

## Performance reviews of SMC under nominal and Uncertain model

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**Abstract.** In this paper, some sliding mode controller strategies are studied for SISO system. An analysis is performed and simulation results obtained after implementation of SMC strategies and its parameter variations in plant, due to uncertainties and external load disturbances. The model used in the study is delay approximated model of the system which is used in the design procedure. The higher order sliding mode approach is compared with the conventional SMC and PID controller. The short information of high order SMC as well as conventional SMC approach are included. The Lyapunov function gives conditions for the stability and robustness behavior of controller design. The known parameters of the switching gain which in turn significantly reduces phenomenon of

chattering. The performance of the control schemes are reviewed and validated by simulation study, and the results are compared to study the advantages and limitations of each of the control strategy over others.

**Keywords:** Sliding mode control; Performance reviews; single variable; stability; robustness.

### Introduction

In real industrial processes, the system has non-linear dynamics and time delays, in addition to other unmeasured or measured external disturbances, also affect system behavior. In literature, the design methods of conventional controllers such as original or muddled form of PI/PID controllers resulted from the research algorithms for typical and common behavior systems are applied to real time systems

from time to time, e.g. [1]-[4]. The PI/PID tuning are applied for nonlinear or delay time systems. The FOPDT and SOPDT plant models [5], [6] are used for design and Analysis. This means that any system is reduced into FOPDT or SOPDT format to apply conventional or advanced controllers by compromising plant-model mismatch upto certain extent. In many cases, it is observed that model order reduction of system introduces some plant-model mismatch and hence the controller obtained by approximated model lacks the important properties such as stability and robustness of the system. There are more chances of instability and poor performance when the controller designed by conventional methods considering lower order model especially in case of presence of unmeasured and unavoidable disturbances. To control system strategy for processes with complex behavior dynamics or plant-model mismatch upto certain extent, sliding mode control strategy (SMC) provides a significant advantages and applications since SMC itself ensures most important factor such as Robustness and stability, Further it is well known that Robustness ensures insensitivity to parameter variations and disturbance rejection of [7]. The functional working

procedure of SMC starts after prescribing the sliding surface passing through the origin of phase plane which reduces the error signal equal to zero. In general it is common that SMC methodology involves two phases, First is reaching phase and second is sliding phase. In case of reaching phase, the variables of the system should start from initial condition and tends towards the sliding surface within finite time while in sliding phase, it travel along the sliding surface and reach the zero error condition with the motion governed by the sliding surface [8]. It is in general, the two terms in control law corresponding to two phases, a continuous control term (equivalent control) brings system variables on sliding surface and a discontinuous control term (switching control) maintains them on the sliding surface, need to be derived separately. However the implementation of SMC introduces high frequency oscillations (also commonly known as chattering phenomena) which are undesired. The main cause of chattering occurs due to discontinuous switching function (signum) which causes control signal to oscillate around the sliding surface [9]. In application, the chattering causes unwanted excessive wear and tear of mechanical components used as signal

controlled elements viz. actuators valves etc. This can also results in excitation and unmolded high frequency dynamics in the system. The effect of chattering problem can be reduced by selection of smooth sliding functions such as hyperbolic tangent or saturation (tanh) instead of signum function. This selection also guarantees ultimate boundedness of the error signal within some predetermined boundary condition [9]-[14]. In literature, many academicians and researchers have developed sliding strategies successfully using FOPDT or SOPDT system model e.g. [10], [11], [15]-[17]. The sliding mode has one of the best advantage of that the sliding strategies can be effective even if there are plant-model differences or disturbances, upto certain limits. The design of control law for system with plant uncertainties and external disturbances can be done using SMC. The main advantages of SMC in cases of uncertainty and external disturbances received more attention and got popularity since the work of Utkin [7]. In literature, many type of SMC theories are available with focus on applied areas. It is found that the early studies were mainly concentrated on conventional approach of design of SMC. In case of

conventional SMC, a design procedure was followed systematically for approximate or original model of the system [14]. Therefore, the conventional SMC methods has been widely used in many applications such as Position and Motion control in Robotics, Aerospace Engineering, Power systems (Converters) [18].

### **Model of DC motor**

The FOPDT model of the DC motor obtained by Furat and Eker [21] is used to study the performance of various controllers, The Model of DC motor is obtained in terms of FOPDT structure as given below,

$$\frac{Y(s)}{U(s)} = \frac{K e^{-t_d s}}{\tau s + 1}$$

Where,  $\tau$ ,  $t_d$  and  $K$  represent time constant, time delay and steady-state gain respectively. If the time delay is so small compared with time constant  $\tau$ , the system model of FOPDT can be approximated and represented as follows:

$$\frac{Y(s)}{U(s)} = \frac{K}{(\tau s + 1)(t_d s + 1)} = \frac{C_n}{s^2 + A_n s + B_n}$$

In this review paper, the model derived by Furat and Eker [21] using first principle method for general electromechanical system is used extensively for detailed study

and performance analysis. The model parameters provided in the literature as per Furat and Eker [21] for system is,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{0.86e^{-0.0035s}}{0.145s + 1}$$

or delay time approximated

$$\frac{Y(s)}{U(s)} = \frac{0.86}{(0.145s + 1)(0.0035s + 1)} = \frac{1694.6}{s^2 + 292.6s + 1970}$$

The transfer function of the system after delay approximation in pole is a second order over-damped system with both poles lies in the left half of s-plane. The model in time domain can be written as

$$\ddot{y}(t) + 292.6\dot{y}(t) + 1970y(t) = 1694.6u(t)$$

Where,  $y(t)$  and  $u(t)$  indicate the system output and control input respectively.

If the uncertainty and external disturbances to the system are introduced into Eq. 5

$$\ddot{y}(t) = -292.6\dot{y}(t) - 1970y(t) + 1694.6u(t) + D(t, u(t))$$

Where,  $D(t; u(t))$  represents lumped uncertainty which is unknown and bounded  $D(t; u(t)) \leq D_{max}$ , where  $\leq D_{max}$  is the upper bound on the uncertainty with  $D_{max} > 0$ . In fact, there are uncertainty and external disturbances in real time experimentation.

### Integral sliding mode controller

In this review, Integral-SMC (I-SMC), type sliding surface studied for the DC motor.

The sliding surface is [21]:

$$\sigma(t) = \left[ k_p + \frac{k_i}{s} + sk_d \right]^{n-1} \Psi(E(s))$$

Where  $\Psi(E(s))$  is the function related to the integral tracking error, which is in S domain,  $\Psi(E(s)) = 1/sE(s)$ . The term 'n0 is equal to the order of approximated system. The terms  $k_p$ ;  $k_{kd}$  represent tuning gains. The tuning gains selected by designer are useful to define the sliding surface  $\sigma(t)$  and the performance of the system is dependent on the chosen values of the gains. The objective in SMC is to design the control law which guarantees that the system output  $y(t)$  remains equal to reference input  $r(t)$  at all times. This implies that the error signal and derivatives must be zero. The function of SMC is to drive the error signal  $e(t)$  towards the sliding surface and thereafter travel along the sliding surface to converge at origin.

Put  $\Psi(E(s)) = 1/sE(s)$  in Eq. 7, it gives

$$\sigma(t) = \frac{1}{s}k_pE(s) + \frac{k_i}{s^2}E(s) + k_dE(s).$$

The error in time domain is,

$$e(t) = r(t) - y(t).$$

where,  $e(t)$  is error signal. Second derivative error signal Eq. 9 is

$$\ddot{e}(t) = \ddot{r}(t) - \ddot{y}(t)$$

$\ddot{y}(t)$  is given by Eq. 6, that is,  $\ddot{y}(t) = -292.6\dot{y}(t) - 1970y(t) + 1694.6u(t) + D(t, u(t))$  or

In general terms  $\ddot{y}(t) = -A_n\dot{y}(t) - B_n y(t) + C_n u(t) + D(t, u(t))$ .

Put  $\ddot{y}(t) = -A_n\dot{y}(t) - B_n y(t) + C_n u(t) + D(t, u(t))$  in Eq. 10

$$\ddot{e}(t) = \ddot{r}(t) - [-A_n\dot{y}(t) - B_n y(t) + C_n u(t) + D(t, u(t))] = \ddot{r}(t) + A_n\dot{y}(t) + B_n y(t) - C_n u(t) -$$

The, second-order derivative can be obtained of the sliding surface given in Eq. 8 by multiplying both side by 's<sup>2</sup>'. Thus 8 can be written as

$$s^2\sigma(t) = sk_p E(s) + k_i E(s) + s^2 k_d E(s).$$

After converting the above Eq. 12 into time domain, it can be represented as

$$\ddot{\sigma}(t) = k_p \dot{e}(t) + k_i e(t) + k_d \ddot{e}(t) \quad (13)$$

. From Eq. 11, we know,  $\ddot{e}(t) = \ddot{r}(t) + A_n\dot{y}(t) + B_n y(t) - C_n u(t) - D(t, u(t))$ . Put this in 13, therefore it can be written as,

$$\ddot{\sigma}(t) = k_p \dot{e}(t) + k_i e(t) + k_d [\ddot{r}(t) + A_n\dot{y}(t) + B_n y(t) - C_n u(t) - D(t, u(t))]. \quad (14)$$

Step 1

As  $\dot{\sigma}(t) = 0$ , put in Eq. 14

$$k_p \dot{e}(t) + k_i e(t) + k_d [\dot{r}(t) + A_n \dot{y}(t) + B_n y(t) - C_n u(t)] = 0, \quad (15)$$

Step 2

Replace  $u$  by  $u_{eq}$  in Eq. 14

$$k_p \dot{e}(t) + k_i e(t) + k_d [\dot{r}(t) + A_n \dot{y}(t) + B_n y(t) - C_n u_{eq}(t)] = 0, \quad (16)$$

Step 3

Obtain  $u_{eq}$  from above Eq. 16

$$u_{eq}(t) = \frac{1}{K_d C_n} [k_p \dot{e}(t) + k_i e(t) + k_d \dot{r}(t) + k_d A_n \dot{y}(t) + k_d B_n y(t)], \quad (17)$$

The above is equivalent controller. The control input has the following form:

$$u(t) = u_{eq}(t) + u_{sw}(t) \quad (18)$$

Substitute above  $u(t)$  in Eq. 14. Thus

$$\ddot{\sigma}(t) = k_p \dot{e}(t) + k_i e(t) + k_d [\dot{r}(t) + A_n \dot{y}(t) + B_n y(t) - C_n [u_{eq}(t) + u_{sw}(t)] - D(t, u(t))] \quad (19)$$

Substitute  $u_{eq}(t)$  in above equation, which gives

$$\ddot{\sigma}(t) = -k_d C_n u_{sw}(t) - k_d D(t, u(t)) \quad (20)$$

Now we have to take switching control.

$$u_{sw}(t) = k_{sw} r^2(t) \ddot{\sigma}(t) \operatorname{sgn}\left(\frac{k_{sf}}{\ddot{\sigma}(t)} \dot{\sigma}(t)\right) + \frac{1}{k_d C_n} \operatorname{sgn}(\sigma(t)) \quad (21)$$

$$\ddot{\sigma}(t) = \epsilon_1 \operatorname{sgn}(\sigma(t)) \quad \text{if } |e(t)| \leq \epsilon_1$$

$$\ddot{\sigma}(t) = e(t) \quad \text{if } |e(t)| \geq \epsilon_1$$

## Stability Analysis

The stability and the performance analysis of any physical system is one of the vital criterion to design control system. The stability of all the controllers can be investigated using Lyapunov stability theorem. The stability concepts of Lyapunov are added in this section is applicable to all sliding surfaces included in this paper. The Let us consider the positive Lyapunov function as follows:

$$V(t) = \frac{1}{2} \dot{\sigma}^2(t) + |\sigma(t)|$$

$$\dot{V}(t) = \dot{\sigma}_m(t)\dot{\sigma}_m(t) + \dot{\sigma}_m(t) \frac{|\sigma_m(t)|}{\sigma_m(t)}$$

Since  $d|\sigma_m(t)|/dt = \dot{\sigma}_m(t)\sigma_m(t)/|\sigma_m(t)|$  is equal to  $\dot{\sigma}_m(t)|\sigma_m(t)|/\sigma_m(t)$  in nitide, the derivative of  $|\sigma_m(t)|$  is taken to be  $\dot{\sigma}(t)|\sigma(t)|/\sigma(t)$  in 25 provid simplification. The Eq. 25, re-written as

$$\dot{V}(t) = \dot{\sigma}_m(t)[\dot{\sigma}_m(t) + \frac{|\sigma_m(t)|}{\sigma_m(t)}]$$

With  $\ddot{\sigma}_m(t) = \ddot{\sigma}(t)$ . From Eqs. 20,

$$\dot{V}(t) = \dot{\sigma}_m(t)[-k_d C_n u_{sw}(t) - k_d D(t, u(t)) + \frac{|\sigma_m(t)|}{\sigma_m(t)}]$$

$$\dot{V}(t) = \dot{\sigma}_m(t) \left[ -k_d C_n k_{sw} r^2(t) \tilde{e}(t) \operatorname{sgn} \left( \frac{k_{sf}}{\tilde{e}(t)} \dot{\sigma}_m(t) \right) - \operatorname{sgn}(\sigma(t)) - k_d D(t, u(t)) + \frac{|\sigma}{\sigma} \right]$$

put  $\operatorname{sgn}(\sigma_m(t)) = \frac{|\sigma_m(t)|}{\sigma_m(t)}$ . The above Eq. 28 can be written as

$$\dot{V}(t) = \dot{\sigma}_m(t) \left[ -k_d C_n k_{sw} r^2(t) \tilde{e}(t) \operatorname{sgn} \left( \frac{k_{sf}}{\tilde{e}(t)} \dot{\sigma}_m(t) \right) - \operatorname{sgn}(\sigma(t)) - k_d D(t, u(t)) + \operatorname{sgn}(\sigma_m(t)) \right]$$

Thus, after simplification,

$$\dot{V}(t) \leq -k_d [\dot{\sigma}_m(t) [C_n k_{sw} r^2(t) |\tilde{e}(t)| - D_{max}] \leq 0$$

$$C_n k_{sw} r^2(t) |\tilde{e}(t)| - D_{max} \leq 0$$

or

$$C_n k_{sw} r^2(t) |\tilde{e}(t)| \leq D_{max}$$

or

$$k_{sw} > \frac{D_{max}}{C_n r^2(t) |\tilde{e}(t)|}$$

The stability proof is said to be completed whenever the above condition is Satisfied. The switching control given in Eq. 21 includes discontinuous terms, 'signum' that may lead to chattering effect in case of real systems. Therefore, hyperbolic tangent function or any other smoother function is preferred to obtain smooth switching. The

switching control law with hyperbolic tangent functions written as

$$u_{sw}(t) = k_{sw} r^2(t) \tilde{e}(t) \operatorname{tanh} \left( \frac{k_{sf}}{\tilde{e}(t)} \dot{\sigma}_m(t) \right) + \frac{1}{k_d C_n} \operatorname{tanh} \left( \frac{\sigma_m(t)}{\Omega} \right)$$

### Conventional SMC

The short algorithm of conventional SMC given by Camacho [14] is given in this section. This method is used to compare the performance of the system with second order integral SMC. The FOPDT model is given by Eq. 1 After delay approximation, the FOPDT model can be written as

$$\frac{Y(s)}{U(s)} = \frac{K}{(\tau s + 1)(t_d s + 1)} = \frac{K}{\tau t_d s^2 + (\tau + t_d)s + 1}$$

The equation in time domain form is

$$\ddot{y}(t) = \frac{1}{\tau t_d} [-(\tau + t_d)\dot{y}(t) - y(t) + K u(t)]$$

The sliding surface used by Camacho [14] is

$$\sigma(t) = \lambda_1 e(t) + \lambda_0 \int_0^t e(t) dt + \frac{d}{dt} e(t)$$

where,  $\lambda_1 = 2\lambda$  and  $\lambda_0 = \lambda^2$  and  $\lambda$  is a tuning parameter, which helps to define the sliding surface. The derivative of 37 is

$$\dot{\sigma}(t) = \lambda_1 \dot{e}(t) + \lambda_0 e(t) + \dot{e}(t)$$

The error is  $e(t) = r(t) - y(t)$ , Thus above Eq. 38 can be written as

$$\dot{\sigma}(t) = \lambda_1 [\dot{r}(t) - \dot{y}(t)] + \lambda_0 e(t) + \dot{r}(t) - \dot{y}(t) = 0$$

We know, from Eq. 36,  $\ddot{y}(t) = \frac{1}{\tau t_d} [-(\tau + t_d)\dot{y}(t) - y(t) + K u(t)]$

$$\lambda_1 [\dot{r}(t) - \dot{y}(t)] + \lambda_0 e(t) + \dot{r}(t) - \frac{1}{\tau t_d} [-(\tau + t_d)\dot{y}(t) - y(t) + K u(t)] = 0.$$

The Equivalent controller can be obtained from above Eq. 40 and given as,

$$u_{eq}(t) = \frac{\tau t_d}{K} \left[ \left( \frac{t_d + \tau}{\tau t_d} - \lambda_1 \right) \dot{y}(t) + \frac{\dot{y}(t)}{\tau t_d} + \lambda_0 e(t) + \dot{r}(t) + \lambda_1 \dot{r}(t) \right] = 0.$$

The  $u_{eq}(t)$  can be simplified by considering

$$\lambda_1 = \frac{t_d + \tau}{\tau t_d}$$

It has been shown that this choice for  $\lambda_1$  is the best for the continuous part of the controller [14]. To assure that the sliding surfaces behave as a critical or over-damped system,  $\lambda_0$  should be

$$\lambda_0 \leq \frac{\lambda_1^2}{4}$$

The switching controller is taken as

$$u_{sw}(t) = K_d \frac{\sigma(t)}{|\sigma(t)| + \delta}$$

As we know, derivative of reference signal of setpoint is zero, that is  $\dot{r}(t) = \ddot{r}(t) = 0$ . The Total control law is

$$u(t) = u_{eq}(t) + u_{sw}(t) = \frac{\tau t_d}{K} \left[ \frac{\dot{y}(t)}{\tau t_d} + \lambda_0 e(t) \right] + K_d \frac{\sigma(t)}{|\sigma(t)| + \delta}$$

where,

$$K_d = \frac{0.51}{|K|} \left( \frac{\tau}{t_d} \right)^{0.76}$$

and

$$\delta = 0.68 + 0.12|K|K_D\lambda_1$$

## Sliding-Mode Control with PID Sliding Surface

In the literature Eker [11] presented the SMC with PID sliding surface for electromechanical systems. This work extensively addresses the issue of robustness to parameter variations and modeling errors. The SMC as being well-known for its property, especially its insensitive to parameter variations, modeling errors, and external load disturbances. Therefore, it can be concluded that appropriate design of SMC is can overcome the above mentioned

de\_iciency ,which are unavoidable in reality and practice. In Eker's work, a PID sliding surface with three

parameters has been introduced to obtain a satisfactory responses in closed-loop system.

The sliding surface with PID is given below

$$s(t) = k_p e(t) + k_i \int_0^t e(t) dt + k_d \frac{d}{dt} e(t)$$

where,  $k_p$ ,  $k_i$  and  $k_d$  are tuning parameters. The Equivalent controller given by Eker [11] is

$$u_{eq}(t) = \frac{1}{k_d C_n} [k_p \dot{e}(t) + k_i e(t) + k_d \ddot{r}(t) + k_d A_n \dot{y}(t) + k_d B_n y(t)].$$

and switching control is

$$u_{sw}(t) = k_{sw} \tanh \left( \frac{s(t)}{\beta} \right).$$

Thus total control law is

$$u(t) = \frac{1}{k_d C_n} [k_p \dot{e}(t) + k_i e(t) + k_d \ddot{r}(t) + k_d A_n \dot{y}(t) + k_d B_n y(t)] + u_{sw}(t) = k_{sw} \tanh \left( \frac{s(t)}{\beta} \right)$$

## Conventional PID controllers

The PID controller structure is

$$G_c(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right)$$

The PID controller by using method of Zeigler-Nicholas is

$$G_c(s) = 58.38 \left( 1 + \frac{1}{0.0055s} + 0.0014s \right) = 58.35 + \frac{10615}{s} + 0.0817s.$$

## Simulation Studies

In this section, parameters of a second order integral sliding mode controller, SMC with PID sliding surface and conventional SMC as well as well known PID controller are used to review performance of the DC motor. It is observe that the second-order integral sliding mode controller has better performance as compared to other SMC approaches and PID controller. The responses of integral second order sliding

mode technique are compared with the reviewed conventional SMC, SMC for stable processes and Zeigler-Nicholas PID controller in terms of transient response and performance error indices.

Table 1. Parameters of second-order integral sliding mode control

Terms	parameters value	Remark
Sliding surface	$k_p = 1.2, k_i = 10^{-4}, k_d = 0.0024$	-
Equivalent control	$r = \text{set point}, y(t) = \text{Output}$	$A_n, B_n, C_n$ plant parameters
Switching Control	$k_{sw} = 40, k_{sf} = 0.025$	$\epsilon_1 = 10^{-20}$

Table 2. Parameters of conventional sliding mode control

Terms	parameters value	Remark
Sliding surface	$\lambda_1 = 292.6, \lambda_0 = 2.1405 \times 10^4$	-
Equivalent control	$r = \text{set point}, y(t) = \text{Output}$	$\tau, t_d, K$ plant parameters [Ref. Eq. 3]
Switching Control	$K_D = 10.05, \delta = 304.2$	-

Table 3. Parameters of PID sliding surface sliding mode control

Terms	parameters value	Remark
Sliding surface	$k_p = 260, k_i = 0.01, k_d = 24$	-
Equivalent control	$r = \text{set point}, y(t) = \text{Output}$	$A_n, B_n, C_n$ plant parameters [Ref. Eq. 2]
Switching Control	$\beta = 25, k_{sw} = 100$	-

### Performance Reviews and Analysis

The performance of the electromechanical system is carried-out with various conditions. The various are taken in the section and performance analysis is obtained.

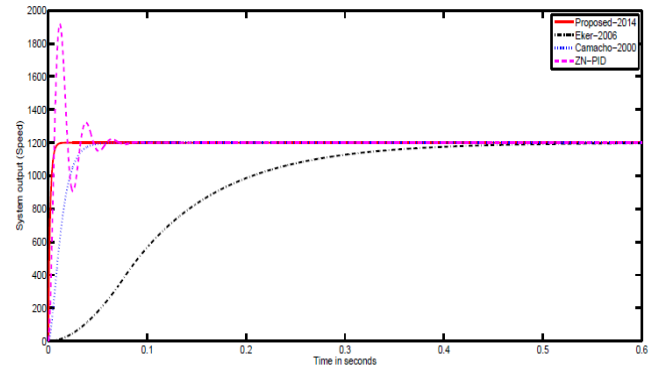


Fig. 1. Case 1: Speed under Nominal Plant Parameters.

### Case 1: Performance with nominal parameters

Table 4. Case 1: Simulation Results Under Nominal Parameters

Approach	Settling time $t_s$	Overshoot %	$M_p$	$ISE \times 10^7$	$IAE \times 10^4$	$ITAE$
Proposed	0.011	0	2.63	2.35	13.23	
Eker-2006	0.4	0	9.45	14.1	17200	
Camacho-2000	0.041	0	3.25	3	103.28	
ZN-PID	0.055	58	2.87	2.75	83.04	

Suppose the parameters of the system given in Eq. 4 are considered. The sliding mode approaches with parameters given in previous section. The sliding mode controllers are implemented using the above parameters and performance of the system is analyzed. The reference speed is 1200 RPM (revolution per minute). The speed of the motor with various approaches is shown in Fig. 1, the input to the system is shown in Fig. 8-8. The sliding surfaces with all SMC approaches are shown in Fig. 3 while the error signal in RPM are shown in Fig.4.



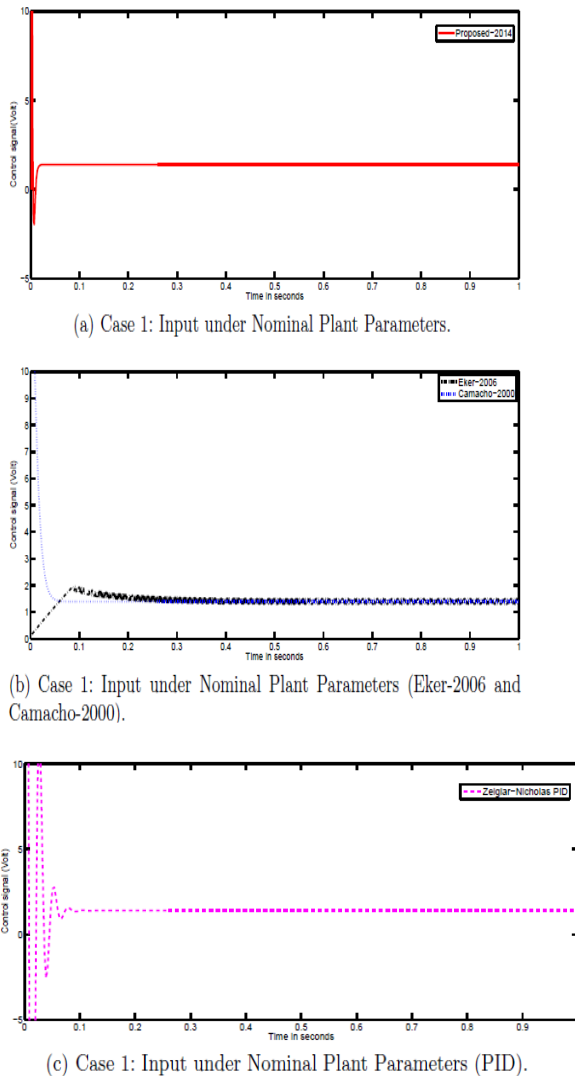


Fig. 2. Input Signal with All Methods

Now in the Fig. 1, it is seen that the response of the I-SMC controller is superior than as compared to the approach of Eker [11], Camacho [14] and conventional PID controller. The rise time in performance of I-SMC is less and there is no overshoot. The conventional PID controller tuned by Zeigler-Nicholas methods results in larger

overshoot and it is concluded that such type of responses are not suitable for applications where DC motor is used as a main controlled

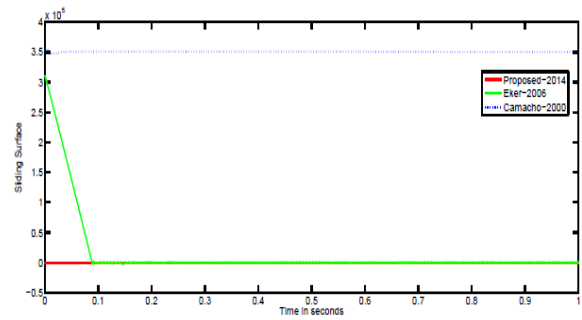


Fig. 3. Case 1: Sliding Surface for Various Approaches.

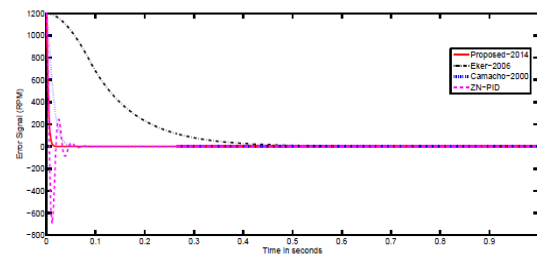


Fig. 4. Case 1: Error Signal for Various Approaches.

The response given by the Eker is seems to be sluggish with higher value of settling time while the response of the Camacho approach is fast compared to Eker but slow as compared to proposed approach. From Fig. 8, it can be concluded that the control effort required by the I-SMC controller is smooth while that of the Eker and Conventional PID controller are continuously oscillatory. The control signal by the method of Camacho is smooth and similar to

the I-SMC approach. Even though the control signal by Camacho approach is smooth, but the sliding surface cannot goes to zero as time tends to infinity as shown in Fig. 8. The sliding surface of the proposed controller goes toward zero as time tends to infinity. The time domain performance measures and error indices are included in Table 4. From Table 4, it is clear that there is effect of the second order integral sliding mode approach on the performance of the motor. As proposed controller results in less values of time domain specification such as settling time and overshoot compared to other approaches. The error performance indices shown justifies that the proposed control scheme is for better than Eker and Camacho controller.

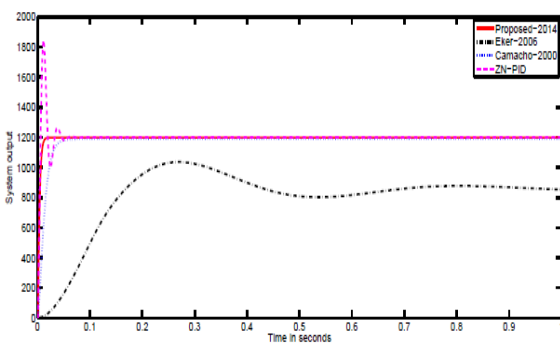


Fig. 5. Case 2: Speed under +10% Parametric Uncertainty.

Case 2: Performance with parametric uncertainty

In this case study the nominal plant parameters are given below

$$\frac{Y(s)}{U(s)} = \frac{1694.6}{s^2 + 292.6s + 1970}$$

is changed to + 10% as follows

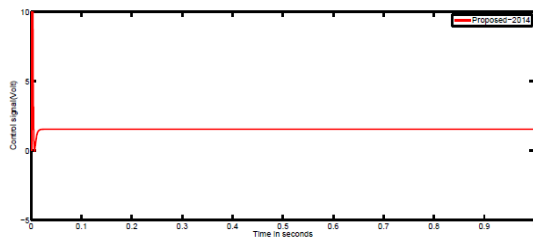
The numerator term 1694.6 is changed with + 10% and new value is  $1694.6 \times 1.1 = 1864.1$ . The poles of the system are at -285.7048 and -6.8952, this poles are changed by +10%, that is  $-285.7048 \times 1.1 = -314.2752$  and  $-6.8952 \times 1.1 = -7.5848$ . Thus the system with +10% parametric uncertainties is

$$\frac{Y(s)}{U(s)} = \frac{1864.1}{(s + 314.2752)(s + 7.5848)} = \frac{1864.1}{s^2 + 321.9s + 2384}$$

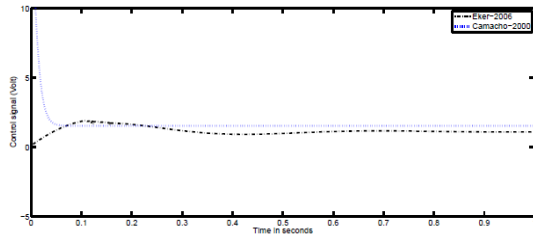
Table 5. Case 2: Simulation Results Under +10% Parametric Uncertainty. (NA:Not Achievable)

Approach	Settling time $t_s$	Overshoot %	$M_p$	$ISE \times 10^7$	$IAE \times 10^4$	$ITAE$
Proposed-2014	0.012	0	2.65	2.40	111.46	
Eker-2006	NA	NA	9.66	13.71	34939	
Camacho-2000	0.046	0	3.29	.32	1190.8	
ZN-PID	0.045	54	2.87	2.75	73.51	

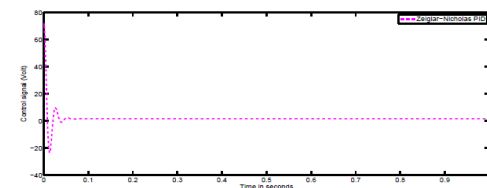
Suppose the parameters of the system given in Eq. 55 are considered and the sliding mode approaches with parameters given in previous section (section 5.2), that is, the parameters of SMC obtained for nominal plant parameters are used to examine the performance of the system, if there are + 10% uncertainties. The sliding mode controllers are implemented using the parameters obtained for parametric uncertain model and performance of the system is analyzed using



(a) Case 2: Input under +10% Parametric Uncertainty.



(b) Case 2: Input under +10% Parametric Uncertainty (Eker-2006 and Camacho-2000).



(c) Case 2: Input under +10% Parametric Uncertainty(PID).

Fig. 6. Input Signal with All Methods under +10% Parametric Uncertainty

the controllers designed for the system with nominal parameters. The reference speed is again 1200 RPM (revolution per minute). The speed of the motor with various approaches under the parametric uncertainties are shown in Fig. 5, the input to the system is shown in Fig. 8-8. The sliding surfaces with all SMC approaches under the parametric uncertainties are shown in Fig. 7 while the error signal in RPM are shown in Fig. 8.

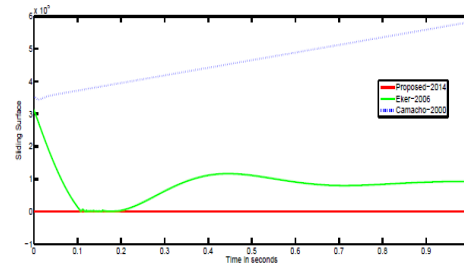


Fig. 7. Case 2: Sliding Surface for Various Approaches (+10% Parametric Uncertainty).

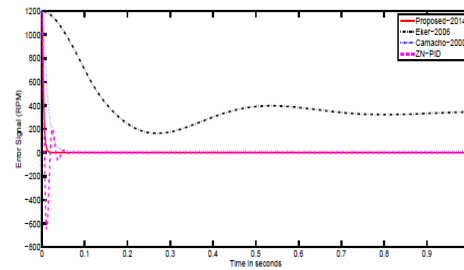


Fig. 8. Case 2: Error Signal for Various Approaches(+10% Parametric Uncertainty).

From the Fig. 5, it is observed that the proposed controller exhibits far superior performance than the approach of Eker [11], Camacho [?] and conventional PID controller under +10% parametric uncertainty. The proposed controller has no overshoots and less rise time. The conventional Zeigler-Nicholas tuned PID controller gives large overshoot and it is concluded that such type of response is not suitable for applications where DC motor is used as a main controlled system. The response given by the Eker is also not suitable under +10% parametric uncertainty while the response of the Camacho approach is acceptable compared to Eker but slow as compared to proposed

approach. From Fig. 8, it can be seen that the control effort by the proposed controller is smooth while that of Conventional PID controller is also smooth under +10% parameteric uncertainty. The control effort due to Eker's approach is not acceptable as it ends with large offset and never the target or reference speed is achieved. The control signal by the Camacho is smooth and similar to the proposed approach. Even though the control signal by Camacho approach is smooth, but The time domain performance measures and error indices under +10% parametric uncertainty are included in Table 5. From Table 5, it is clear that there is acceptable as well as better performance of the second order integral sliding mode approach on the speed control of the DC motor. As proposed controller gives less time domain parameters such as settling time and overshoot compared to other approaches. The error performance indices also shown the proposed control approach is better than the Eker and Camacho controller. the sliding surface cannot goes to zero as time tends to infinity as shown in Fig.8. The sliding surface of the proposed controller goes toward zero as time tends to infinity.

## Conclusions

In this paper, the model of DC motor in terms of FOPDT is used extensively to study the performance of sliding strategies. The delay term of the model is used to get second order model, which it-turn used to obtain the control laws. The second order integral sliding mode approach and conventional SMCs are reviewed and in each case, equivalent control for typical electromechanical system is designed. The switching control is selected in such way that the chattering effect will be minimum. The performance of second order integral SMC is compares

with the conventional SMC and SMC of stable processes. The pioneer method of Zeigler-Nicholas for PID controller tuning is used and the parameters of the PID controller is obtained.

The simulation results carried out in Mathworks MATLABR2009a to study the performances. For all the SMC methods following points are reviews and included in the paper:

1. Sliding surface and its simulation,
2. Equivalent control law and its implementation ,
3. Switching control law and its implementation,

4. Unit step responses are studied for sliding control strategies.
5. Performance analyses under nominal parameters and parameters under uncertainties.

The results obtained for various approaches gives the conclusion that the performance given by the second order integral sliding mode approach is far superior than that of the other approaches. It is concluded that the further work can be extended for higher order sliding mode for higher order systems to improve the system performance.

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