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Stability of Vibrations of Vibroprotected Plate with Hysteresis Type Characteristics under White Noise Excitations

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Annotation. *This paper dedicated for that investigation of stability of stationary vibrations of plate with hysteresis type characteristics and dynamic absorber with elastic characteristics under white noise excitations. Hysteresis type characteristics of plate is according to Pisorenko-Boginich's hypothesis*[1].

Results and discussion. Mathematical model of vibrations of plate with hysteresis type characteristics and dynamic absorber with elastic characteristics is received as following[2]:

$$\ddot{x}_{ik} + (\epsilon_1 + j\epsilon_2 sign(\omega))p_{ik}^2 x_{ik} - d_{3ik}u_{ik0}x_2 = -d_{ik}W_0;$$
$$u_{ik0}\ddot{x}_{ik} + \ddot{x}_2 + n^2 x_2 = -W_0, \quad (1)$$

where x_{ik} = displacement of (ik) point of plate; x_2 = displacement of dynamic absorber; $j^2 = -1$; $\epsilon_1 = 1 - (c_0 + T_1^*)\eta_1^* - (T_2^* + T_3^*)\nu_1^*$; $\epsilon_2 = (c_0 + T_1^*)\eta_2^* + (T_2^* + T_3^*)\nu_2^*$; $T_1^* = T_1(\langle x_{ika}\rangle)$; $T_2^* = T_2(\langle x_{ika}\rangle)$; $T_3^* = T_3(\langle x_{ika}\rangle)$; $\eta_1^* = \eta_1(\langle x_{ika}\rangle)$; $\eta_2^* = \eta_2(\langle x_{ika}\rangle)$; $\nu_1^* = \nu_1(\langle x_{ika}\rangle)$; $\nu_2^* = \nu_2(\langle x_{ika}\rangle)$; $\eta_1,\ \eta_2,\ \nu_1,\ \nu_2$ = statistical linearization coefficients;

$$T_1 = \frac{3D}{d_{2ik}\rho h p_{ik}^2}\sum_{i_1=1}^{r} c_{i_1} x_{ika}^{i_1}\frac{h^{i_1}}{2^{i_1}(i_1+3)}\Big[\iint_s u_{ik}\Big(\frac{\partial^2}{\partial x^2}(\alpha_{11}|\alpha_{11}|^{i_1}) + \frac{\partial^2}{\partial y^2}(\alpha_{22}|\alpha_{22}|^{i_1})\Big)dxdy\Big];$$

$$T_2 = \frac{6D(1-\mu)}{d_{2ik}\rho h p_{ik}^2}\sum_{i_2}^{s_2} k_{i_2} x_{ika}^{i_2}\frac{h^{i_2}}{2^{i_2}(i_2+3)}\iint_s u_{ik}\Big(\frac{\partial^2}{\partial x\partial y}(\alpha_{33}|\alpha_{33}|^{i_3})\Big)dxdy;$$

$$T_3 = \frac{2D(1-\mu)}{d_{2ik}\rho h p_{ik}^2}k_0\iint_s u_{ik}\frac{\partial^2}{\partial x\partial y}(\alpha_{33})dxdy;$$ c_{i_1} $(i_1 = 0,\ldots,r)$ and k_{i_2} $(i_2 = 0,\ldots,s_2)$ parameters depend on material of plate and they are investigated from experiment[1]; $\alpha_{11} = \dfrac{\partial^2 u_{ik}}{\partial x^2} + \mu\dfrac{\partial^2 u_{ik}}{\partial y^2}$; $\alpha_{22} = \dfrac{\partial^2 u_{ik}}{\partial y^2} + \mu\dfrac{\partial^2 u_{ik}}{\partial x^2}$; $\alpha_{33} = \dfrac{\partial^2 u_{ik}}{\partial x\partial y}$; μ = Poisson's ratio;

$D=\frac{Eh^3}{12(1-\mu^2)}$ - cylindrical stiffness; E = Young's module; $x_{ika} = x_{ika}(t)$ = amplitude of x_{ik}; $\langle x_{ika}\rangle$= mathematical expectations of amplitude of x_{ik}; $d_{ik}=\frac{d_{1ik}}{d_{2ik}}$; $d_{3ik}=\frac{c}{\rho h d_{2ik}}$; ρ = density of plate; h= width of plate; $d_{1ik}=\iint_s u_{ik}dxdy$; $d_{2ik}=\iint_s u_{ik}^2 dxdy$; $u_{ik}=u_{ik}(x,y)$ = natural vibration form of x_{ik}; $u_{ik0}=u_{ik}(x_0,y_0)$ = quantities of at a point (x_0,y_0) dynamic absorber situated of natural vibration form of x_{ik}; p_{ik} = natural frequency of plate; $n=\sqrt{\frac{c}{m}}$ = natural frequency of dynamic absorber; c, m = stiffness and mass of dynamic absorber; $W_0=\xi cos\omega t$ = acceleration of foundation; ξ = maximum quantity of W_0; ω = frequency of system.

It is known us that, a lot of cases characters of random vibrations are learned by Ito's equations[3-5]. We will get solutions of system of differential equations (1) as following:

$$x_{ik}=\sigma_{ik}(t)e^{j\omega t}+\xi_{ik}(t)e^{-j\omega t};$$
$$x_2=\sigma_2(t)e^{j\omega t}+\xi_2(t)e^{-j\omega t}, \quad (2)$$

where $\sigma_{ik}(t)=\sigma_{ik}$, $\xi_{ik}(t)=\xi_{ik}$, $\sigma_2(t)=\sigma_2$, $\xi_2(t)=\xi_2$, are amplitudes of x_{ik} and x_2, they are counted slowly changing functions;

We will count derivatives of solutions (2) and put in (1). After doing that it is possible to get system of normal type differential equations.

$$\dot{\sigma}_{ik}=\frac{1}{2j\omega b_1}((\omega^2-p_{ik}^2(\epsilon_1+j\epsilon_2))b_1\sigma_{ik}+(\omega^2-p_{ik}^2(\epsilon_1-j\epsilon_2))b_1\xi_{ik}e^{-2j\omega t}+$$
$$+d_{3ik}u_{ik0}b_1(\sigma_2+\xi_2e^{-2j\omega t})-b_1d_{ik}W_0e^{-j\omega t});$$
$$\dot{\xi}_{ik}=-\frac{1}{2j\omega}((\omega^2-p_{ik}^2(\epsilon_1+j\epsilon_2))\sigma_{ik}e^{2j\omega t}+(\omega^2-p_{ik}^2(\epsilon_1-j\epsilon_2))\xi_{ik}+$$
$$+d_{3ik}u_{ik0}(\sigma_2e^{2j\omega t}+\xi_2)-d_{ik}W_0e^{j\omega t}); \quad (3)$$
$$\dot{\sigma}_2=\frac{1}{2j\omega}((\omega^2-b_3)(\sigma_2+\xi_2e^{-2j\omega t})+u_{ik0}p_{ik}^2(\epsilon_1+j\epsilon_2)\sigma_{ik}+$$
$$+u_{ik0}p_{ik}^2(\epsilon_1-j\epsilon_2)\xi_{ik}e^{-2j\omega t}-b_2W_0e^{-j\omega t});$$
$$\dot{\xi}_2=-\frac{1}{2j\omega}((\omega^2-b_3)(\sigma_2e^{2j\omega t}+\xi_2)+u_{ik0}(\epsilon_1+j\epsilon_2)p_{ik}^2\sigma_{ik}e^{2j\omega t}+$$

$$+u_{ik0}p_{ik}^2(\epsilon_1 - j\epsilon_2)\xi_{ik} - b_2 W_0 e^{j\omega t}),$$

where $b_1 = n^2 d_{ik} + u_{ik0} d_{3ik}$; $b_2 = 1 - u_{ik0} d_{ik}$; $b_3 = n^2 + u_{ik0}^2 d_{3ik}$.

Ito's system of differential equations[5] can be written as following by using stochastic averaging method from (3).

$$d\sigma_{ik} = \frac{1}{2j\omega}\Big(\big(\omega^2 - p_{ik}^2(\epsilon_1 + j\epsilon_2)\big)\sigma_{ik} + d_{3ik}u_{ik0}\sigma_2\Big)dt + \frac{1}{4\omega^2}\sum_{l=1}^{4}\gamma_{1l}\,dw_w(t);$$
$$d\xi_{ik} = \frac{-1}{2j\omega}((\omega^2 - p_{ik}^2(\epsilon_1 - j\epsilon_2))\xi_{ik} + d_{3ik}u_{ik0}\xi_2)dt + \frac{1}{4\omega^2}\sum_{l=1}^{4}\gamma_{2l}\,dw_w(t); \quad (4)$$
$$d\sigma_2 = \frac{1}{2j\omega}((\omega^2 - b_3)\sigma_2 + u_{ik0}p_{ik}^2(\epsilon_1 + j\epsilon_2)\sigma_{ik})dt + \frac{1}{4\omega^2}\sum_{l=1}^{4}\gamma_{3l}\,dw_w(t);$$
$$d\xi_2 = -\frac{1}{2j\omega}((\omega^2 - b_3)\xi_2 + u_{ik0}p_{ik}^2(\epsilon_1 - j\epsilon_2)\xi_{ik})dt + \frac{1}{4\omega^2}\sum_{l=1}^{4}\gamma_{4l}\,dw_w(t),$$

where $w_w(t) =$ Weiner's process and it has $\langle w_w(t)\rangle = \langle W_0(t)\rangle = 0$ property; $\sum_{1l=1}^{4}\gamma_{1l}, \sum_{2l=1}^{4}\gamma_{2l}, \sum_{3l=1}^{4}\gamma_{3l}, \sum_{4l=1}^{4}\gamma_{4l}$ are investigated from following matrix[5-6];

$$\|\gamma\gamma^T\| = \left\|\begin{matrix} 0 & \frac{d_{ik}^2 W_a}{2} & 0 & \frac{d_{ik}b_2 W_a}{2} \\ \frac{d_{ik}^2 W_a}{2} & 0 & \frac{d_{ik}b_2 W_a}{2} & 0 \\ 0 & \frac{d_{ik}b_2 W_a}{2} & 0 & \frac{b_2^2 W_a}{2} \\ \frac{d_{ik}b_2 W_a}{2} & 0 & \frac{b_2^2 W_a}{2} & 0 \end{matrix}\right\|, \quad (5)$$

W_a = amplitude of $W_0(t)$.

Stability problem of solution of Ito's differential equations are learned[7] for mechanical systems with hysteresis characteristics. According to [7] we will calculate mathematical expectations of (4).

$$\frac{d\langle\sigma_{ik}\rangle}{dt} = \frac{1}{2j\omega}((\omega^2 - p_{ik}^2(\epsilon_1 + j\epsilon_2))\langle\sigma_{ik}\rangle + d_{3ik}u_{ik0}\langle\sigma_2\rangle);$$
$$\frac{d\langle\xi_{ik}\rangle}{dt} = -\frac{1}{2j\omega}((\omega^2 - p_{ik}^2(\epsilon_1 - j\epsilon_2))\langle\xi_{ik}\rangle + d_{3ik}u_{ik0}\langle\xi_2\rangle); \quad (6)$$
$$\frac{d\langle\sigma_2\rangle}{dt} = \frac{1}{2j\omega}((\omega^2 - b_3)\langle\sigma_2\rangle + u_{ik0}p_{ik}^2(\epsilon_1 + j\epsilon_2)\langle\sigma_{ik}\rangle);$$
$$\frac{d\langle\xi_2\rangle}{dt} = -\frac{1}{2j\omega}((\omega^2 - b_3)\langle\xi_2\rangle + u_{ik0}p_{ik}^2(\epsilon_1 - j\epsilon_2)\langle\xi_{ik}\rangle).$$

We can see from (6) that stability of vibrations of this system is not depend on spectral density of acceleration of foundation. If excitations are parametric, stability of vibrations will be depend on spectral density of acceleration of foundation.

Characteristic equations of (6) will be as fellowing:

$$\lambda^{(1)}{}_{1,2}=\frac{1}{4j\omega}\left(2\omega^2-b_3-\epsilon_1 p_{ik}^2-\epsilon_2 p_{ik}^2 j\right)+\frac{\pm\lambda^{(1*)}+j\lambda^{(1**)}}{4\omega};$$
$$\lambda^{(1)}{}_{3,4}=\frac{1}{4j\omega}\left(-2\omega^2+b_3+\epsilon_1 p_{ik}^2-\epsilon_2 p_{ik}^2 j\right)+\frac{\pm\lambda^{(1*)}-j\lambda^{(1**)}}{4\omega}, \quad (7)$$

where

$$\lambda^{(1*)}=\sqrt{\frac{-\kappa_{1**}+\sqrt{\kappa_{1**}^2+\kappa_{1*}^2}}{2}};\lambda^{(1**)}=sign(\kappa_{1*})\sqrt{\frac{\kappa_{1**}+\sqrt{\kappa_{1**}^2+\kappa_{1*}^2}}{2}};$$
$$\kappa_{1*}=2\left(b_3-\epsilon_1 p_{ik}^2-2d_{3ik}u^2{}_{ik0}\right)\epsilon_2 p_{ik}^2;$$
$$\kappa_{1**}=(b_3-\epsilon_1 p_{ik}^2)^2-\epsilon_2^2 p_{ik}^4{}^2+4d_{3ik}u^2{}_{ik0}p_{ik}^2\epsilon_1. \quad (8)$$

Solutions of (6) will be following according to received (7):

$$\langle\sigma_{ik}\rangle=\langle C_{10}^{(1)}\rangle e^{\lambda_1^{(1)}t}+\langle C_{10}^{(2)}\rangle e^{\lambda_2^{(1)}t};$$
$$\langle\xi_{ik}\rangle=\langle D_{10}^{(1)}\rangle e^{\lambda_3^{(1)}t}+\langle D_{10}^{(2)}\rangle e^{\lambda_4^{(1)}t};$$
$$\langle\sigma_2\rangle=\langle C_{20}^{(1)}\rangle e^{\lambda_1^{(1)}t}+\langle C_{20}^{(2)}\rangle e^{\lambda_2^{(1)}t};$$
$$\langle\xi_2\rangle=\langle D_{20}^{(1)}\rangle e^{\lambda_3^{(1)}t}+\langle D_{20}^{(2)}\rangle e^{\lambda_4^{(1)}t}, \quad (9)$$

where

$$\langle C_{10}^{(1)}\rangle=\frac{d_{3ik}u_{ik0}}{\sqrt[4]{z_{10}^2+z_{20}^2}};\ \langle C_{20}^{(1)}\rangle=\frac{\sqrt{(\epsilon_2 p_{ik}^2+\lambda^*)^2+(\epsilon_1 p_{ik}^2-b_3-\lambda^{**})^2}}{\sqrt[4]{z_{10}^2+z_{20}^2}};$$
$$\langle C_{10}^{(2)}\rangle=\frac{d_{3ik}u_{ik0}}{\sqrt[4]{z_{30}^2+z_{40}^2}};\ \langle C_{20}^{(2)}\rangle=\frac{\sqrt{(\epsilon_2 p_{ik}^2-\lambda^*)^2+(\epsilon_1 p_{ik}^2-b_3-\lambda^{**})^2}}{\sqrt[4]{z_{30}^2+z_{40}^2}};$$
$$\langle D_{10}^{(1)}\rangle=\frac{\sqrt{(\epsilon_2 p_{ik}^2-\lambda^*)^2+(\epsilon_1 p_{ik}^2-b_3+\lambda^{**})^2}}{\sqrt[4]{z_{50}^2+z_{60}^2}};\langle D_{20}^{(1)}\rangle=\frac{p_{ik}^2 u_{ik0}\sqrt{\epsilon_1^2+\epsilon_2^2}}{\sqrt[4]{z_{50}^2+z_{60}^2}};$$
$$\langle D_{10}^{(2)}\rangle=\frac{\sqrt{(\epsilon_2 p_{ik}^2+\lambda^*)^2+(\epsilon_1 p_{ik}^2-b_3+\lambda^{**})^2}}{\sqrt[4]{z_{70}^2+z_{80}^2}};\langle D_{20}^{(2)}\rangle=\frac{p_{ik}^2 u_{ik0}\sqrt{\epsilon_1^2+\epsilon_2^2}}{\sqrt[4]{z_{70}^2+z_{80}^2}};$$

$z_{10}=(\epsilon_2 p_{ik}^2+\lambda^*)^2-\left(\epsilon_1 p_{ik}^2-b_3-\lambda^{**}\right)^2-4d_{3ik}^2 u_{ik0}^2; z_{20}=-2\left(\epsilon_2 p_{ik}^2+\lambda^*\right)\left(\epsilon_1 p_{ik}^2-b_3-\lambda^{**}\right); z_{30}=(\epsilon_2 p_{ik}^2-\lambda^*)^2-\left(\epsilon_1 p_{ik}^2-b_3-\lambda^{**}\right)^2-4d_{3ik}^2 u_{ik0}^2; z_{40}=$

$$-2\left(\epsilon_2 p_{ik}^2-\lambda^*\right)\left(\epsilon_1 p_{ik}^2-b_3-\lambda^{**}\right); z_{50}=\left(\epsilon_2 p_{ik}^2-\lambda^*\right)^2-\left(\epsilon_1 p_{ik}^2-b_3+\lambda^{**}\right)^2- 4p_{ik}^4 u_{ik0}^2(\epsilon_1^2-\epsilon_2^2); z_{60}=-2\left(\epsilon_2 p_{ik}^2-\lambda^*\right)\left(\epsilon_1 p_{ik}^2-b_3+\lambda^{**}\right)+8\epsilon_1\epsilon_2 p_{ik}^4 u_{ik0}^2; z_{70}= (\epsilon_2 p_{ik}^2+\lambda^*)^2-\left(\epsilon_1 p_{ik}^2-b_3+\lambda^{**}\right)^2-4p_{ik}^4 u_{ik0}^2(\epsilon_1^2-\epsilon_2^2); z_{80}=-2\left(\epsilon_2 p_{ik}^2+\lambda^*\right)\left(\epsilon_1 p_{ik}^2-b_3+\lambda^{**}\right)+8\epsilon_1\epsilon_2 p_{ik}^4 u_{ik0}^2.$$

It is known us averaged square quantity of generalized coordinates gives stationary vibrations of mechanical systems[8]. Averaged square quantity of (2) is content following condition:

$$\langle x_{ika}^2\rangle = 4\langle\sigma_{ik}\rangle_0\langle\xi_{ik}\rangle_0, \tag{10}$$

where $\langle\sigma_{ik}\rangle_0$ and $\langle\xi_{ik}\rangle_0$ are quantities of $\langle\sigma_{ik}\rangle$ and $\langle\xi_{ik}\rangle$ when $t=0$.

Averaged square quantity of x_{ik} will be following according to (9) and (10):

$$\langle x_{ika}^2\rangle = 4(\langle \mathrm{C}_{10}^{(1)}\rangle+\langle \mathrm{C}_{10}^{(2)}\rangle)(\langle D_{10}^{(1)}\rangle+\langle D_{10}^{(2)}\rangle). \tag{11}$$

Roots of characteristic equations (7) should be negative sign[9] for stationary vibrations of our system being stable. According to this fact we can write following:

$$-\epsilon_2 p_{ik}^2 \pm \lambda^{(1*)} < 0. \tag{12}$$

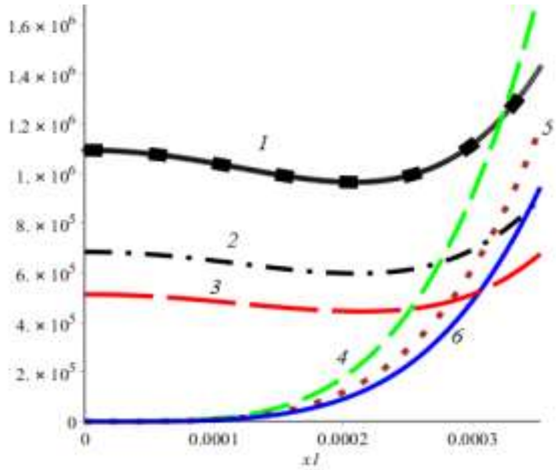
ϵ_2 is positive function which has characteristic of losing the energy of material of plate, κ_{1*} κ_{1**} are positive sign always for small $\langle x_{lka}\rangle$. They give $\lambda^{(1*)}>0$. We can say that second inequality of (12) is exist always. The first inequality of (12) is counted condition of stability for our problem.

$$\lambda^{(1*)} < \epsilon_2 p_{ik}^2. \tag{13}$$

Simplifying (13) gives following:

$$\epsilon_2{}^2 p_{ik}^2 d_{3ik} u_{ik0}^2 > 0. \tag{14}$$

Analysis and calculations. We will analize and calculate received results for steal plate 45-mark[10]:



$h = 0.001\text{m}; a = b = 0.19\text{ m}; \text{E} = 2.08 \cdot 10^{11}\, N/\text{m}^2; \text{G} = 0.8 \cdot 10^{11}\, N/\text{m}^2; \mu = 0.3; \rho = 7810\, kg/\text{m}^3; x_0 = a/2; y_0 = b/2; (i = k = 1); u_{ik}(x,y) = u_{11}(x,y) = sin\frac{\pi x}{0.19} sin\frac{\pi y}{0.19}; d_{1ik} = d_{111} = 4 \cdot 0.19^2/\pi^2; d_{2ik} = d_{211} = 0.19^2/4; \eta_1 = 142.717 x_{11a} + 22660477 x_{11a}^2 - 1.74149 \cdot 10^{12} x_{11a}^3; \eta_2 = 60.57097 x_{11a} + 9617405 x_{11a}^2 - 27.389243 \cdot 10^{11} x_{11a}^3; \nu_1 = 0.613245 x_{11a} - 62.7862 x_{11a}^2 + 4240.29 x_{11a}^3;\ \nu_2 = 0.26027 x_{11a} - 26.6473 x_{11a}^2 + 1799.635 x_{11a}^3; p_{11} = 847.48\, c^{-1}.$

Fig.-1. Changing of $\lambda^{(1*)}$ and $\lambda^{(1**)}$ functions according to natural frequency of plate.

We can see the changing of $\lambda^{(1*)}$ function (6, 5, 4 lines) and $\lambda^{(1**)}$ function (3, 2, 1 lines) according to natural frequency (847, 947, 1147) of plate in Fig.-1. Received results depend on that $\lambda^{(1*)}$ and $\lambda^{(1**)}$ should be positive sign as showen Fig.-1. As a result of the fact, the vibration is always stable.

If $\lambda^{(1*)}$ and $\lambda^{(1**)}$ functions are unknown sign we can not separate the real and abstract parts of roots of chatacteristic equation (7). It brings that stability problem can not be solved by Silvestre's criteria.

We can say that additionally, if natural frequency of plate is increase, quantity of $\lambda^{(1*)}$ and $\lambda^{(1**)}$ will be also increasing. It gives that $\lambda^{(1*)}$ and $\lambda^{(1**)}$ are always positive sign.

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