

# Performance Analysis of Bit Error Rate (BER) in OFDM System by Integrating with the Discrete Fractional Fourier Transform (DFrFT)

Gajam Navya (PG Scholar)

Bolishetti Satish Chandra (Ph.D Scholar, Associate Professor)

Department of ECE, Malla reddy Institute of Engineering and Technology, Hyderabad, Telangana.

## Abstract:

*Orthogonal system division multiplexing is one of the most commonly used LTE in today's world. It provides high data rate, high speed and low complexity compared to conventional methods such as CDMA, TDMA and FDMA. Thus OFDM also offers some of the disadvantages such as bit error rate, PAPR, ICI, ISI etc.,. In this paper, the performance of BER is evaluated by integrating the Orthogonal system division multiplexing (OFDM) with Discrete Fractional Fourier Transform (DFrFT). The performance of the signal is evaluated by considering the total number of subcarriers as 64 where 48 of them are data carriers with a predefined bandwidth of 20MHz. The generated random signal is transmitted through Additive White Gaussian Noise (AWGN) channel to the receiver section. The performance of the Bit Error Rate (BER) is evaluated by the ratio of a function of energy per bit to noise power spectral density  $E_b/N_0$ . Simulation results shows the comparison of BER performance for booth the conventional method as well as the proposed scheme.*

**Keywords:** OFDM, Discrete Fractional Fourier Transform, Bit Error Rate, Time-System Bandwidth Product

## 1. Introduction

Fractional Fourier Transform is a generalization of Fourier Transform, and was initially introduced Namias[13]. But it's introduction to Signal Processing delayed until early 90s simultaneously with the advent of powerful

digital computing. Different versions and methods for DFrFT can be found in the literature [14],[17],[16]. In this paper we will use DFrFT method developed by Santhanam et al. Since then DFrFT has been widely used in a variety of signal processing applications that includes image compression[7], encryption [8], remote sensing and detection[20] and in communications [10], [12] and etc. The works that has been published on the subject of application of DFrFT in communication applications mostly fail to address clearly the reasons of why different results and achievements have been observed which leaves the readers uncertain about the conditions that DFrFT can be applied and how it needs to be implemented in order to meet the design goals. OFDM is very well know in communications and has been widely used in numerous digital and wireless applications and standards[3]. Increasing demand for higher data rates and on the other hand constraints on power and frequency bandwidth have been a dilemma for researchers, and this problem has been discussed from different points of view. Efficient use of bandwidth frequency can increase in achieving higher data rates. We will show that replacing DFT with DFrFT in OFDM systems results in better BER. We call the new OFDM system Fr-OFDM. This paper discusses that the improvement is because of the better Time-Bandwidth Product achieved by DFrFT Moazzeni et al [11] have evaluated the TBP for OFDM in case of gauss-hermite functions and on the other hand Bhata et al [2] shows that DFrFT that have been introduced by [18] has better time-

bandwidth product. The remainder of the paper is organized as follows: in section II there is a brief discussion and introduction of DFrFT, section III presents the general overview of an OFDM system and section IV and V present simulation results, discussion and conclusion accordingly and the last section includes the references.

## 2. Related Work

### 2.1 Quadrature Amplitude Modulation

QAM, Quadrature amplitude modulation is widely used in many digital data radio communications and data communications applications. A variety of forms of QAM are available and some of the more common forms include 16 QAM, 32 QAM, 64 QAM, 128 QAM, and 256 QAM. Here the figures refer to the number of points on the constellation, i.e. the number of distinct states that can exist.

QAM is a method for sending two separate (and uniquely different) channels of information. The carrier is shifted to create two carriers namely the sine and cosine versions. The outputs of both modulators are algebraically summed, the results of which is a single signal to be transmitted, containing the In-phase (I) and Quadrature-phase (Q) information. The set of possible combinations of amplitudes (A) and phases ( $\theta$ ), as shown on an x-y plot, is a pattern of dots known as a QAM constellation as shown in Figure 1.

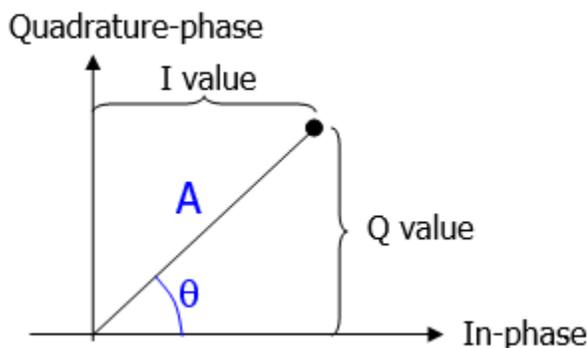


Figure 1: diagram of I-Q Constellation

Consider the 16 QAM modulation schemes, in which 4 bits are processed to produce a single vector. The resultant constellation consists of four different amplitudes distributed in 12 different phases as shown in Figure 2.

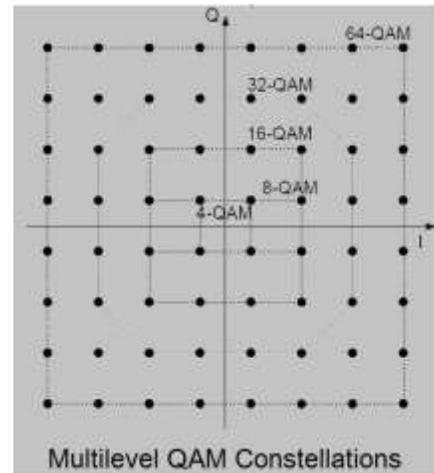


Figure 2: 4,8,16,32,64 QAM Constellation

### Bit Error Rate

In digital transmission, the number of bit errors is the number of received bits of a data stream over a communication channel that has been altered due to noise, interference, distortion or bit synchronization errors. The bit error rate or bit error ratio (BER) is the number of bit errors divided by the total number of transferred bits during a studied time interval. BER is a unitless performance measure, often expressed as a percentage. The bit error probability  $p_e$  is the expectation value of the BER. The BER can be considered as an approximate estimate of the bit error probability. This estimate is accurate for a long time interval and a high number of bit errors.

## 3. Literature Review

### 3.1 FFT Implementation

Initially, orthogonal system generations are done through a bank of analog Nyquist filters or Transmultiplexer [Akansu et al. (1998)], be that as it may, these days different changes are accessible. Practically, Discrete Fourier Transform (DFT) and Inverse DFT (IDFT) forms are being utilized to execute these orthogonal signs. Note that DFT and IDFT can be executed productively by utilizing Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT), separately. These Transforms are basic because of its capacity to uncover the concealed data from the system.

### **3.2 Existing Convolution & Product Theorem for FRFT**

Studies reported in above sections reveals that almost all the properties of FRFT are already derived except convolution theorem, product theorem and correlation theorem. The handiness of convolution theorem can be best clarified by its application in filtering. Since, filtering can be performed in both ways i.e., time domain filtering and system domain filtering. All the while, if the computational complexity is a premise parameter then it can be demonstrated that under various information conditions one kind of separating has advantage over other and the other way around. For non-stationary signals and noise, filtering of the signal from the clamor can be performed by outlining a channel in FRFT domain with this optimum angle parameter value.

FRFT domain convolution and product theorems were extensively investigated previously.

### **3.3 Existing Correlation Theorems and Related Theories**

The fractional domain correlation theorem became analyzed and said these days in many articles. Firstly, Akay et al had defined the fractional correlation crucial by means of the

usage of fractional shift operator, however an special identification for comparing the correlation become no longer provided. Similarly, if the FRFT of correlation quintessential is decided then it outcomes in a totally complicated expression. Subsequently, Tao et al had tried to formulate the correlation theorem especially for FRFT, but of their definition the correlation vital includes handiest one chirp. It shows that after FRFT of this correlation fundamental may be accomplished, the converted amount will not fulfill the variable dependability constraint. Cottone used fractional spectral moments to represents the fractional correlation and energy spectral density however the correlation theorem for FRFT changed into no longer documented concisely. Torres had documented any other correlation theorem for the FRFT by means of using fractional translational operator.

By definition, the FRFT is a subclass of integral transformations characterised with the aid of quadratic complicated exponential kernels. These complex exponential kernels often introduce very rapid oscillations. Hence, it is not possible to evaluate these differences through direct numerical integration considering those rapid oscillations require excessively large sampling quotes. A possible technique is to decompose these integral trans- formations into sub-operations. This results in more time of computation, large numerical inaccuracies, and the need for more memory. Also with the growing trend of studying the sign in discrete shape to make use of the benefits of virtual structures over analog systems, the need of discretization of FRFT, i.E. Definition or algorithms to obtain discrete FRFT (DFRFT), arises swiftly. Next segment is dedicated to the analysis of present DFRFT algorithms.

## 2.5 DISCRETE FRACTIONAL FOURIER TRANSFORM (DFRFT)

Many researchers have tried to provide you with a few method of comparing DFRFT. On this adventure, many methods and algorithms had come into lifestyles. But none were given acclamation because of deficiency of both not fulfilling some of the top houses that it's continuous kind posses or nonexistence of a closed form expression. In this section, all of the strategies available within the literature are covered and divided into six lessons on the premise in their technique of evaluation.

### 3.4 Sampling Type DFRFT

The sampling theorem for the FRFT of band-restricted and time-restricted alerts is follows from Shannon's sampling theorem. With this approach, the only manner to derive the DFRFT is sampling the continuous FRFT and computing it immediately from the samples, but by sampling the continuous FRFT at once the resultant discrete transform obtained will lose many important properties. The maximum critical problem with this type DFRFT will be non-compliance of unitary and reversibility houses. In addition, the DFRFT obtained by direct sampling of the FRFT lacks closed form expressions and is non-additive, which affects its software area. In order to maintain some of the FRFT residences, a sort of DFRFT become derived as a special case of the continuous FRFT. Specifically, the enter feature was assumed as a periodic, equally spaced impulse train. Since this form of DFRFT is a special case of continuous FRFT, many properties of the FRFT exist and have the short set of rules. However, this kind of DFRFT can not be described for all values of  $\alpha$  because of various imposed constraints.

## 4. Proposed Method

## 4.1 Discrete Fractional Fourier Transform

In the conventional Fourier transform, time and system pivot axis are orthogonal to each other, and system domain analysis of stationary signs is exceptionally helpful with Fourier transform. In instance of signs that are not stationary and their system content differs with time then conventional Fourier change can't fulfill the system investigation requests, which implies that this kind of signs can't be completely spoken to bu simply taking a gander at it's Fourier Transform, for example, Chirp Signals. In any case, On the other hand, DFrFT is another straight change that can be instructed of as n-th energy of the DFT. This time if the system of the system is changing directly after some time then DFrFT of the system with fitting edge (or n-th energy of DFT) will end up comparable of a DFT of an unadulterated sinusoid which is based on the trill rate of the system. There are a few portrayals of DFrFT in writing [2], yet not every one of them are appropriate for this examination, since in a FrOFDM system, for example, ordinary OFDM system we require a hermitian and unitary transform. At the end of the day we require a change that is invertible and preserve energy. Among those present in writing the one created by Santhanam et al. [19] fits in this application. DFrFT change of system  $x[n]$  is given by:

$$X_k[r] = \sum_{p=0}^{N-1} z_k[p] e^{-j\frac{2\pi}{N}pr} \quad (1)$$

In equation 1  $z_k[p]$  is an intermediate variable which is given by:

$$z_k[p] = v_{kp} \sum_{n=0}^{N-1} v_{kp} \sum_{n=0}^{N-1} x[n] v_{np} e^{-j\frac{2\pi}{N}pr} \quad (2)$$

where  $v_p$  is the  $p$ th eigenvector of the commuting matrix as given in [4]. The angle  $\alpha_r$  of the DFrFT is related to index  $r$  as  $\alpha_r = 2\pi r N$ . It is to show

that for any given angle  $\alpha_r$  the inverse DFrFT transform can be evaluated by taking DFrFT at  $-\alpha_r$ .

#### 4.2 ORTHOGONAL SYSTEM DIVISION MULTIPLEX OFDM

Fig. 1 demonstrates the fundamental outline of an OFDM system which is basic in all varieties of OFDM systems[15]. The N-point IFFT takes reverse Fourier change of the yield waveforms of the Modulation unit where is 16-QAM in show paper. The primary part of Fourier Transform in OFDM systems is to expel Inter Carrier Interference and Inter Symbol Interference when cyclic expansion is included appropriately[9]. Bunches of research have been done on various pieces of an OFDM systems. For instance COFDM (Coded OFDM) [1] or Wavelet OFDM[6] are diverse varieties of OFDM system keeping in mind the end goal to build the execution and BER of the OFDM systems. Portrayal Fig. 3. Graph of a standard OFDM system including Transmitter and Receiver

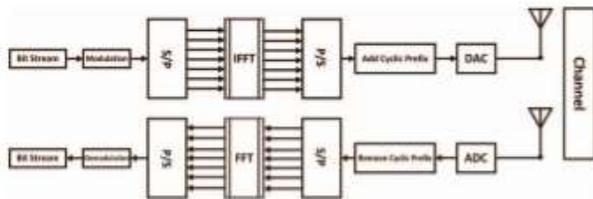


Figure. 3. Diagram of a standard OFDM system including Transmitter and Receiver

$$s(t) = \sum_{k=0}^{N-1} s_k e^{j2\pi f_k t} \quad (3a)$$

$$f_k = f_0 + k\Delta f \quad (3b)$$

Equation 3 is also called time domain OFDM Subcarrier.  $\Delta f$  is the system spacing we get for N-point DFT system and is the minimum system spacing between waveforms that keeps the orthogonality among them.  $f_0$  is the central

carrier system. Fig. 4 shows the OFDM Subcarriers spectra and it is easy

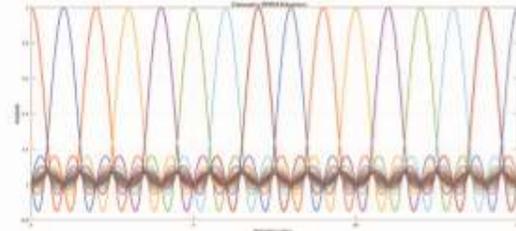


Fig. 4. OFDM Subcarriers Signal System Spectra

to see that inner product of any two sub carriers result in zero. Intuitively speaking if we could have narrower main lobes therefore more subcarriers could have been packed in the same system bandwidth.

#### 5. Simulation Results

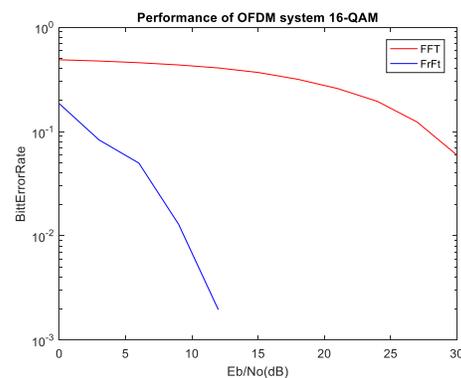


Figure 5. BER of 16-QAM OFDM System using DFrFT and DFT

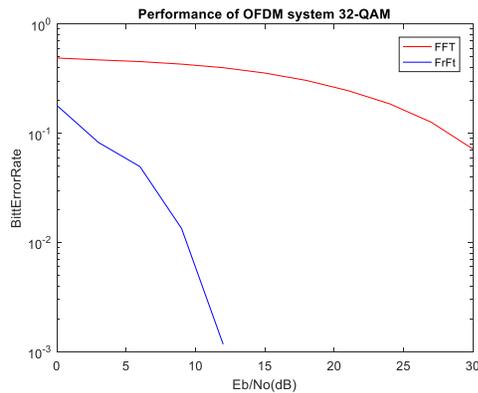


Figure 6. BER of 32-QAM OFDM System using DFrFT and DFT

## 6. Conclusion

This paper actualizes the Discrete Fractional Fourier Transform in an OFDM system DFrFT. The simulation results about favours the performance enhancement of proposed OFDM system over conventional OFDM systems. The outcomes confirm that revolution in time- system accomplished by DFrFT by and large upgrades the execution of the OFDM system. As the pivot of time-system hub is nearer to  $\alpha = 90^\circ$  the Bit Error Rate execution turns out to be more like a conventional OFDM system in light of the fact that at angle  $\alpha = 90^\circ$  DFrFT comes down to FFT. The kind of Gauss-Hermite capacities utilized as a part of DFrFT have littler time-data bandwidth capacity item and this is the hidden explanation behind accomplishing higher execution in Fr-OFDM systems. smaller time-bandwidth speed item is similar to higher mainlobe to sidelobe proportion that abatements the inter carrier interference.

The BER of the Fr-OFDM outflanks conventional OFDM systems as SNR increments, yet at the same time falls behind the hypothetical BER. The accomplished constriction in time-data bandwidth capacity item in DFrFT space can affect OFDM system in various ways. By

expanding mainlobe to sidelobe proportion it diminishes the Inter Carrier Interference and then again smaller mainlobes leaves more space in the distributed transmission capacity of OFDM along these lines more subcarriers can be stuffed together in the same assigned data bandwidth capacity in contrast with traditional OFDM systems. Future work incorporate the investigation of limit of Fr-OFDM systems, Optimizing the time-frequency revolution edge in DFrFT space to accomplish the most extreme BER and Capacity execution and furthermore examination of the system for various sorts of channels and different tweaks other than Quadrature Amplitude Modulation.

## Future Scope:

Future work include the analysis of capacity of Fr-OFDM systems, Optimizing the time-frequency rotation angle in DFrFT domain to achieve the maximum BER and Capacity performance and also analysis of the system for different types of channels and other modulations other than Quadrature Amplitude Modulation.

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**Bolishetti Satish Chandra**  
Ph.D research scholar from India and working as Associate professor, in ECE Department from Malla reddy institute of Engineering and technology, Hyd. He studied B.Tech in ECE from

Nagarjuna University, Guntur, AP and M.Tech in Systems and signal processing from JNTU college of engineering, JNTUH, Hyd, Telangana. He is having 17 + years of work experience in Academics, teaching and research. He participated and presented research papers in both

national and international conference, seminars and workshops also published 7 research papers in national and international peer review journals.



**Gajam Navya** pursuing M.Tech in malla reddy institute of engineering and technology, Hyderabad , Telangana specialization in wireless and mobile communication and B.Tech in Nalla Narasimha Reddy Education society's group of

institutions, Hyderabad, Telangana specialization of electronics and communication engineering and passed in 2015. Her Area of interest includes Digital Communication, signal processing, satellite communication.