

Implementation of Principal Component Analysis Technique in Peak to Average Power Ratio

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Abstract:

Wireless communication has become one of the most rapid technology evolvments in the world. Orthogonal Frequency Division Multiplexing (OFDM) has become the most efficient multi carrier modulation technique for achieving high data rate transmission in wireless communication. OFDM offers advantages such as immunity to selective fading, spectrum efficiency, and various more. Regardless of these beneficial advantages, the performance of OFDM is limited as Peak Power to Average Power Ratio (PAPR) is significantly high. This would reduce the performance of Power Amplifier (PA). Principle Component Analysis (PCA) technique is proposed to implement in the high PAPR as it identifies the directions of most variation in the data set and it reduces the data down into its basic component, dispossess any unnecessary parts. A novel framework using Principal Component Analysis in PAPR is studied. A simple framework using Covariance Matrix in Principal Component Analysis is developed in the PAPR. An evaluation of the overall performances is made using the proposed Principal Component Analysis scheme in PAPR in OFDM system. The project starts with initializing the parameter used in the OFDM system. The input data is generated randomly. Next, serial to parallel the subcarriers ready for the processing of IFFT. Then, the IFFT takes place. After that, PCA technique save the largest eigenvector and eigenvalue. Lastly, the optimal transmitted signal with the lowest PAPR is chosen. It has been observed from the obtained results that PCA technique increased the value of PAPR even when it suppressed the amplitude of the signal. When implementing PCA technique with clipping, PAPR is reduced significantly. It can be concluded that PCA technique alone do not reduced PAPR, but when combining with other methods, notable PAPR reduction can be achieved.

Index Terms—PCA, PAPR, OFDM, CCDF

I. Introduction

The revolution of telecommunications has evolved dramatically, transforming wired communications to wireless communication system.

Within recent years, the use of wireless communications has expanded remarkably worldwide, as more and more users are sending and receiving data and image applications. The early application of Frequency Division Multiplexing (FDM) is used which is the telegraphs. It carried low rate signal using a separate carrier frequency for each signal. Throughout time, the Orthogonal Frequency Division Multiplexing or commonly referred to as OFDM scheme is focused on the synthesis of band-limited orthogonal signals for multi-channel data transmission. New scheme of transmitting signals synchronously over a band-limited channel without inter-carrier interface (ICI) and inter-symbol interface (ISI) is also introduced. The OFDM contradict from FDM as the multiple carriers or also known as subcarriers carry the flow of information, the subcarriers are orthogonal, and a guard interval is included to each symbol to decrease the channel delay spread and inter symbol interference.

OFDM acted as multi-carrier modulation method to achieve high speed communication. It has seen as a robust signal processing attributable to its high spectral-efficiency [1]. Apart from that, other notable characteristics that OFDM can offers are simpler channel equalization, resilience against Selective Fading, cost effective equalization process, better spectrum efficiency, and immunity to narrowband effects. OFDM is said to be the most promising candidate for flexible spectrum pooling in communication systems[2].

Whilst OFDM furnishes many advantages, there is still one disadvantage to its use which need to be addressed when considering its use. The most significant drawback is high peak-to-average-power ratio (PAPR). PAPR can be defined as the relationship between the maximum power of a sample in a transmit OFDM symbol and its average power[3]. The use of large number of subcarriers introduces high PAPR in OFDM systems. High PAPR occurs due to several sinusoids get added coherently in inverse fast Fourier transform (IFFT) at a particular instant[2]. The output envelope has a sudden growth and as a result, a very high peaks is constructed.

II. Orthogonal frequency division multiplexing (OFDM)

The OFDM concept is based on spreading the data to be transmitted over a large number of carriers, each being modulated at a low rate. These sub-carriers or sub-channels divide the available bandwidth and are sufficiently separated in frequency or is known as frequency spacing so that they are orthogonal. The carriers are made orthogonal to each other by appropriately choosing the frequency spacing between them[4]. As the subcarriers are orthogonal, it ensures that the subcarriers do not interfere with each other, and at the same time, for higher system efficiency, they are packed densely in the spectrum without any frequency guard bands [5].

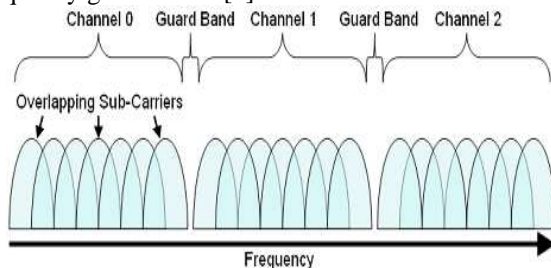


Figure 1 Spectrum of an OFDM System

From Figure 1, there are seven sub-carriers for each individual channel. As the symbol rate increases as the channel bandwidth increases, this implementation allows for a greater data throughput than with an FDM system. The orthogonality of the carriers means that each carrier has an integer number of cycles over a symbol period. Due to this, the spectrum of each carrier has a null at the center frequency of each of the other carriers in the system. This results in no interference between the carriers, although their spectra overlap.

III. Peak to Average Power Ratio (PAPR)

The main issue when implementing OFDM in wireless communication technologies is its high PAPR. The input symbol stream of the IFFT should possess a uniform power spectrum, but the output of the IFFT may result in a nonuniform or spiky power spectrum. Most of transmission energy would be allocated for a few instead of the majority subcarriers. This problem can be quantified as the PAPR measure. It causes many problems in the OFDM system at the transmitting end. PAPR can be defined as the ratio between instantaneous powers to average power[6] represented as follows

$$PAPR = \frac{\text{Instantaneous or peak power}}{\text{average power}} \quad (1)$$

The Root Mean Square (RMS) of the OFDM signal is defined as the root of the time average of the envelope power ($\sqrt{\bar{P}}$) where \bar{P} is denoted by

$$\bar{P} = \frac{1}{T} \int_{t=0}^T |x(t)|^2 dt = \frac{1}{N} \sum_{n=0}^{N-1} |X_n|^2 \quad (2)$$

where, $x(t)$ is the OFDM signal as shown in equation (2.4). The value \bar{P} corresponds to a single OFDM symbol and relies on the sequence of information consisting coefficients $\{X_n\}$. The average power of OFDM symbols can be described as

$$P_{av} = E\{X_n\} \quad (3)$$

The PAPR of an OFDM signal will then can be defined as

$$PAPR = \frac{\max_{0 \leq t \leq T} [|x(t)|^2]}{P_{av}} \quad (4)$$

$$PAPR = \frac{\max_{0 \leq t \leq T} [|x(t)|^2]}{E\{|x(t)|^2\}} \quad (5)$$

When the input data power is normalized, then $E\{|x(t)|^2\} = 1$. Thus, the equations obtained are stated by

$$PAPR = \max_{0 \leq t \leq T} [|x(t)|^2] \quad (6)$$

$$PAPR = \max_{0 \leq t \leq T} \left[\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi kn/N} \right]^2 \quad (7)$$

The maximum PAPR for an OFDM signal $x(n)$ in a system is defined by [7],

$$PAPR = 10 \log \frac{\max |x(n)|^2}{\text{Avg} |x(n)|^2} \quad (8)$$

Crest factor (CF) is the ratio of the maximum magnitude to the root mean square of the signal that is corresponding to the square root of PAPR by[8]

$$\text{Crest Factor, CF} = \sqrt{PAPR} \quad (9)$$

IV. Principal Component Analysis

Principal Component Analysis (PCA) is used as a pre-processing step in reducing the PAPR by identifies the directions of most variation in the data set and it reduces the data down into its basic component. PCA method is a statistical technique that converts a set of correlated variables into a set of linearly uncorrelated variables that is known as principal component[9]. Principal Component Analysis (PCA) is widely used for decreasing the dimensionality of datasets, improving interpretability

and reducing loss of information. This is because great datasets are progressively common and difficult to interpret [10].

PCA can be generalized as correspondence analysis (CA) in order to handle qualitative variables and as multiple factor analysis (MFA) in order to handle heterogeneous sets of variables. The goals of PCA are to extract the most important information from the data table. It then compressed the size of the data set by keeping only this important information. Next, PCA simplify the description of the data set. Lastly, it analyze the structure of the observations and the variables[11]. In order to achieve these goals, PCA computes new variables called principal components which are obtained as linear combinations of the original variables.

To transform the correlated data to uncorrelated set of data, there are two methods that can be done which are by singular value decomposition (SVD) of input data and by eigenvalue decomposition of the covariance matrix. These two methods are used to calculate the principal components.

A. Covariance Matrix Method

Given the data matrix $X = [x_1, x_2, \dots, x_n]$ where N is the total number of samples and x_i represents the i^{th} sample. Compute the mean, μ of all samples as

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i \quad (10)$$

Then, after calculating the mean of each variable in the data matrix, the mean-centring data are calculated by subtracting the mean from all samples by,

$$D = \{d_1, d_2, \dots, d_n\} = \sum_{i=1}^N x_i - \mu \quad (11)$$

where D denotes the Mean-Centring Data. The covariance matrix, Σ is then calculated as follows

$$\Sigma = \frac{1}{N-1} DD^T \quad (12)$$

Next, the eigenvectors and eigenvalues of the covariance matrix are computed. The eigenvectors are sorted according to their corresponding eigenvalues. Then, the eigenvectors that have the largest eigenvalues is selected.

B. Singular Value Decomposition (SVD)

To calculate SVD, R^T and S are first calculated by diagonalizing $X^T X$ as follows where $L^T L = I$,

$$X^T X = (LSR^T)^T (LSR^T) = RS^T L^T LSR^T = RS^2 R^T \quad (13)$$

The left singular vectors (L) is then calculated as follows,

$$L = XRS^{-1} \quad (14)$$

The columns of the right singular vectors (R) represent the eigenvectors of $X^T X$ or the principal components of the PCA. As the number of principal components and their eigenvalues are equal to q, thus the dimension of the original data matrix must be reversed to be compatible with SVD method. Thus, the mean-centring matrix is transposed before calculating the SVD method.

C. SVD vs. Covariance Matrix

The eigenvalues and eigenvectors of the covariance matrix are calculated, where the covariance matrix is the product of DD^T , where $D = \{d_i\}_{i=1}^N, d_i = x_i - \mu$. Using Equation (15) that is used to calculate SVD, the covariance matrix can be calculated as follows, where $L^T L = I$.

$$DD^T = (LSR^T)^T (LSR^T) = RS^T L^T LSR^T \quad (15)$$

Thus,

$$DD^T = RS^2 R^T = (SVDD^T)^2 \quad (16)$$

where S^2 represents the eigenvalues of DD^T or $D^T D$ and the columns of the right singular vector (R) represent the eigenvectors of DD^T . To conclude, the square root of the eigenvalues that are calculated using the covariance matrix method are equal to the singular values of SVD method. Moreover, the eigenvectors of Σ are equal to the columns of R. Thus, the eigenvalues and eigenvectors that are calculated using the two methods are equal.

V. Results and discussion

Simulation is done for OFDM signal with PCA technique using MATLAB software based on the parameters described in Table 1.

PARAMETER	VALUE
Modulation	QAM
Size of Symbol	16, 64
Number of transmitting symbols	64, 128, 256, 512
Oversampling factor	1
Number of runs	1000

A. PCA

PCA technique was applied to the original OFDM with both 16-QAM and 64-QAM. Both modulation schemes were tested with number of subcarriers of 64, 128, 256, and 512. Table 4.1 shows the

performance comparison of original OFDM and PCA method. Figure 2 shows simulation results when PCA technique is applied to the OFDM of 16-QAM with 64 subcarriers.

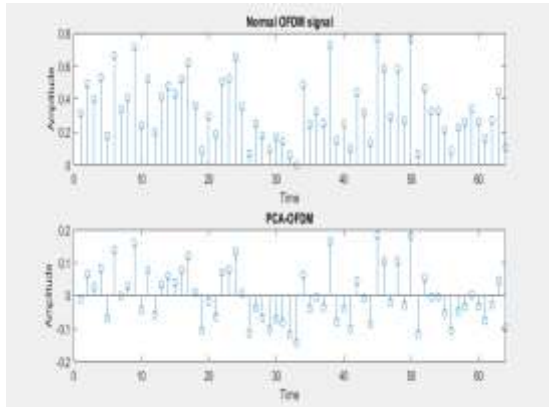


Figure 2 Variation in OFDM Signal while applying PCA technique



Figure 3 Screenshot of the MATLAB Window showing PAPR using the PCA method

Modulation	Number of subcarrier, N	Original OFDM (dB)	PCA Technique (dB)
16-QAM	64	13.7	16.2
	128	14.5	18.2
	256	15.2	20.1
	512	16.9	22.9
64-QAM	64	13.9	17.8
	128	14.9	18.7
	256	15.6	20.6
	512	17.3	23.5

Figure 4 PAPR performance in original OFDM and PCA Technique

Figure 3 shows the screenshot of the MATLAB Window showing PAPR using the PCA technique. Figure 4 shows the performance comparison of original OFDM and PCA method in 16-QAM and 64-QAM. Normal OFDM consists of several peaks. PCA method, it is seen that several peaks get suppress. It is shown that PCA did not reduced the value of PAPR as it is increased for about 0.3dB higher than its original value. Figure 3 shows the Screenshot of the MATLAB Window showing PAPR using the PCA technique

B. PAPR CCDF

To analyze the PAPR reduction efficiency of the proposed scheme complimentary cumulative distribution function graph is plotted. The CCDF curve is a plot of relative power levels and probability. The x-axis shows the threshold values and in y-axis probability of PAPR value is greater than the threshold is plotted.

From Figure 5, the performance of PAPR CCDF shows the increase of PAPR value when implementing the PCA technique.

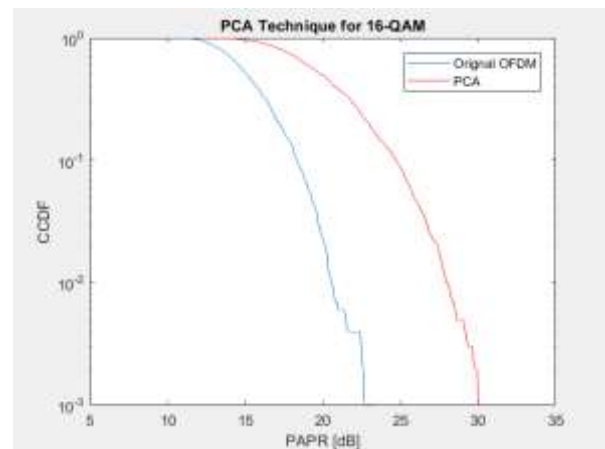


Figure 5 PAPR CCDF of original OFDM and PCA Technique

From a research conducted by [12], PCA with clipping can be conclude to be the best reduction technique in PAPR rather than using PCA technique alone. Thus, a comparison was made with PAPR value of PCA technique and PCA-Clipping technique to verify that combination of PCA technique with other method is a better reduction technique for PAPR. Figure 6 shows the PAPR CCDF of original OFDM and PCA-Clipping Technique.

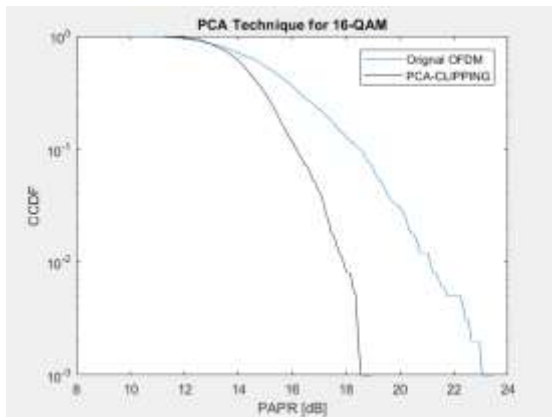


Figure 6 PAPR CCDF of original OFDM and PCA-Clipping Technique

As the results obtained from simulation of both 16-QAM and 64-QAM, PCA with clipping reduced the PAPR value significantly. Clipping technique clipped 70% of the peaks of the signal after PCA technique were applied. PCA with clipping technique improved the results of PAPR by decreased the value for about 0.3 dB to 0.6 dB from its original OFDM value.

Modulation	No. of Subcarriers, N	Original OFDM (dB)	PCA Method (dB)	PCA-Clipping Method (dB)
16-QAM	64	12.9	16.1	6.5
	128	14.7	18.5	8.0
	256	16.5	20.4	12.1
	512	18.3	23.7	15.7
64-QAM	64	13.2	17.6	10.3
	128	15.1	19.4	11.5
	256	17.0	22.3	14.9
	512	19.4	24.3	16.6

Several factors that may contributed on why PCA technique did not reduced PAPR value are identified. Removing more than three eigenvectors will commonly worsen the results. [13]. It is best to eliminate only the first two eigenvectors or principal component. Apart from that, data signal may be nonlinear. PCA is a linear method[14]. Thus, using linear PCA in nonlinear problems can sometimes be incompetent. Minor components are not usually consisting of noise or unimportant variance, but they also contain important information. If the minor components are eliminated, hence the important information will also be lost. This results in inconsistency of PCA technique

VI. Conclusion

When the PCA technique is implemented in original OFDM signal, it is seen that several peaks get suppress compared to the original OFDM signal, but the value of PAPR did not reduced. With the implementation of clipping to the PCA technique, the PAPR value experienced significant reduction. It can be concluded that using PCA technique alone did not affect the value of high PAPR, but when combining PCA technique with other method, notable improvement of PAPR value can be achieved. The PCA technique did not improved the reduction of PAPR as it may be caused by several factors. Factors that could affect the deteriorated of PAPR value are the elimination of more than three eigenvectors, and data is nonlinear.

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