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Application Of Kalman Filtering Technique To Estimate Biochemichal Oxygen Demand And Dissolved Oxygen Content Of A River Stream.

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Abstract

The quality of water in a stream is measured by the in stream biochemical oxygen demand(BOD) and the dissolved oxygen(DO).If DO falls below certain levels or BOD rises above certain levels, fishes die. In this paper a discrete dynamic model of first order difference equation is described for the dynamics of BOD and DO in a two reach river system. Kalman filtering technique is applied to the first order discrete dynamic model to estimate the concentrations of BOD and DO in reaches of a non-tidal river, subject to the measured value of DO. This methodology was applied to Warri River in Delta state of Nigeria. The system was simulated, the concentration of BOD in the river ranges from 1.41mg/l to 16.99mg/l and the concentration of DO in the river ranges from 1.67mg/l to 6.98mg/l.

Key: Kalman filter, Estimation, DO, BOD, River.

1.0 Introduction

According to Wikipedia: 'DO is the capital of water, and the indication of water self-purification. DO is consumed by reducibility material, such as sulphide, nitrite and ferrous material. DO contents is reduced as a result of water polluted by organic matter and reducing material. Breathing of microbes in the water and oxidative decomposition of aerobic microbes to organic matter can also consume DO. Do in stream may vary from 0 mg/l to 18 mg/l, readings above 18 mg/l are physically impossible. BOD is an important measure of water quality. It is a measure of the amount of oxygen needed (in milligrams per litre) by bacteria and other microorganisms to oxidize the organic matter present in a water sample over a period of 5 days. The quality of water in a stream is measured by the in stream BOD and DO, if DO falls below certain levels or BOD rises above certain levels, fishes die. The BOD of drinking water should be less than 1 mg/l i.e unpolluted Rivers have a BOD below 1 mg/l, moderately polluted Rivers vary between 2 to 8 mg/l of BOD.'

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There were many researchers on stream and water quality modelling. The first water quality model was developed by Streeter and Phelps (1925). The basic principles behind this model include (i) DO is supplied by reaeration and photosynthesis and demanded by respiration and BOD, and, and (ii) BOD is due to emission from point and nonpoint sources and could be reduced by oxidation, sedimentation and absorption processes. Guo et al (2003) developed a stochastic water-quality forecasting system that reflect random characteristics of many parameters and based on Kalman filtering and self- adaptive techniques. The developed system was used for predicting DO and BOD levels in the Yiluo River. Tamura and Kawaguchi (1980) presented real-time parameter estimation procedures for BOD parameters in four reaches of the Yomo River, Japan. The procedure was based on a method fitting n-dimensional time series data to an autoregressive model of finite order, using a steady-state model. Other researchers were: Beck(1975); Beck and Young(1976); Bowles and Grenney(1978); DiCola et al(1976); Gnauk et al(1976); Koivo and Philips(1976); Renaldi et al(1979); Moore and Jones(1978); Lettenmaier and Burges(1976).

2.0 **Definitions of Terms and Concepts**

(i)Biochemical oxygen demand (BOD)-a parameter used to measure the rate of absorption of oxygen by decomposing organic matter. The unit is milligram per litre (mg/l)

(ii)Dissolve Oxygen (DO)- is the amount of oxygen dissolved (and hence available to sustain marine life) in a body of water such as lake, river, or stream. The unit is in milligram/ litre (mg/l).

(iii)A reach of river-is defined as a stretch of a river of some convenient length which receives one major effluent discharged from a sewage facility (fig. 1).

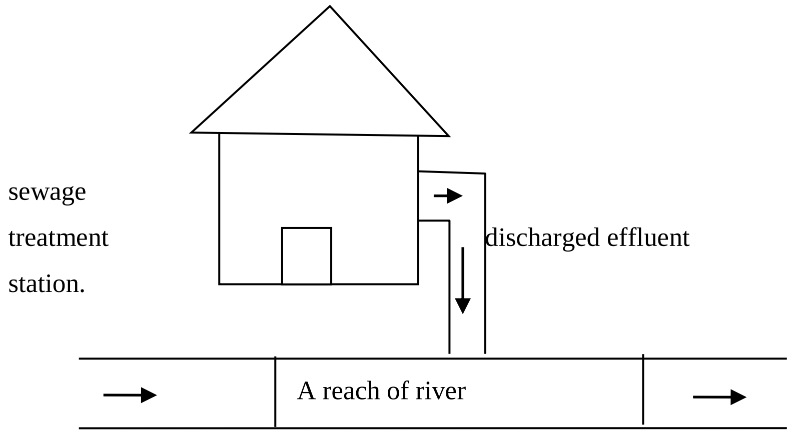

Fig.1**:** A reach of river

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(iv)A two reach river system-is defined as a stretch of a river of some convenient length divided into two sections such that each section receives one major effluent discharged from sewage facility (fig.2).

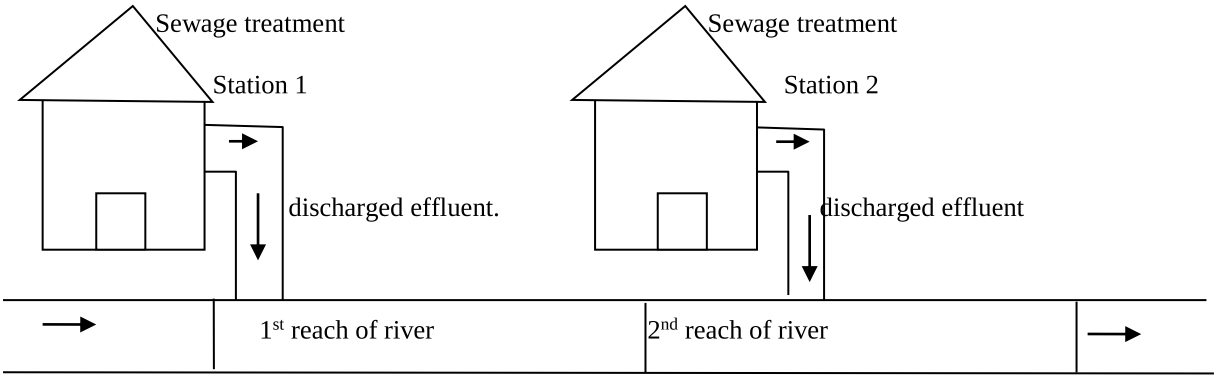

Fig.2: A two reach river system

2.0 Materials and methods

2.1 Kalman filter

According to Wikipedia Kalman filter known as linear quadratic estimation (LQE) is an algorithm that uses series of measurements observed over time which contains noise(random variations) and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on single measurement alone, by using Bayesian inference and estimating a joint probability distribution over the variables for each timeframe.

The main problem according to Kalman (1960) is given as:

Consider the dynamic model

$$\left.\begin{aligned} X_{k+1} &= \Phi X_k + \xi_k \\ Y_k &= HX_k + \eta_k \end{aligned}\right\} \quad (1)$$

where ξ_k, η_k are independent Gaussian random process of (n x 1) and (r x 1) vectors respectively with zero mean , X_k is an n-vector, Y_k is a r-vector ($r \leq n$), Φ, H are n x n, r x n matrices respectively whose elements are non-random functions of time.

Given the observed values of $Y_0,\ldots,Y_k$, find an estimate $\hat{X}_{k|k}$ of X_k which minimizes the expected loss. The solution of the problem above is given in terms of theorem as:

2.2 Theorem on kalman filter.

Given prior estimate $\hat{X}_{k|k-1}$ and its covariance $P_{k|k-1}$ together with the observed values$Y_0,\ldots,Y_k$, the optimal estimate $\hat{X}_{k|k}$ of X_k is given by

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k(Y_k - H\hat{X}_{k|k-1}) \qquad (2)$$

where ,

$$K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T+R)^{-1} \qquad (3)$$

The matrices $P_{k|k}$, $\hat{X}_{k+1|k}$, $P_{k+1|k}$ are given by recursion relations:

$$P_{k|k} = P_{k|k-1} - K_kHP_{k|k} \qquad (4)$$

$$\hat{X}_{k+1|k} = \Phi\hat{X}_{k|k} \qquad (5)$$

$$P_{k+1|k} = \Phi P_{k|k}\Phi^T + Q \qquad (6)$$

Where , Q is the covariance of the process noise, R is the covariance of the observation noise; for each time step, k. The proof of the theorem is found in Singh and Titli (1978).

According to Robert and Patrick (1992), the Kalman filter is represented by Kalman filter Loop as in figure 3.

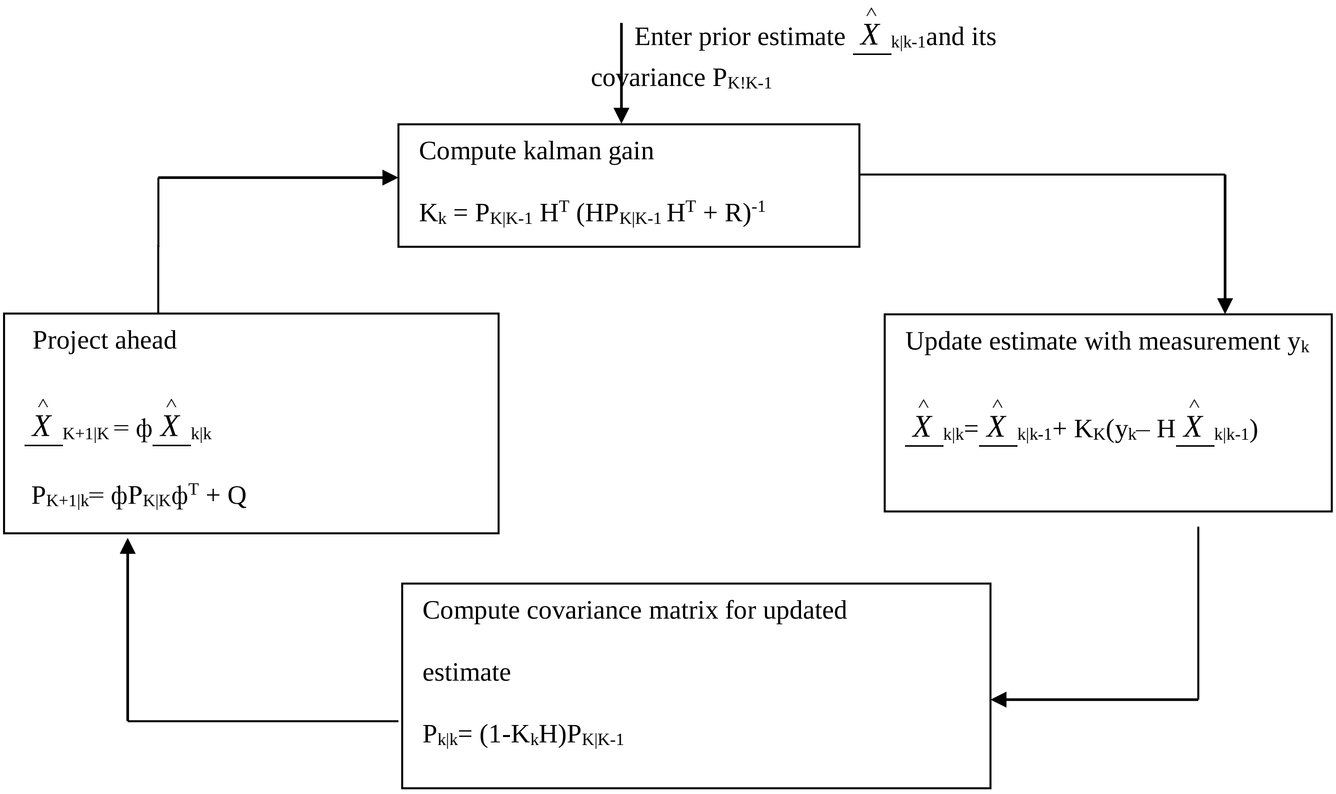

Fig.3: The Kalman filter loop (Robert and Patrick, 1992)

In order to use Kalman filter to estimate the internal state of a process, one must model the process in accordance with the framework of the Kalman filter. This means specifying the following matrices: Φ, the state-transition model, H, the observation model, Q, the covariance of the process noise, R, the covariance of the observation noise; for each time step, k. Where k = 0, 1,2, ...,N-1 and $N \geq 1$

3.0 The Warri River

The Warri river in Delta State of Nigeria is an example of inland water receiving sewage from several industries, factories and markets. These wastes often contain significant spectrum of organic and inorganic substances capable of producing adverse effects on the physical, chemical and biotic components of the environment either directly or indirectly on human health (Aghoghovwia, 2008). The rapid increase in human population, inadequate infrastructural facilities, lack of good/proper facilities for waste disposal as well as problem of refuse collection and disposal, have contributed to the environmental decay

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(Odiete, 1990). Warri River flows through the adjourning mangrove swamp forest area of the southern part of Nigeria, where the drainage and catchment areas are probably very rich in decaying organic matter and humus. Warri River stretches within latitude $5^0 21'$ to $6^0 00'$N and longitude $5^0 24'$to $6^0 2'$E. Its source is around Utagba Uno, covering a surface area of above 255 km^2 with a length of about 150 km. From its source, the river flows South-westerly direction passing between Oviorie and Ovu-inland and southwards at Odiete through Agbarho to Otokutu and Ugbolokposo (Egborge, 2001) It turns southwards to Effurun and forms a 'W' between Effurun and Warri. Important land marks in this River stretch are Enerhen, Igbudu, Ovwian and Aladja(steel town),Warri ports, main Warri market, NNPC Refinery, Globe star, etc.

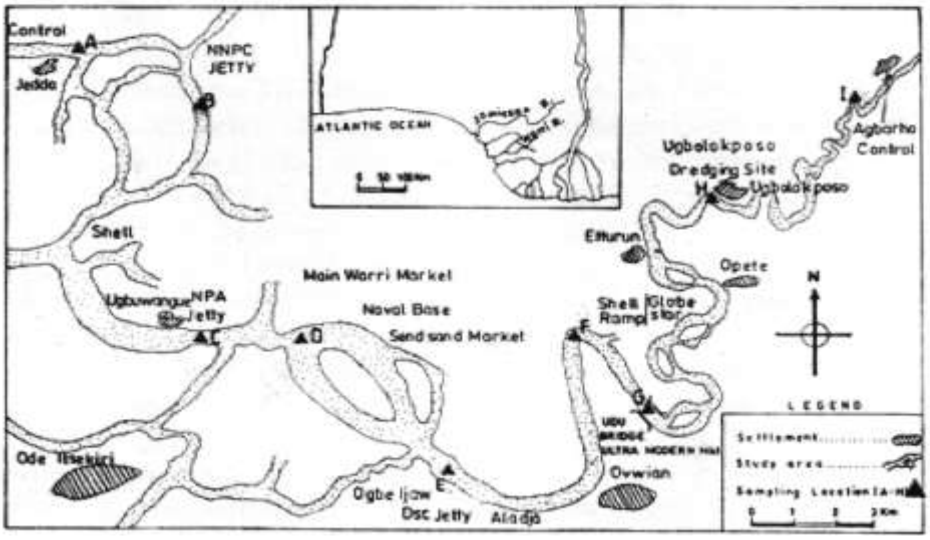

Fig.4: Map showing Warri River (Aghoghovwia, 2008)

4.0 Formulation of the two reach River model.

Beck (1978) presented a single reach river model given as:

$$\left.\begin{aligned} \dot{z}_i &= -k_{1i} z_i + \frac{Q_{i-1}}{v_i} z_{i-1} - \frac{Q_i + Q_E}{v_i} z_i + \frac{\vartheta_i Q_E}{v_i} \\ \dot{q}_i &= k_{2i}(q_i^{\,s} - q_i) + \frac{Q_{i-1}}{v_i} q_{i-1} - \frac{Q_i + Q_E}{v_i} q_i - k_{1i} z_i - \frac{\mu_i}{v_i} \end{aligned}\right\} \qquad (7)$$

Where,

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z_i , z_{i-1} are the concentrations of B.O.D. in reaches i and i-1 in mg/litre

q_i , q_{i-1} are the concentrations of D.O. in reaches i and i-1 in mg/litre

v_i is the volume of water in reach i in million gallons

Q_E is the flow rate of the effluent to reach i in million gallons/day

k_{li} is the oxygen consumption coefficient$(day)^{-1}$ in reach i

k_{2i} is the oxygen recovery coefficient$(day)^{-1}$ in reach I

Q_i , Q_{i-1} are the stream flow rates in reaches i and i-1 in million gallons/day

q_i^s is the D.O. saturation level for the i^{th} reach (mg/litre)

$\frac{\mu_i}{v_i}$ is the removal of D.O. due to bottom sludge requirements $(mg/litre(day)^{-1})$

ϑ_i is the concentration of B.O.D. in the effluent in mg/litre

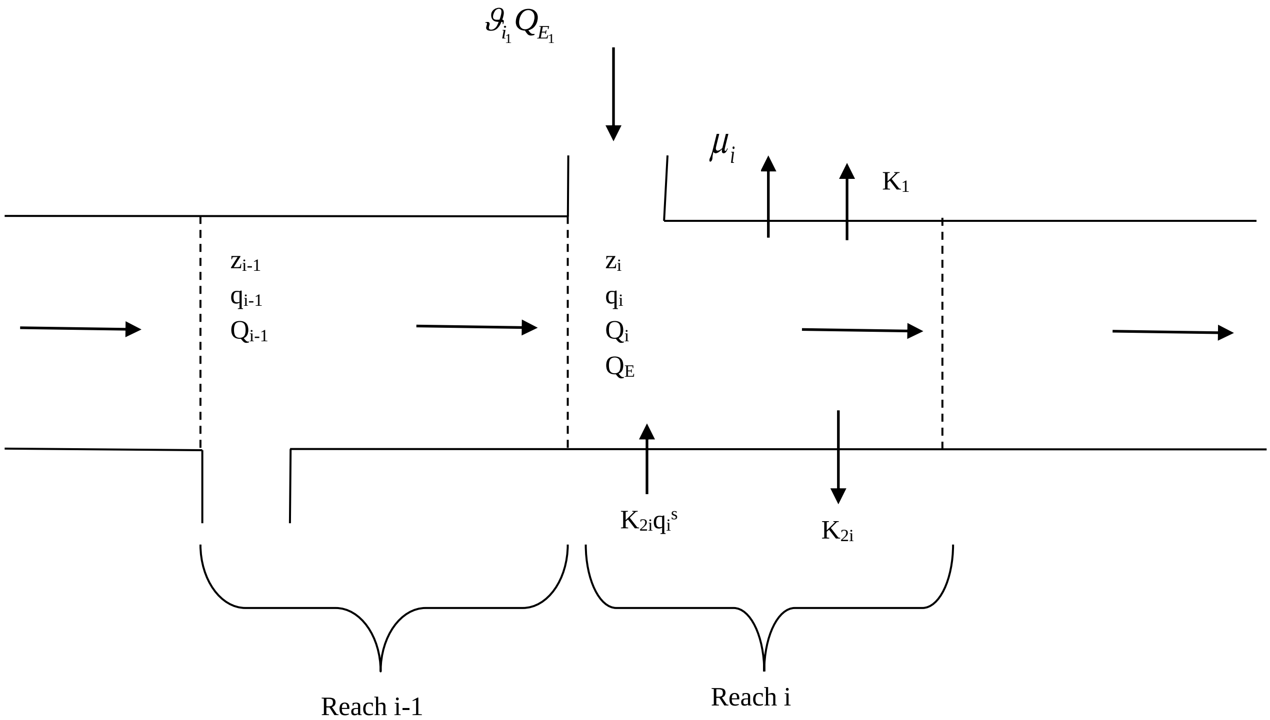

Fig.5: Flow diagram for Becks model.

A two reach river model is formulated as an extension of a single reach model of Beck (1978). The river is divided into two reaches (reach 1 and reach 2). Model equations for reach 1 and reach 2 were formulated independently and then merged together to form four order state space equation. Reach 1 is designated as i_1 and Reach 2 is designated as i_2.

4.1 Model equation for Reach 1

$$\left.\begin{aligned} \frac{dz_{i_1}}{dt} &= -k_{1i_1} z_{i_1} + \frac{Q_{i_{1-1}}}{v_{i_1}} z_{i_{1-1}} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} z_{i_1} + \frac{\vartheta_{i_1} Q_{E_1}}{v_{i_1}} \\ \frac{dq_{i_1}}{dt} &= k_{2i_1} q_{i_1}{}^{s} - k_{2i_1} q_{i_1} + \frac{Q_{i_{1-1}}}{v_{i_1}} q_{i_{1-1}} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} q_{i_1} - k_{1i_1} z_{i_1} - \frac{\mu_{i_1}}{v_{i_1}} \end{aligned}\right\} \quad (8)$$

Where,

z_{i_1}, $z_{i_{1-1}}$ are the concentrations of B.O.D. in reaches i_1 and i_{1-1} in mg/litre

q_{i_1}, $q_{i_{1-1}}$ are the concentrations of D.O. in reaches i_1 and i_{1-1} in mg/litre

v_{i_1} is the volume of water in reach i_1 in million gallons

Q_{E_1} is the flow rate of the effluent to reach i_1 in million gallons/day

k_{1i_1} is the oxygen consumption coefficient(day)$^{-1}$ in reach i_1

k_{2i_1} is the oxygen recovery coefficient (day)$^{-1}$ in reach i_1

Q_{i_1}, $Q_{i_{1-1}}$ are the stream flow rates in reaches i_1 and i_{1-1} in million gallons/day

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$q_{i_1}{}^{s}$ is the D.O. saturation level for the $i_1{}^{th}$ reach (mg/litre)

μ_{i_1} is the removal of D.O. due to bottom sludge requirements (mg/litre(day)$^{-1}$)

ϑ_{i_1} is the concentration of B.O.D. in the effluent in mg/litre

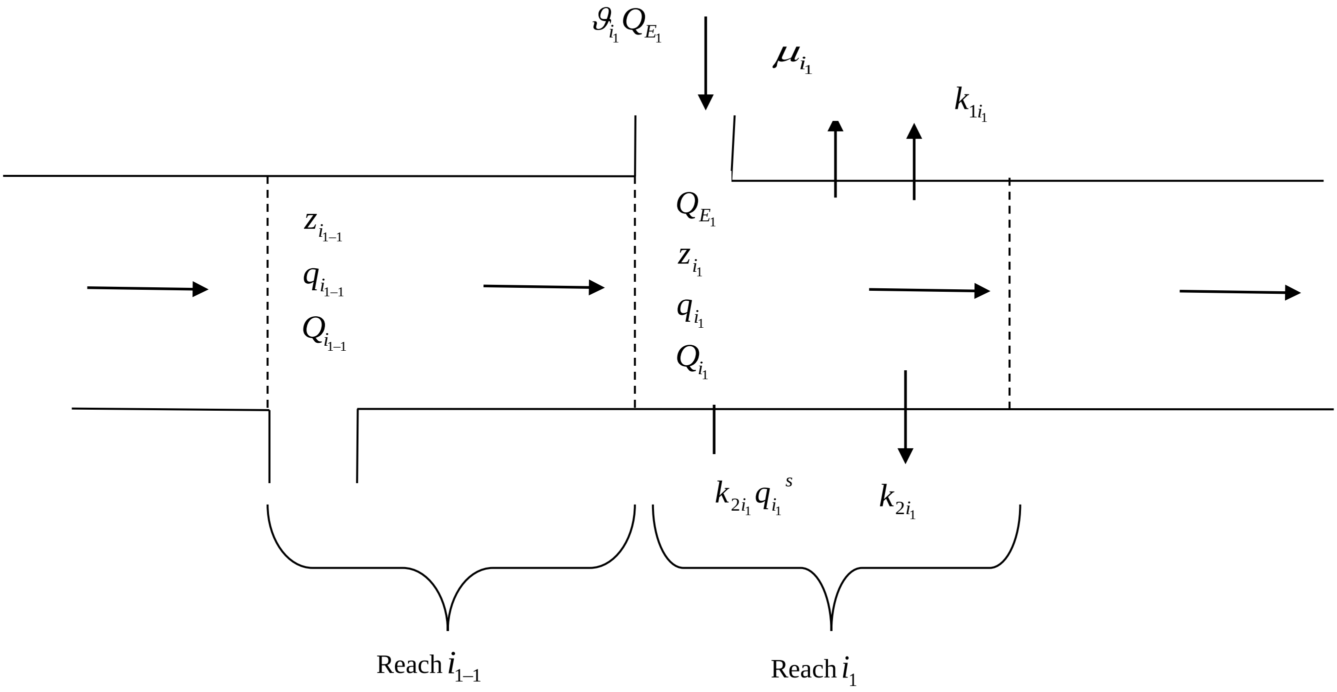

Fig.6:Flow diagram for reach 1

4.2 Model equation for Reach 2

$$\left.\begin{aligned}\frac{dz_{i_2}}{dt} &= -k_{1i_2} z_{i_2} + \frac{Q_{i_{2-1}}}{v_{i_2}} z_{i_{2-1}} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} z_{i_2} + \frac{\vartheta_{i_2} Q_{E_2}}{v_{i_2}} \\ \frac{dq_{i_2}}{dt} &= k_{2i_2} q_{i_2}{}^{s} - k_{2i_2} q_{i_2} + \frac{Q_{i_{2-1}}}{v_{i_2}} q_{i_{2-1}} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} q_{i_2} - k_{1i_2} z_{i_2} - \frac{\mu_{i_2}}{v_{i_2}}\end{aligned}\right\} \quad (9)$$

Where,

z_{i_2}, $z_{i_{2-1}}$ are the concentrations of B.O.D. in reaches i_2 and i_{2-1} in mg/litre

q_{i_2}, $q_{i_{2-1}}$ are the concentrations of D.O. in reaches i_2 and i_{2-1} in mg/litre

v_{i_2} is the volume of water in reach i_2 in million gallons

Q_{E_2} is the flow rate of the effluent to reach i_2 in million gallons/day

k_{1i_2} is the oxygen consumption coefficient(day)$^{-1}$ in reach i_2

k_{2i_2} is the oxygen recovery coefficient(day)$^{-1}$ in reach i_2

Q_{i_2}, $Q_{i_{2-1}}$ are the stream flow rates in reaches i_2 and i_{2-1} in million gallons/day

$q_{i_2}{}^s$ is the D.O. saturation level for the $i_2{}^{th}$ reach (mg/litre)

μ_{i_2} is the removal of D.O. due to bottom sludge requirements (mg/litre(day)$^{-1}$)

ϑ_{i_2} is the concentration of B.O.D. in the effluent in mg/litre

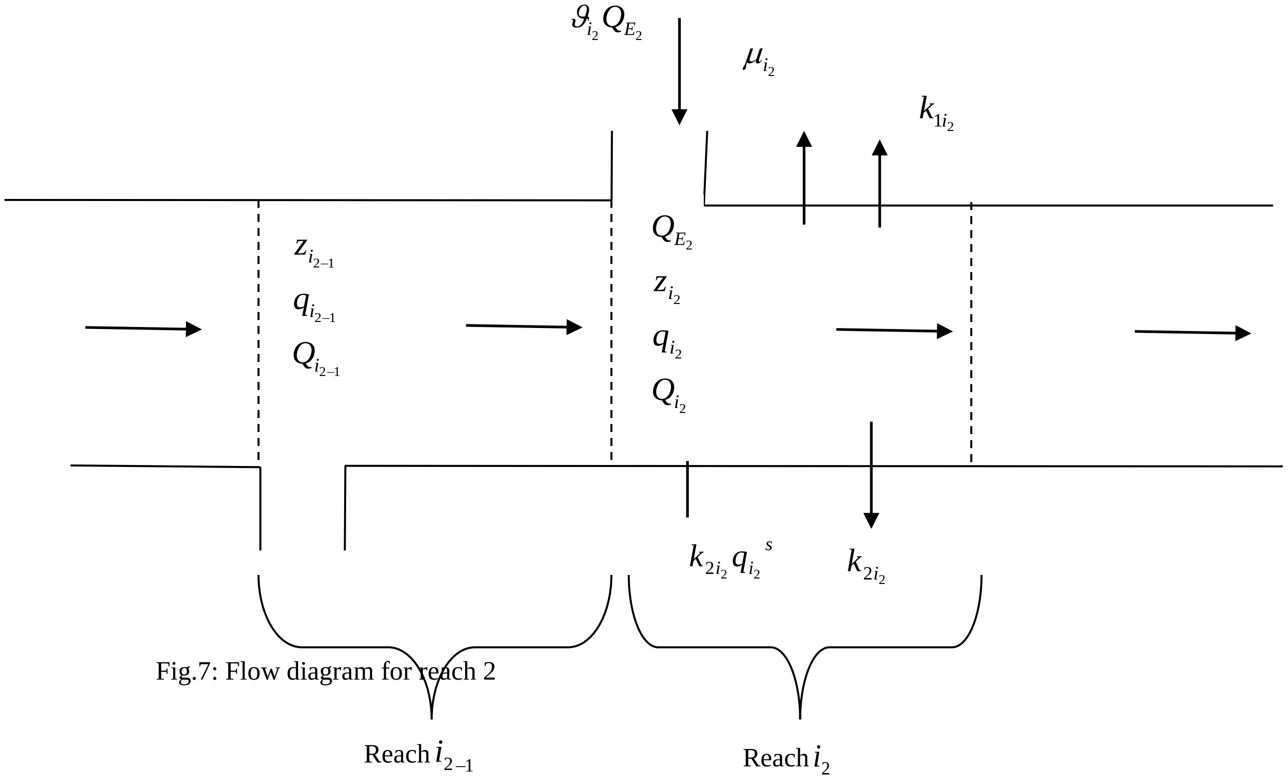

Fig.7: Flow diagram for reach 2

Putting eqns. (8) and (9) together into a matrix gives:

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$$\begin{bmatrix} \frac{dz_{i_1}}{dt} \\ \frac{dq_{i_1}}{dt} \\ \frac{dz_{i_2}}{dt} \\ \frac{dq_{i_2}}{dt} \end{bmatrix} = \begin{bmatrix} -k_{1i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} & 0 & 0 & 0 \\ -k_{1i_1} & -k_{2i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} & 0 & 0 \\ 0 & 0 & -k_{1i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} & 0 \\ 0 & 0 & -k_{1i_2} & -k_{2i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} \end{bmatrix} \begin{bmatrix} z_{i_1} \\ q_{i_1} \\ z_{i_2} \\ q_{i_2} \end{bmatrix} +$$

$$\begin{bmatrix} \frac{Q_{E_1}}{v_{i_1}} & 0 \\ 0 & 0 \\ 0 & \frac{Q_{E_2}}{v_{i_2}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \vartheta_{i_1} \\ \vartheta_{i_2} \end{bmatrix} + \left[\begin{bmatrix} \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 & 0 \\ 0 & \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 \\ 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} & 0 \\ 0 & 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} \end{bmatrix} \begin{bmatrix} z_{i_{1-1}} \\ q_{i_{1-1}} \\ z_{i_{2-1}} \\ q_{i_{2-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ k_{2i_1} q_{i_1}{}^{s} - \frac{\mu_{i_1}}{v_{i_1}} \\ 0 \\ k_{2i_2} q_{i_2}{}^{s} - \frac{\mu_{i_2}}{v_{i_2}} \end{bmatrix} \right] \quad (10)$$

Equation (10) is represented by first order deference equation of the form:

$$X_{k+1} = \Phi X_k + BU_k + C \quad (11)$$

Equation (11) is in conformity with the framework of the Kalman filter.

Where:

$$\Phi = \begin{bmatrix} -k_{1i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_1}} & 0 & 0 & 0 \\ -k_{1i_1} & -k_{2i_1} - \frac{(Q_{i_1} + Q_{E_1})}{v_{i_2}} & 0 & 0 \\ 0 & 0 & -k_{1i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} & 0 \\ 0 & 0 & -k_{1i_2} & -k_{2i_2} - \frac{(Q_{i_2} + Q_{E_2})}{v_{i_2}} \end{bmatrix} \quad (12)$$

$$X_k = \begin{bmatrix} z_{i_1} \\ q_{i_1} \\ z_{i_2} \\ q_{i_2} \end{bmatrix} \qquad (13)$$

$$B = \begin{bmatrix} \frac{Q_{E_1}}{v_{i_1}} & 0 \\ 0 & 0 \\ 0 & \frac{Q_{E_2}}{v_{i_2}} \\ 0 & 0 \end{bmatrix} \qquad (14)$$

$$U_k = \begin{bmatrix} \vartheta_{i_1} \\ \vartheta_{i_2} \end{bmatrix} \qquad (15)$$

$$C = \left[\begin{bmatrix} \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 & 0 \\ 0 & \frac{Q_{i_{1-1}}}{v_{i_1}} & 0 & 0 \\ 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} & 0 \\ 0 & 0 & 0 & \frac{Q_{i_{2-1}}}{v_{i_2}} \end{bmatrix} \begin{bmatrix} z_{i_{1-1}} \\ q_{i_{1-1}} \\ z_{i_{2-1}} \\ q_{i_{2-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ k_{2i_1} q_{i_1}{}^{s} - \frac{\mu_{i_1}}{v_{i_1}} \\ 0 \\ k_{2i_2} q_{i_2}{}^{s} - \frac{\mu_{i_2}}{v_{i_2}} \end{bmatrix} \right] \qquad (16)$$

Φ, B, X_k, U_k, is the transition model, the control input model, the process state vector and the control vector respectively. C is a constant matrix.

5.0 Estimation problem

Given

$$\left. \begin{aligned} X_{k+1} &= \Phi X_k + BU_k + C + \xi_k \\ Y_k &= HX_k + \eta_k \end{aligned} \right\} \qquad (17)$$

From the observed values Y_0,.............,Y_k, find an estimate $\hat{X}_{k|k}$ of X_k which minimizes the expected loss.

Where:

Φ = (4× 4) constant matrix obtained from the transition model

B = (4× 2) control input matrix which is applied to the control vector $\underline{U}_k$.

C = (4 x 4) constant matrix

$\underline{Y}_k$ = (2× 1) output vector (vector measurement at time t_k), $\underline{Y}_k = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, y_1 is the measured D.O in reach 1 and y_2 is the measured D.O in reach 2 of the stream.

H = (2× 4) constant matrix giving the ideal connection between the measurement and the State vector at time t_k

$\underline{X}_k$= (4× 1) process state vector at time t_k , i.e.,$\underline{X}_k$ =$\underline{X}$ (t_k) = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, x_1 and x_3 are the concentrations of B.O.D.(mg/l) in reaches 1 and 2 of the stream respectively, while x_2 , x_4 are the concentrations of D.O in reaches 1 and 2 of the stream respectively.

$\underline{U}_k$ = (2× 1) control vector.

$\underline{\eta}_k$ = (2× 1) measurement error –assumed to be white noise sequence with known co-Variance R and having zero cross correlation with $\underline{\xi}_k$-sequence.

$\underline{\xi}_k$ = (4× 1) vector-assumed to be white noise sequence with known co-variance Q

Note that x_2 and x_4 are the estimates of the true value of D.O in Reach 1 and 2 of the stream, y_1 and y_2 are the measured value of D.O in reach 1 and 2 of the stream.

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Table1: **Parameter values for Warri river model at 29ºC(Ogwola,2017)**

Symbol	Value	Source
L_1	500 metres	Ogwola,2017
L_2	500 metres	,,
W_1	150 metres	,,
W_2	150 metres	,,
D_1	7.5 metres	,,
D_2	7.5 metres	,,
Q_{i_1}	$0.643m^3/s$(14672937.5 gallons/day)	,,
Q_{i_2}	$0.643m^3/s$(14672937.5 gallons/day)	,,
v_{i_1}	$562500m^3$(148597242.04 gallons)	,,
v_{i_2}	$562500m^3$(148597242.04 gallons)	,,
	Assumptions[with respect to Guo et al(2003)]	
Q_{E_1}	$0.659m^3/s$(15046510.9 gallons/day)	Assumption
Q_{E_2}	$0.659m^3/s$(15046510.9 gallons/day)	,,
k_{1i_1}	1	,,
k_{1i_2}	2	,,
k_{2i_1}	2	,,
k_{2i_2}	3	,,

The value of Φ is obtained from (12) with respect to table 1

$$\Phi = \begin{bmatrix} -1.2 & 0 & 0 & 0 \\ -1.0 & -2.2 & 0 & 0 \\ 0 & 0 & -2.2 & 0 \\ 0 & 0 & -2.0 & -3.0 \end{bmatrix}$$

The process variance Q and the measurement variance R were tuned to:

$$Q = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \text{and } R = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

With respect to Kalman filter Tank filling the initial values $X_{0|0}$ and $P_{0|0}$ are given by:

$$X_{0|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} , \qquad P_{0|0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H which is the constant matrix that gives ideal connection between the measurement and the state vector and it is given by:

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6.0 D.O Measurements.

The measured DO values are given by

$$\underline{Y_k} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad (18)$$

where y_1 and y_2 are the measured values of DO in reach 1 and reach 2 at time t_k respectively.

Table 2: **D.O Measurements, $\underline{Y}_k$ at time t_K** (Aghoghovwia, 2011).

K	**t_k**	$\underline{Y}_k = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$
0	t_0	$\underline{Y}_0 = \begin{bmatrix} 5.0 \\ 5.7 \end{bmatrix}$
1	t_1	$\underline{Y}_1 = \begin{bmatrix} 6.0 \\ 5.3 \end{bmatrix}$
2	t_2	$\underline{Y}_2 = \begin{bmatrix} 5.5 \\ 6.3 \end{bmatrix}$

3	t_3	$\underline{Y}_3=\begin{bmatrix}5.4\\6.6\end{bmatrix}$
4	t_4	$\underline{Y}_4=\begin{bmatrix}6.7\\7.3\end{bmatrix}$
5	t_5	$\underline{Y}_5=\begin{bmatrix}7.0\\8.0\end{bmatrix}$
6	t_6	$\underline{Y}_6=\begin{bmatrix}6.5\\6.1\end{bmatrix}$
7	t_7	$\underline{Y}_7=\begin{bmatrix}6.2\\7.5\end{bmatrix}$

The computer program in Ogwola (2017) was used to simulate the system. The result is given on table 3.

Table 3:**The Simulation Results for the System**

K	t_k	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$
0	t_0	0	1.6665	0	1.8998
1	t_1	1.4103	3.653	3.3751	3.7581
2	t_2	2.7214	3.6092	6.4619	5.3346
3	t_3	2.9887	3.8324	12.3551	5.8718
4	t_4	4.3429	5.1018	13.5198	6.6294
5	t_5	5.0655	5.0841	16.992	6.9818
6	t_6	4.3844	4.6026	10.5232	5.0013
7	t_7	4.313	4.5266	11.6564	6.7248

Where

$\hat{x}_1$ is the optimal estimate of BOD in Reach 1 of the river in mg/l

$\hat{x}_2$ is the optimal estimate of DO in Reach 1 of the river in mg/litre

$\hat{x}_3$ is the optimal estimate of BOD in Reach 2 of the river in mg/litre

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$\hat{x}_4$ is the optimal estimate of DO in Reach 2 of the river in mg/litre.

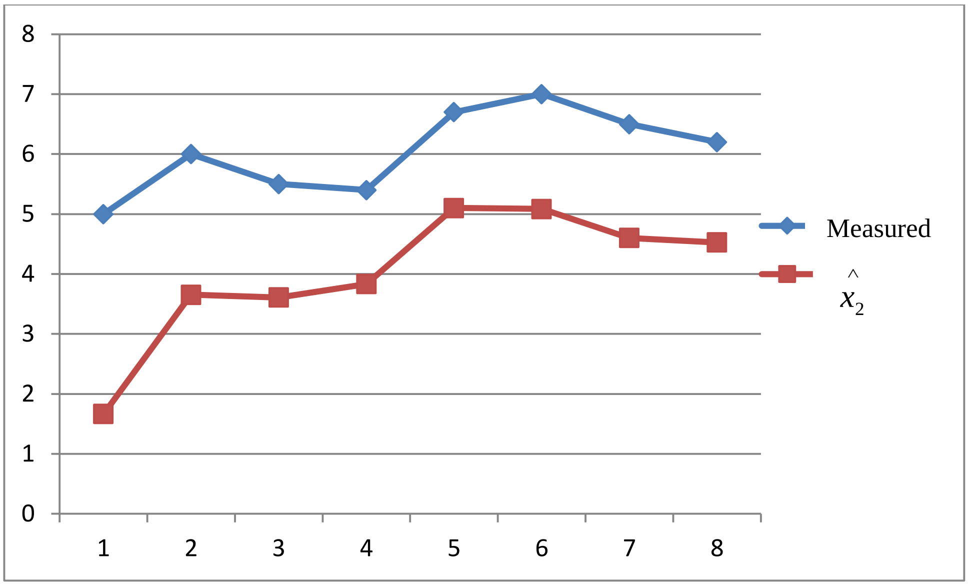

Fig. 8: Graph of the measured and of the estimated values of DO in Reach 1

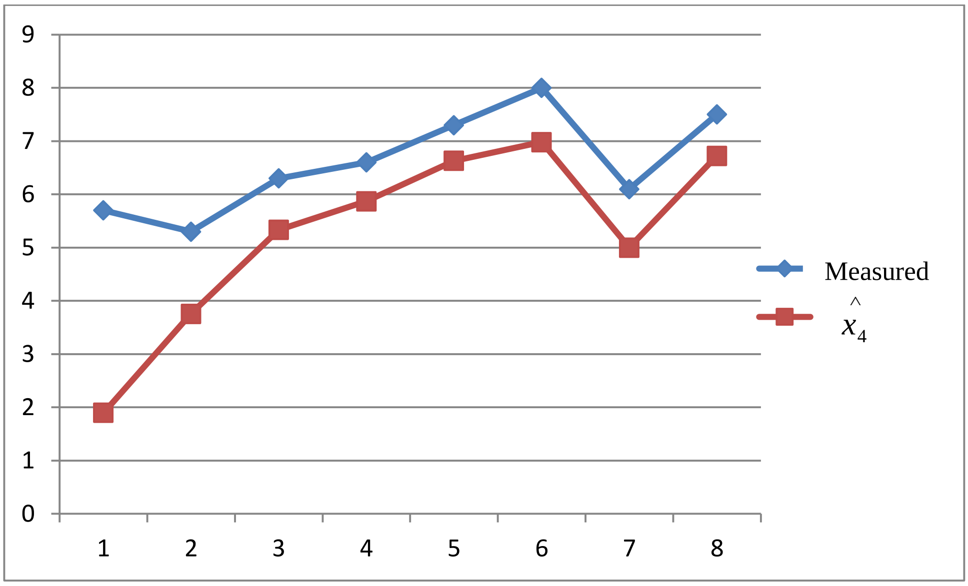

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Fig. 9: Graph of the measured and of the estimated values of DO in Reach 2

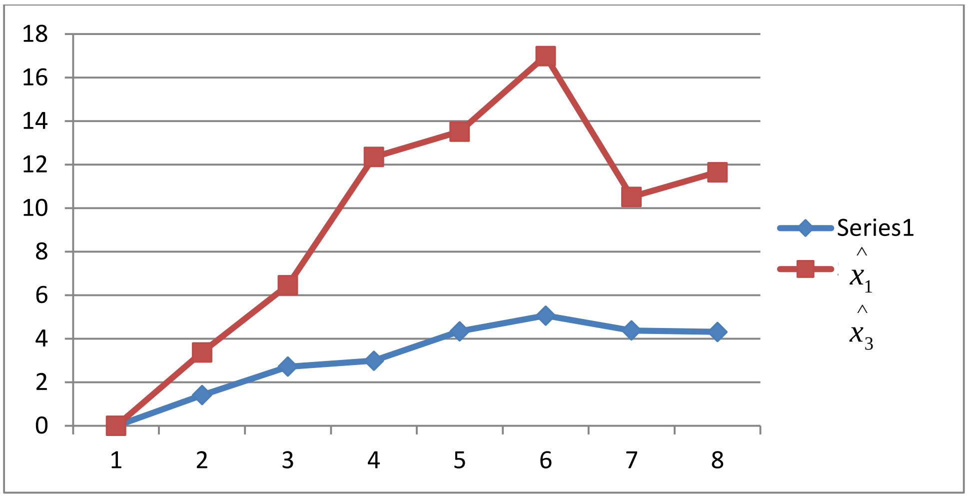

Fig. 10: Graph of the estimated values of BOD in Reach 1and Reach 2

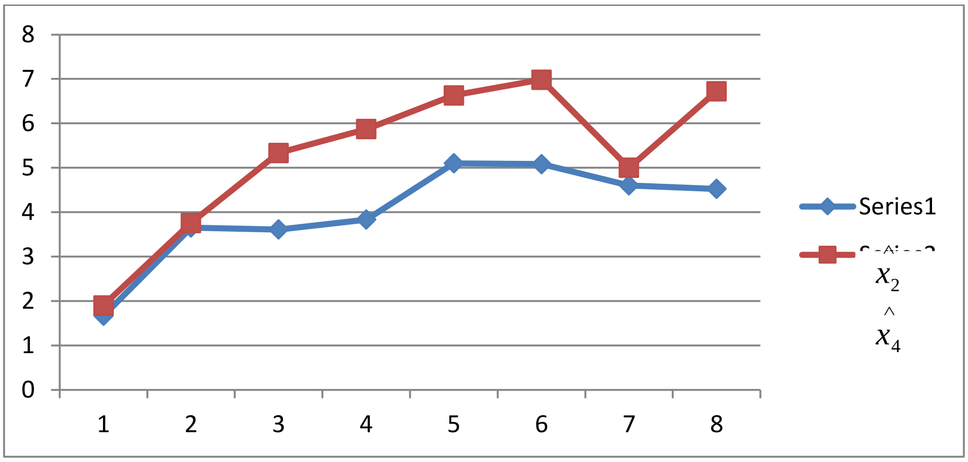

Fig. 11: Graph of the estimated values of DO in Reach 1 and Reach 2

7.0 Discussion

In figure 8 and 9, it is observed that the trajectories of the measured and estimated values of DO were of similar shape and not too far from each other, this shows that the estimation was satisfactory. The estimated values become closer to the measured values as the time increases.

In figure 10, it is observed that most of the values of BOD in Reach 1 and Reach 2 were above 2mg/l as the time increases. According to Wikipedia, 'polluted Rivers have a BOD greater than 1mg/l'.

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Since the concentrations of BOD were above 2mg/l it shows that the River is polluted which is in agreement with Aghoghovwia (2011).

In figure 10, it is also observed that the estimates of the concentration of BOD in Reach 2 were above that of Reach 1as time increases. This may be as the result of the effect of pollution in Reach 1 which flows down to Reach 2 as time increases. The estimated value of BOD was obtained from the measured value of DO as there were no measurements of BOD taken.

In figure 11, the difference in the concentration of DO in Reach 1 and Reach 2 was not much even though the concentration of BOD in Reach 2 rose up with time it does not reduce the concentration of DO in Reach 2. This is because DO is an indication of water self-purification as given in Wikipedia. According to Aghoghovwia (2011), 'the concentration of BOD differed significantly between seasons'.

8.0 Conclusion

The developed system was used to estimate DO and BOD values in the Warri River. The result indicated that the randomness in many system parameters had been effectively handled by Kalman filtering techniques. The accuracy of the state estimation was satisfactory.

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