

A Note on Modified Vogel's Approximation Method for Solving Transportation Problems

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Abstract

This paper is a criticism of the paper on modified Vogel's approximation method given by Soomro et al in which they said that modified Vogel's approximation method is same as Vogel's approximation method and in some problems it is better than Vogel's approximation method for solving ordinary transportation problems. Numerical examples are given in support of the claim.

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1. Introduction

Transportation problem was first formulated in 1941 by Hitchcock [2], is a special case of linear programming problem in which our objective is to satisfy the demand at destinations from the supply at the minimum transportation cost. It was further developed in 1949 by Koopman [3] and in 1951 by Dantzig [1].

A certain class of linear programming problem known as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contexts of determining optimum shipping pattern. For example: A product may be transported from factories to retail stores. The factories are the sources and the stores are the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origin and it is required that the given quantities of the product be shipped to each of n destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. Many authors give their methods to find initial feasible solution of a transportation problem e.g. north west corner rule, least cost entry method and Vogel's approximation method etc. In 2015, Soomro et al [5], gives modified Vogel's approximation method to find initial solution of a transportation problem. In this method they have used penalty of each row of maximum numbers (difference between two maximum costs) but kept same penalty for each column as in Vogel's method. According to them modified Vogel's method is almost same as Vogel's method and in some cases it is better than Vogel's method. In present paper we have given two examples taken from [4] which shows that Vogel's method is superior than modified Vogel's method.

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This paper is organized as follows: Section 2, contains Mathematical Formulation. Numerical examples are given in Section 3, to justify the claim. In section 4, Conclusion is given.

2. Mathematical Formulation

2.1. Transportation Problem

The problem can be formulated as:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n$$

$$x_{ij} \geq 0, \text{ for all } i, j$$

For each supply point $i, (i = 1, 2, \dots, m)$ and demand point $j, (j = 1, 2, \dots, n)$

c_{ij} =unit transportation cost from i^{th} source to j^{th} destination

x_{ij} =amount of homogeneous product transported from i^{th} source to j^{th} destination

a_i =amount of supply at i^{th} source.

b_j =amount of demand at j^{th} destination.

where a_i and b_j are given non-negative numbers and assumed that total supply is equal to total demand, i.e. $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$, then transportation problem is called balanced otherwise it is called unbalanced. The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost.

Because of the special structure of the transportation model, the problem can also be represented as Table 1.

Table 1: Tabular representation of model (α)

Destination \rightarrow source \downarrow	D_1	D_2	\dots	D_n	supply(a_i)
S_1	c_{11}	c_{12}	\dots	c_{1n}	a_1
S_2	c_{21}	c_{22}	\dots	c_{2n}	a_2
\vdots	\vdots	\vdots	\dots	\vdots	\vdots
S_m	c_{m1}	c_{m2}	\dots	c_{mn}	a_m
Demand (b_j)	b_1	b_2	\dots	b_n	

3. Numerical Examples

Numerical example: Input data and initial solution obtained by applying modified Vogel's method and by Vogel's method for different examples is given in tables 2 and 3.

Table 2: Input data and initial solution

Ex.	Input Data	Obtained cost by modified method	Obtained Cost by Vogel's method	optimal solution
1	$[c_{ij}]_{3 \times 3} = [50 \ 30 \ 220; \ 90 \ 45 \ 170; \ 250 \ 200 \ 50]$; $[a_i]_{3 \times 1} = [1, 3, 4]$; $[b_j]_{1 \times 3} = [4, 2, 2]$	850	820	820

Table 3: Input data and initial solution

Ex.	Input Data	Obtained cost by modified method	Obtained Cost by Vogel's method	optimal solution
2	$[c_{ij}]_{4 \times 5} = [3 \ 4 \ 6 \ 8 \ 8; \ 2 \ 10 \ 1 \ 5 \ 30; \ 7 \ 11 \ 20 \ 40 \ 15; \ 2 \ 1 \ 9 \ 14 \ 18]$; $[a_i]_{3 \times 1} = [20, 30, 15, 13]$; $[b_j]_{1 \times 3} = [40, 6, 8, 18]$	350	321	321

4. Conclusion

Thus from the above discussion we have concluded that there are some transportation problems where Modified Vogel's method may be inferior than Vogel's method.

References

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