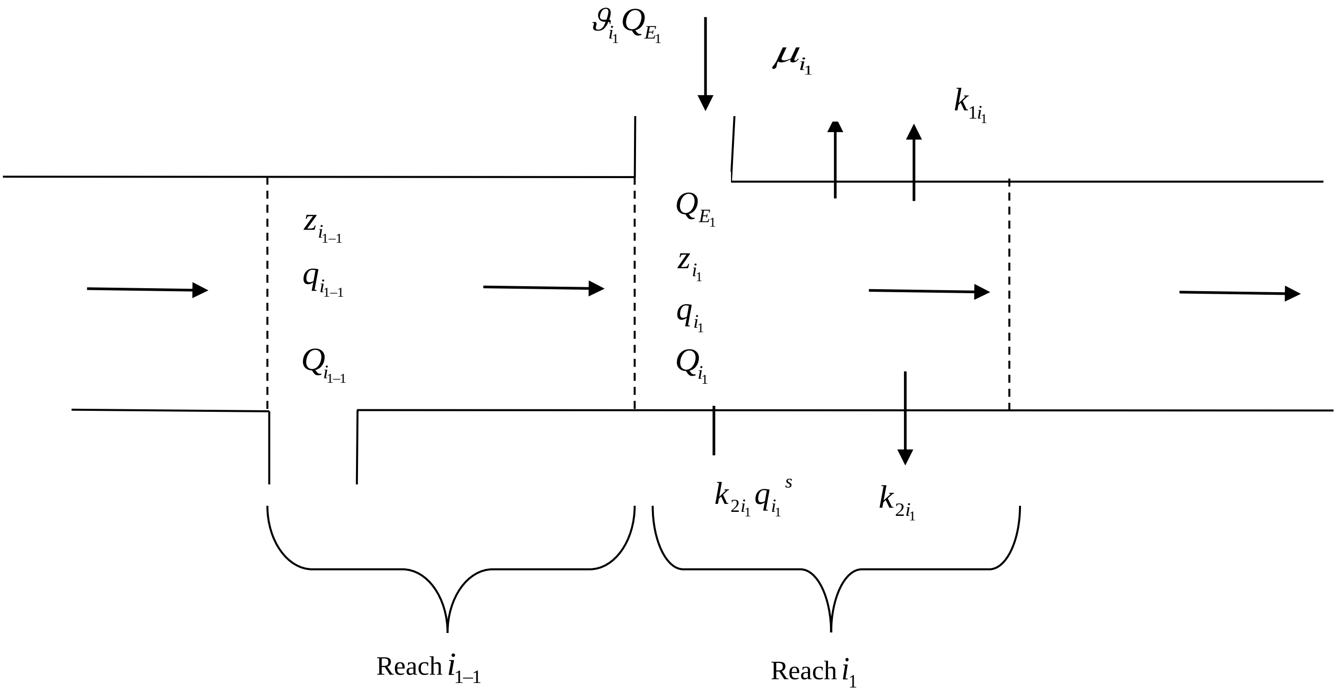

Fig.7: Flow diagram for reach 1

7.2 **Model Equation for Reach 2**

$$\left.\begin{aligned}\frac{dz_{i_2}}{dt} &= -k_{1i_2} z_{i_2} + \frac{Q_{i_{2-1}}}{v_{i_2}} z_{i_{2-1}} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} z_{i_2} + \frac{\vartheta_{i_2} Q_{E_2}}{v_{i_2}} \\ \frac{dq_{i_2}}{dt} &= k_{2i_2} q_{i_2}{}^{s} - k_{2i_2} q_{i_2} + \frac{Q_{i_{2-1}}}{v_{i_2}} q_{i_{2-1}} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} q_{i_2} - k_{1i_2} z_{i_2} - \frac{\mu_{i_2}}{v_{i_2}}\end{aligned}\right\} \quad (13)$$

Where,

z_{i_2}, $z_{i_{2-1}}$ are the concentrations of B.O.D. in reaches i_2 and i_{2-1} in mg/litre

q_{i_2}, $q_{i_{2-1}}$ are the concentrations of D.O. in reaches i_2 and i_{2-1} in mg/litre

v_{i_2} is the volume of water in reach i_2 in million gallons

Q_{E_2} s the flow rate of the effluent in reach i_2 in million gallons/day

k_{1i_2} is the oxygen consumption coefficient(day)$^{-1}$ in reach i_2

k_{2i_2} is the oxygen recovery coefficient(day)$^{-1}$ in reach i_2

Q_{i_2}, $Q_{i_{2-1}}$ are the stream flow rates in reaches i_2 and i_{2-1} in million gallons/day

$q_{i_2}{}^{s}$ is the D.O. saturation level for the $i_2{}^{th}$ reach (mg/litre)

$\frac{\mu_{i_2}}{v_{i_2}}$ is the removal of D.O. due to bottom sludge requirements (mg/litre(day)$^{-1}$)

ϑ_{i_2} is the concentration of B.O.D. in the effluent in mg/litre

Putting eqns. (12) and (13) together in matrix form gives:

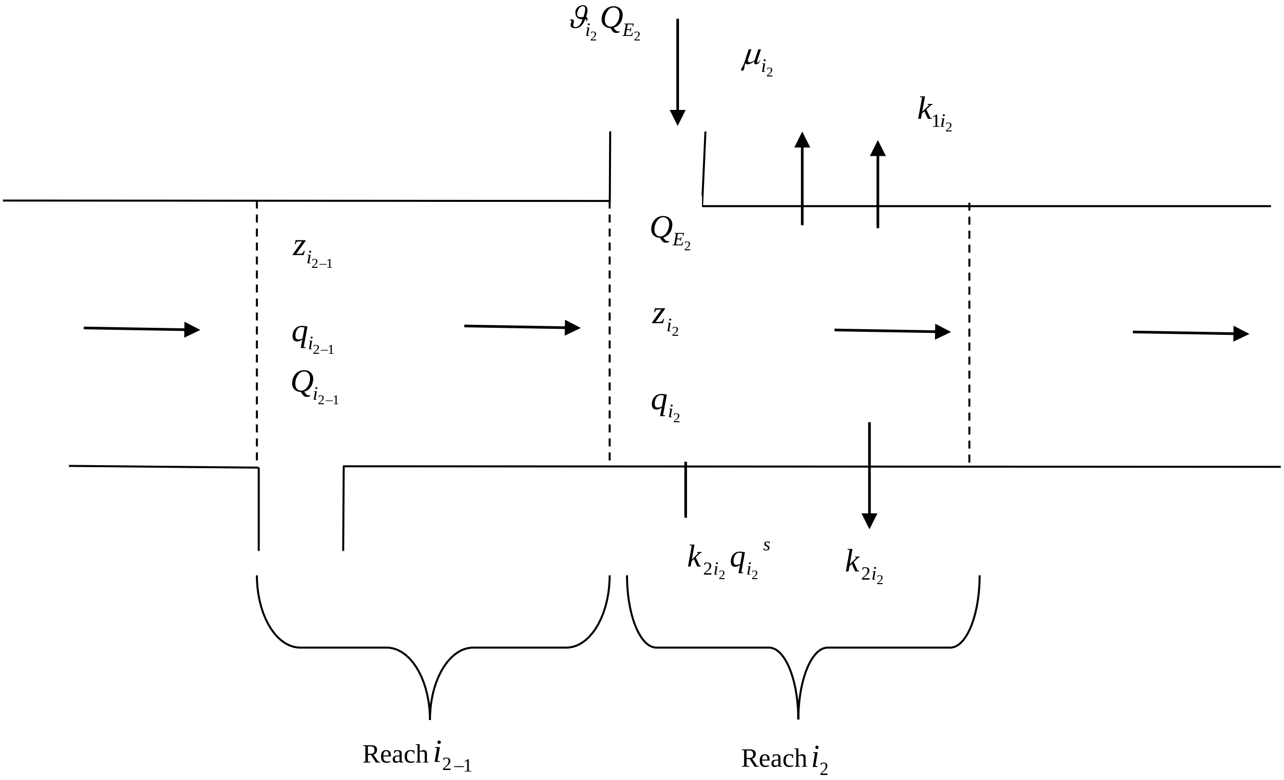

Fig. 8:Flow diagram for reach 2

Putting eqns. (11) and (12) together into a matrix form gives:

$$\begin{bmatrix} \frac{dz_{i_1}}{dt} \\ \frac{dq_{i_1}}{dt} \\ \frac{dz_{i_2}}{dt} \\ \frac{dq_{i_2}}{dt} \end{bmatrix} = \begin{bmatrix} -k_{1i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} & 0 & 0 & 0 \\ -k_{1i_1} & -k_{2i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} & 0 & 0 \\ 0 & 0 & -k_{1i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} & 0 \\ 0 & 0 & -k_{1i_2} & -k_{2i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} \end{bmatrix} \begin{bmatrix} z_{i_1} \\ q_{i_1} \\ z_{i_2} \\ q_{i_2} \end{bmatrix} +$$

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$$\begin{bmatrix} \frac{Q_{E_1}}{v_{i_1}} & 0 \\ 0 & 0 \\ 0 & \frac{Q_{E_2}}{v_{i_2}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \vartheta_{i_1} \\ \vartheta_{i_2} \end{bmatrix} + \left[\begin{bmatrix} \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 & 0 \\ 0 & \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 \\ 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} & 0 \\ 0 & 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} \end{bmatrix} \begin{bmatrix} z_{i_{1-1}} \\ q_{i_{1-1}} \\ z_{i_{2-1}} \\ q_{i_{2-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ k_{2i_1} q_{i_1}{}^{s} - \frac{\mu_{i_1}}{v_{i_1}} \\ 0 \\ k_{2i_2} q_{i_2}{}^{s} - \frac{\mu_{i_2}}{v_{i_2}} \end{bmatrix} \right] \tag{14}$$

Equation (14) is represented by first order deference equation of the form:

$$X_{k+1} = \Phi X_k + BU_k + C \tag{15}$$

Where:

$$\Phi = \begin{bmatrix} -k_{1i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} & 0 & 0 & 0 \\ -k_{1i_1} & -k_{2i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_2}} & 0 & 0 \\ 0 & 0 & -k_{1i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} & 0 \\ 0 & 0 & -k_{1i_2} & -k_{2i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} \end{bmatrix} \tag{16}$$

$$X_k = \begin{bmatrix} z_{i_1} \\ q_{i_1} \\ z_{i_2} \\ q_{i_2} \end{bmatrix} \tag{17}$$

$$B = \begin{bmatrix} \frac{Q_{E_1}}{v_{i_1}} & 0 \\ 0 & 0 \\ 0 & \frac{Q_{E_2}}{v_{i_2}} \\ 0 & 0 \end{bmatrix} \tag{18}$$

$$U_k = \begin{bmatrix} \vartheta_{i_1} \\ \vartheta_{i_2} \end{bmatrix} \tag{19}$$

$$C = \left[\begin{bmatrix} \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 & 0 \\ 0 & \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 \\ 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} & 0 \\ 0 & 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} \end{bmatrix} \begin{bmatrix} z_{i_{1-1}} \\ q_{i_{1-1}} \\ z_{i_{2-1}} \\ q_{i_{2-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ k_{2i_1} q_{i_1}{}^{s} - \frac{\mu_{i_1}}{v_{i_1}} \\ 0 \\ k_{2i_2} q_{i_2}{}^{s} - \frac{\mu_{i_2}}{v_{i_2}} \end{bmatrix}\right] \quad (20)$$

Φ, B, X_k, U_k, is the transition model, the control input model, the process state vector and the control vector respectively. C is a constant matrix.

8.0 **Optimal Control Problem.**

The optimal control problem is then stated as follows:

$$\left.\begin{aligned} &\text{Minimize } J = \sum_{k=0}^{N-1} X_k{}^T Q X_k + U_k{}^T R U_k \\ &\text{Subject to: } \mathbf{X}_{k+1} = \Phi X_k + BU_k + \xi_k \\ &\qquad Y_k = HX_k + \eta_k \end{aligned}\right\} \quad (21)$$

Where:

Φ = (4× 4) constant matrix obtained from the transition model

B = (4× 2) control input matrix which is applied to the control vector $\underline{U}_k$.

$\underline{Y}_k$ = (2× 1) output vector (vector measurement at time t_k), $\underline{Y}_k = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, y_1 is the measured D.O in reach 1 and y_2 is the measured D.O in reach 2 of the stream.

H = (2× 4) constant matrix giving the ideal connection between the measurement and the State vector at time t_k

$\underline{X}_k$= (4× 1) process state vector at time t_k , i.e.,$\underline{X}_k$ =$\underline{X}$ (t_k) = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, x_1 and x_3 are the concentrations of B.O.D.(mg/l) in reaches 1 and 2 of the stream and x_2 , x_4 are the concentrations of D.O in reaches 1 and 2 of the stream.

$\underline{U}_k$ = (2× 1) control vector, $\underline{U}_k$= $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, where u_1 and u_2 are the concentrations of B.O.D in the effluent to reaches 1 and reach 2 of the stream.

$\underline{\eta}_k$ = (2× 1) measurement error –assumed to be white noise sequence with known co-

Variance R and having zero cross correlation with $\underline{\xi}_k$sequence.

$\underline{\xi}_k$= (4× 1) vector-assumed to be white noise sequence with known co-variance Q

Note that x_2 and x_4 are the estimates of the true value of D.O in Reach 1 and 2 of the stream, y_1 and y_2 are the measured value of D.O in reach 1 and 2 of the stream.

9.0 D.O Measurements

The DO measurements $\underline{Y}_k$ = $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ at t_k , where $y_1$and y_2 are the concentrations of DO in reach 1 and reach 2 of the river as given in Table 2.

Table 2: **D.O Measurements, $\underline{Y}_k$ at time t_K** (Aghoghovwia, 2011).

K	**t_k**	$\underline{Y}_k$= $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$
0	t_0	$\underline{Y}_0$= $\begin{bmatrix} 5.0 \\ 5.7 \end{bmatrix}$

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1	t_1	$\underline{Y}_1 = \begin{bmatrix} 6.0 \\ 5.3 \end{bmatrix}$
2	t_2	$\underline{Y}_2 = \begin{bmatrix} 5.5 \\ 6.3 \end{bmatrix}$
3	t_3	$\underline{Y}_3 = \begin{bmatrix} 5.4 \\ 6.6 \end{bmatrix}$
4	t_4	$\underline{Y}_4 = \begin{bmatrix} 6.7 \\ 7.3 \end{bmatrix}$
5	t_5	$\underline{Y}_5 = \begin{bmatrix} 7.0 \\ 8.0 \end{bmatrix}$
6	t_6	$\underline{Y}_6 = \begin{bmatrix} 6.5 \\ 6.1 \end{bmatrix}$
7	t_7	$\underline{Y}_7 = \begin{bmatrix} 6.2 \\ 7.5 \end{bmatrix}$

Table1: **Parameter Values for Warri river Model at 29°C(Ogwola,2017)**

Symbol	Value	Sources
L_1	500 metres	Ogwola,2017
L_2	500 metres	,,
W_1	150 metres	,,
W_2	150 metres	,,
D_1	7.5 metres	,,
D_2	7.5 metres	,,
Q_{i_1}	0.643m^3/s(14672937.5 gallons/day)	,,
Q_{i_2}	0.643m^3/s(14672937.5 gallons/day)	,,
v_{i_1}	562500m^3(148597242.04 gallons)	,,
v_{i_2}	562500m^3(148597242.04 gallons)	,,

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	ASSUMPTIONS[with respect to Guo et al (2003)]	
Q_{E_1}	0.659m^3/s(15046510.9 gallons/day)	Assumption
Q_{E_2}	0.659m^3/s(15046510.9 gallons/day)	,,
k_{1i_1}	1	,,
k_{1i_2}	2	,,
k_{2i_1}	2	,,
k_{2i_2}	3	,,

With respect to table 1, values of Φ and B were obtained from (15) and (17) respectively.

$$\Phi = \begin{bmatrix} -1.2 & 0 & 0 & 0 \\ -1.0 & -2.2 & 0 & 0 \\ 0 & 0 & -2.2 & 0 \\ 0 & 0 & -2.0 & -3.0 \end{bmatrix} \qquad B = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \\ 0 & 0.1 \\ 0 & 0 \end{bmatrix}$$

The process variance Q and the measurement variance R were tuned to:

$$Q = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \text{and } R = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

The control system tool box function [G,S] = LGR(Φ ,B,Q,R) is used to compute the optimal gain matrix G.

$$G = \begin{bmatrix} 0.0471 & -0.0067 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0280 & -0.0058 \end{bmatrix}$$

With respect to Kalman filter Tank filling the initial values $X_{0|0}$ and $P_{0|0}$ are:

$$X_{0|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad P_{0|0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H which is the constant matrix that gives ideal connection between the measurement and the state vector is given by:

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 3:**The Simulation Results for the System**

K	t_k	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$	u_1	u_2
0	t_0	0	1.6665	0	1.8998	0.0112	0.011
1	t_1	1.4103	3.653	3.3751	3.7581	-0.042	-0.0727
2	t_2	2.7214	3.6092	6.4619	5.3346	-0.104	-0.15
3	t_3	2.9887	3.8324	12.3551	5.8718	-0.1151	-0.3119
4	t_4	4.3429	5.1018	13.5198	6.6294	-0.1704	-0.3401
5	t_5	5.0655	5.0841	16.992	6.9818	-0.2045	-0.4353
6	t_6	4.3844	4.6026	10.5232	5.0013	-0.1757	-0.2656
7	t_7	4.313	4.5266	11.3439	6.3318	-0.1728	-0.2809

Where

$\hat{x}_1$ is the optimal estimate of BOD in Reach 1 of the river in milligram per litre(mg/L)

$\hat{x}_2$ is the optimal estimate of DO in Reach 1 of the river in mg/litre

$\hat{x}_3$ is the optimal estimate of BOD in Reach 2 of the river in mg/litre

$\hat{x}_4$ is the optimal estimate of DO in Reach 2 of the river in mg/litre

u_1 is the concentration of BOD in the effluent to Reach 1in mg/litre

u_2 is the concentration of BOD in the effluent to Reach 2 in mg/litre

10.0 **Discussion**

The results of this research showed that the average estimated values of BOD (x_1) in Reach 1(column 3 of Table 3) is 3.1533 mg/l and the average estimated values of BOD (x_3) in Reach 2(column 5 of Table 3) is 9.3214 mg/l. This shows that the BOD varies with respect to locations in this river.

According to Wikipedia, 'unpolluted rivers have BOD below 1 mg/l'. The average concentration of BOD in Reach 1 and Reach 2 is 3.1533 mg/l and 9.3214 mg/l respectively, therefore Warri river is polluted and not suitable for drinking.

With respect to the Water Quality Assessment and Pollution Control ,Reach 2 is not a suitable environment for fish growth since the BOD value of 9.3214 mg/l is greater than 4mg/l. The BOD values in Reach 2 of the stream exceeded stipulated permissible limit for drinking and for protection of health in fish which is also in line with Aghoghovwia (2011).

Effluent is a treated wastewater flowing from treatment plant into a river. The concentration of BOD in the effluent determines the level of treatment of the sewage. If an effluent has BOD of less than 1 mg/l then it is unpolluted and suitable for discharging into the stream.

With respect to the assumption that the flow rate of the effluent to the river is 15046510.9 gallons/day the results of this research show that the maximum concentration of BOD in the effluent to Reach1(u_1) is 0.0112 mg/l(seventh column of table 3) and the maximum concentration of BOD in the effluent to Reach2(u_2) is 0.0110 mg/l(eighth column of table 3). These values are less than 1 mg/l of BOD and therefore the effluents are unpolluted and safe for discharging into the stream.

Therefore, the sewage must be treated to a level such that the concentration of effluent from sewage must not exceed 0.011 mg/l of BOD.

11.0 **Conclusion**

The fact that emerged from this study is that the level of BOD in Warri river exceeded the stipulated permissible limit for drinking water and to protect health of fish, sewages need to be treated to the desired level before discharging into the river to avoid stream pollution.

12.0 **References**

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