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Simulation of State Variable Compensator for Linearized Mechanical System

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***Abstract**:*

The development of state variable approach has been made possible by mathematical modelling of a system which, alongside the output, provides information on the state of the system variables at certain fixed points along the flow of signals. The paper has presented simulation of state variable compensator for linearized mechanical system. The dynamic model of linearized mechanical system representing engine idle speed in the form of a transfer function was obtained. The transfer function was converted into state space equation and a feedback gain matrix is developed. Simulink model of the system was developed and was used for simulation in MATLAB. The step response results reveal the performance of the state variable for open loop and closed loop conditions.

Keywords: *Linearization, Engine Idle Speed, Step Response, State variable*

1. Introduction

In the analysis and design of feedback system for engineering application, several methods are considered such as the use of root locus and frequency response [1]. In these methods, the physical system can be represented using transfer function model. According to [1], the transfer function model offers the benefit of being a simple and powerful analysis and design techniques. However, disadvantages exist in using transfer function models. For example, they are only defined considering zero initial conditions. Also, transfer function models are only applicable to linear- time-invariant (LTI) systems and are restricted to single-input-single-out (SISO) system [1].

The dynamic characteristics of a physical system can be represented using the state variables equation. Nevertheless, the concept of system state is not only applicable to the analysis of physical systems but also particularly useful in analyzing biological, social and economic systems [2]. The development of state variable approach has been made possible by mathematical modelling of a system which, alongside the output, provides information on the state of the system variables at certain fixed points along the flow of signals. The state variable is directly a time domain technique that offers a basis for modern control theory and system optimization. It also serves as a very powerful scheme for the analysis and design of linear, and nonlinear, time-invariant or time varying multi-input-multi-output [MIMO] system [1]. The advantages of state variable analysis notwithstanding, the transfer function approaches serve to give the control engineer a deep physical knowledge of the system and largely help in providing a pilot design of system. In such case a complex system is approximated and represented by more simplified and manageable model.

This paper intends to study the state variable analysis of a linearized mechanical system. In order to realize the objective of the paper, the remaining parts of the paper will be devoted to system setup and state variable equations, construction of state space model in Simulink, simulation, discussion and conclusion.

2. System Setup and State Equation

There are various equations governing the dynamics of engine Idle Speed control model in engineering. An idle speed engine model is obtained from a set of complex nonlinear equations which consists of throttle mass flow, intake manifold, engine air mass flow, torque generation, and engine rotational dynamics as presented in [3]. Nevertheless, under certain assumptions, these equations can be linearized at nominal speed to obtain a linear system model. According to[3], a consideration of deviation in throttle position in degrees as input and deviation in engine speed in revolution per minute (rpm) as output for the model.

In the field of control systems engineering, several techniques have been used to model the engine idling speed. In Xiaocheng et al [4], fuzzy proportional integral and derivative (FPID) model-based study on idle control of gas engine was presented. Wong et al [5] presented a technique on modelling and optimization of engine idle speed system. A linearized engine idle speed model was exploited in Eze et al [6] using the approach in Cook and Powel [7].

The transfer function for a typical idle speed model and the nominal parameters are given by [3]:

$$G(s) = k\frac{s^2+n_1s+n_2}{s^3+d_1s^2+d_2s+d_3}e^{-0.15s} \quad (1)$$

The time delay and the transfer function parameters k, n_1, n_2, d_1, d_2, d_3, represent various parameters of the combustion engine model. Using the approach in [8] gives:

$$\frac{X(s)}{U(s)} \times \frac{Y(s)}{X(s)} = \frac{ks^2+n_1s+n_2}{s^3+d_1s^2+d_2s+d_3} \quad (2)$$

$$\frac{X(s)}{U(s)} = \frac{k}{s^3+d_1s^2+d_1s+d_3} \quad (3)$$

Assuming zero initial conditions:

$$s^3X(s)+21.2s^2X(s)+51.3sX(s)+ 189.5X(s) = kU(s) \quad (4)$$

Let

$$x = x_1 \quad (5)$$

$$\dot{x}_1 = x_2 \quad (6)$$

$$\dot{x}_2 = x_3 \quad (7)$$

$$\dot{x}_3 = -21.2x_3 - 51.3x_2 - 189.5x_1 + 29.8u \quad (8)$$

Transforming Eq. (4) to (8) into state space form, gives:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -189.5 & -51.3 & -21.2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 29.8\end{bmatrix}u \quad (9)$$

Now,

$$\frac{Y(s)}{X(s)} = s^2+n_1s+n_1 \quad (10)$$

$$Y(s) = s^2X(s)+n_1sX(s)+n_2X(s) \quad (11)$$

Solving Eq. (12) and transforming it into state space form gives:

$$y = \begin{bmatrix}833 & 50 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad (12)$$

Eq. (9) and (12) match the general, linear state space form [9]:

$$\dot{x} = Ax + Bu \quad (13)$$

$$y = Cx + Du \quad (14)$$

where $D = 0$ for simulation in this paper.

Given the state variable feedback matrix K, such that:

$$K = \begin{bmatrix}K_1 & K_2 & K_2\end{bmatrix} \quad (15)$$

and,

$$u = -Kx \quad (16)$$

To determine K, we use:

$$(A-BK) = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -189.5-K_1 & -51.3-K_2 & -21.2-K_3\end{bmatrix} \quad (17)$$

Solving $\det[\lambda I-(A-BK)] = 0$, gives:

$$\det[\lambda I-(A-BK)] = \lambda^3+(21.2+K_3)\lambda^2+ (51.3+K_2)\lambda+(189.5+K_1) \quad (18)$$

A desired characteristics equation is chosen using:

$$E_{ch} = (\lambda^2+2\xi\omega_n\lambda+\omega_n^2)(\lambda+\xi\omega_n) \quad (19)$$

where ξ, is the damping factor and ω_n, is the natural frequency.

Choosing $\xi = 0.8$ for minimal overshoot, and making the settling time, $t_s = 1s$, then:

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.8\omega_n} \approx 1 \quad (20)$$

If we choose $\omega_n = 6$ and then substitutes these values into Eq. (19) gives:

$$E_{ch} = \lambda^3+14.4\lambda^2+82.1\lambda+172.8 \quad (21)$$

Equating Eq. (18) and (21), gives:

$$K = \begin{bmatrix}K_1 & K_2 & K_3\end{bmatrix} = \begin{bmatrix}-16.7 & 30.8 & -6.8\end{bmatrix} \quad (22)$$

The block diagram representation of the state space model for Simulink simulation in open loop is shown in Fig. 1. In Fig. 2, the closed loop simulation with the inclusion of a feedback gain matrix.

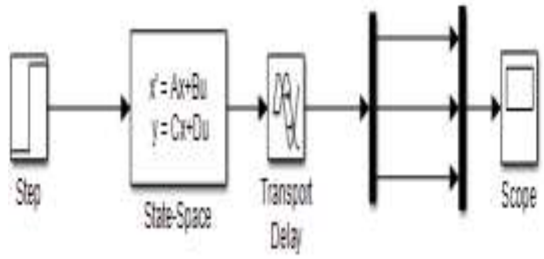

Fig. 1 State space simulation programme

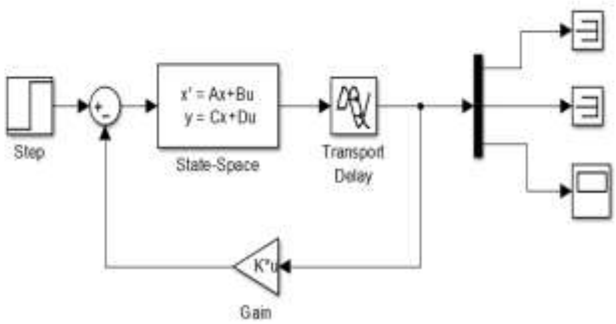

Fig. 2 Closed loop simulation programme

In Fig. 2, Terminator blocks from the Simulink/Sinks library are added to the two signals of the Demux block that is being plotted.

3. Simulation Results and Discussion

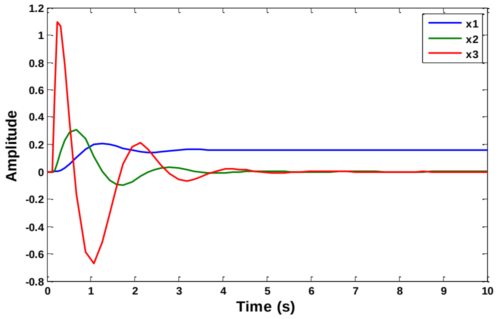

Fig. 3 Step response of system state variables

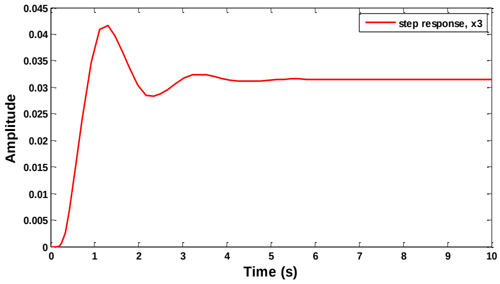

Fig. 4 Step response of closed loop for state variable x_3

The simulation plots in Fig. 3 represents the open loop step response of the state variables, x_1, x_2, and x_3. The focus is on the idle speed which is the state variable, x_3. A state feedback matrix K is included in the loop as a compensator and simulation is performed and a stable step response plot is obtained as shown in Fig.4. The step response plot of state variable x_3 is that of engine idle speed.

4. Conclusion

The paper has presented simulation of state variable compensator for linearized mechanical system. The linearized system represents the dynamic model of an engine idle speed in form of a transfer function. The transfer function model was transformed into state space model to study the dynamic response of the variables. The simulation plots reveal the response pattern and characteristics of the state variables. A stable response is obtained with the inclusion of the gain matrix for the idle speed.

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