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Two special types of Binomial distribution

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Abstract

Binomial distribution is generalised form of Bernoulli distribution. There are many types of binomial distribution. In this research paper, we will discuss two special types of binomial distribution, namely the negative binomial distribution and the geometric distribution in detail. We will also discuss the difference between negative binomial distribution and geometric distribution.

Keywords: random variable, bernoulli, binomial, negative binomial, geometric, sequence

Introduction

The Trials of a random experiment are said to be Bernoulli trials if they satisfy the following condition:-

1. The trials should be independent.
2. Each trial has exactly two outcomes that is success or failure.
3. The probability of success remains the same in each trial.

We define Y as a random variable associated with Bernoulli trials as follows:-

Y (Success)=1

Y (Failure)=0.

The probability mass function of Y can be written as:-

$P(y)=p^{y}(1-p)^{1-y}$ where y=0,1 (1.1) and 0 otherwise

Here we say that Y has a Bernoulli distribution.

We consider n Bernoulli trials, where trials are independent and probability of success and failure on each trial are p and 1-p respectively. Let a random variable X equals to number of observed success in n Bernoulli trials, then possible values of X are 0,1,2,……,n.

If x (x=1,2,3,….,n) success occur, then n-x failure occur. Then number of ways of selecting the x positions for x successes in n trials is ${}^{n}C_{x.}$

In this case, formula (1.1) will changed into a new formula as:-

${}^{n}C_{x.}\, p^{x}(1-p)^{n-x}$ where x=0,1,2,3,…..,n

Let random variable X that has a probability mass function of the type

$p(x) = {}^{n}C_{x.}\, p^{x}(1-p)^{n-x}$ where x=0,1,2,3,…..,n (1.2)
and 0 otherwise.

If X has probability mass function of the above type, then it X is said to have binomial distribution.

Method

For negative binomial distribution, consider a sequence of independent trials of a random experiment. Here probability p of success in each trial should same. The experiment under the same conditions continues till a definite number of successes are obtained. Here we have randomly placed failures F and

Successes S in a sequence. The last two bold typed **S** denotes the (r-1)th success and rth success respectively.

See this:-

F, F, F, F, F, S, F, F, F, F, F……………. F, F, F, F, F, S, F, F, F, F, F, **S**, F, F, F, F, F, **S** (1.3)

↑ ↑

(r-1)th success rth success

Let Z denotes the number of failures in this sequence before rth success (r is a fixed positive integer), then Z+r is the total number of trials, which should be completed before rth success. Let us take sequence (1.3) without rth success:-

F, F, F, F, F, S, F, F, F, F, F……………. F, F, F, F, F, S, F, F, F, F, F, **S**, F, F, F, F, F (1.4)

Here we have dropped rth success from (1.3) and get a new sequence (1.4). Since Z denotes the number of failures before rth success, therefore (1.4) has Z failures and r-1 success and hence we have total Z+r-1 trials, with (r-1) successes. In this case formula (1.2) reduces to

$^{z+r-1}C_{r-1}\, p^{r-1}\, (1-p)^{(z+r-1)-(r-1)}$

$= {}^{z+r-1}C_{r-1}\, p^{r-1}\, (1-p)^{z}$ (1.5)

Now we will include 1 success in the sequence (1.4) at the last place:-

F, F, F, F, F, S, F, F, F, F, F……………. F, F, F, F, F, S, F, F, F, F, F, S, F, F, F, F, F, S

This is the sequence same as (1.3). It means that last included success is rth success.

Then by multiplication rule of probability, formula (1.5) reduces to

$^{z+r-1}C_{r-1}\, p^{r-1}\, (1-p)^{z}\, p$

Which is equals to $^{z+r-1}C_{r-1}\, p^{r}\, (1-p)^{z}$

Thus the probability mass function of Z is

$g(z) = {}^{z+r-1}C_{r-1}\, p^{r}\, (1-p)^{z}$ where z=0,1,2….. (1.6)

and 0 otherwise.

This distribution with a probability mass function of the form g(z) is called negative binomial distribution.

Now we will restrict ourselves only to first success. It means the experiment under the same conditions continues till the first success is obtained. Therefore sequence (1.3) reduces to:-

F, F, F, F, F, S (first success) (1.7)

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As Z denotes the total number of failure before first success, therefore a player has to perform Z+1 trials to get first success. When we had sequence (1.3), then total numbers of trials were Z+r-1. If we compare Z+r-1 with Z+1, then we get r=1.Put it in formula (1.6), we get

$p(1-p)^z$

Let a random variable which has probability mass function of the type:-

$h(z) = p(1-p)^z$, where z=0,1,2,….

And 0 otherwise

A random variable which has probability mass function of this type is said to have geometric distribution.

Therefore geometric distribution is special case of negative binomial distribution with r=1.

Proposition 1: Consider a school basketball player .He is a 80% free throw shooter. What is the probability that he makes his 8th free throw on his 15th shot?

Solution- Here free throw by player is a success. We have to find probability of getting 8th success in 15th shot. There are 15-8=7 failures before 8th success. Let Z denotes the failures before 8th success. Since total trials before rth success is Z+r-1, therefore total trials before 8th success are Z+r-1=7+8-1=14.Hence we have,

${}^{z+r-1}C_{r-1}\, p^r\, (1-p)^z$ (1.8) $= {}^{14}C_7(0.80)^8(1-0.80)^7$

Which is our required result.

Proposition 2: Consider a school basketball player .He is a 80% free throw shooter. What is the probability that he makes his first free throw on his 15th shot?

Solution. Let Z denotes the failures before first success, therefore Put r=1, p=0.80 and z=14 in formula (1.8), we get

$(0.80)(1-0.80)^{14}$

Which is our required result.

Results and Discussion

In negative binomial distribution, the experiment under the same conditions continues till a definite number of successes are obtained. In geometric distribution, the experiment under the same conditions continues till the first success is obtained. There is not much difference between negative binomial distribution and geometric distribution. Geometric distribution is just a special case of negative binomial distribution.

Conclusion

Geometric distribution is a special case of negative binomial distribution; negative binomial distribution is special case of binomial. Binomial is generalised form of Bernoulli distribution. Bernoulli distribution ->Binomial distribution - >negative Binomial distribution ->Geometric distribution

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