Non-linear oscillation of inextensible cable-connected satellites system at the equilibrium position near the main resonance for small eccentricity e of the orbit with oblateness of the earth and air resistance as perturbative forces

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ABSTRACT
This paper is devoted to study the non-linear main resonant oscillation of the system about stable position of equilibrium where it oscillates like a dumb-bell satellite in the phase plane $(a, \theta)$. B.K.M. method has been exploited to get the general solution valid at and near the main resonance $n=1$.
Key word: Satellite, B.K.M. Method, parametric resonance, oblateness.

INTRODUCTION
The Russian mathematics Belestky; V. V. (1960) made significant studies on the effect of perturbative forces on cable-connected satellites system. Similar problems have been studied by Singh, R.B. and Sharma, B; B.R.A.Bihar University, Muzaffarpur, India.
Non-linear oscillation of the system about stable position of equilibrium for small eccentricity near the main resonance $n=1$.

The differential equation of motion of inextensible cable-connected satellites system in the central gravitational field of oblate earth with air resistance in polar form is obtained in the form:

$$(1 + ecossin)v'' + 2e' \sin v + 3 sin^2 \cos \Psi$$

$$= 2esin + 5B (1 + ecossin)^2 \sin \Psi \cos \Psi - f(1 + ecossin)^2 \cos \Psi$$

$$+ fe (1 - 3ecossin)\sin \sin \Psi$$

(1)

Where
- $B =$ oblateness force parameter
- $f =$ Air resistance force parameter
- $e =$ eccentricity of orbit
- $v =$ True anomaly of the centre of mass of the system

Here dashes denote differentiation w.r. to $v$.
The Equilibrium position are given by

$$\phi = \phi_0 \text{ and } \Psi = \Psi_0 = \sin^{-1} \left( \frac{-f}{3 - 5B} \right) = A_0$$

(3)

The equation of small oscillation about the position of equilibrium is obtained by putting $\Psi = \Psi_0 + \delta$ in (1) and considering expansion only up to third order infinitesimals in the form:
\[
\delta'' + n^2 \delta = e \left[ (2 + A_0 f) \sin v + 2\delta' \sin v - \delta'' \cos v + 10A_0 B \sqrt{1 - A_0^2} C \right]
\]

Now let us construct the general solution of the oscillation system based on B.K.M. method which will be valid at and near the main resonance \( n=1 \). Assuming \( e \) to be a small parameter, the solution in the first approximation can be sought in the form:

\[
\delta = a \cos k \quad \text{where} \quad k = v + \theta
\]

Here the amplitude \( a \) and phase \( \theta \) must satisfy the system of ordinary differential equations

\[
\begin{align*}
\frac{da}{dv} &= e A_1(a, \theta) \\
\frac{d\theta}{dv} &= n - 1 + e B_1(a, \theta)
\end{align*}
\]

(7)

Where \( A_1(a, \theta) \) and \( B_1(a, \theta) \) are the periodic solutions periodic with respect to \( \theta \) of the system of partial differential equations:

\[
(n - 1) \frac{\partial A_1}{\partial \theta} - 2 a n B_1 = \frac{1}{\pi} \int_0^{2\pi} \sin k dk
\]

and

\[
a(n - 1) \frac{\partial B_1}{\partial \theta} + 2 n A_1 = -\frac{1}{\pi} \int_0^{2\pi} \cos k \sin k dk
\]

(7)

Where \( f_0(v, \delta, \delta', \delta'') \) can be easily obtained in the form:

\[
f_0(v, \delta, \delta', \delta'') = (2 + A_0 f) \sin v - 2 a n \sin v + a n^2 \cos v \cos k
\]

\[
+ \left( 10A_0 B \sqrt{1 - A_0^2} + 2f \sqrt{1 - A_0^2} \right) \cos v
\]

\[
+ \left[ 10B(1 - 2A_0^2) - 2A_0 f \right] a \cos k \cos v - 2 B A_0 \sqrt{1 - A_0^2} + f + A_0^2 \frac{20}{3} B(1 - 2A_0^2) \right) a^2 \cos^2 k \cos v
\]

\[
+ \frac{A_0 f}{3} - \frac{20}{3} B(1 - 2A_0^2) \right) a^2 \cos^2 k \cos v
\]

\[
+ f \sqrt{1 - A_0^2} a \cos k \sin v - \frac{A_0}{2} f a^2 \cos^3 k \sin v
\]

\[
- \sqrt{1 - A_0^2} f a^3 \cos^3 k \sin v
\]

(9)
Now, substituting the value of \( f_0(v, \delta, \delta', \delta'') \) from (9) in (8), we get on integrating.

\[
(n - 1) \frac{\partial A_1}{\partial \theta} - 2 \alpha n B_1 = \mu \cos \theta - \nu \sin \theta \tag{10}
\]

and

\[
\alpha (n - 1) \frac{\partial B_1}{\partial \theta} + 2 \alpha A_1 = -\nu \cos \theta - \mu \sin \theta \tag{11}
\]

where

\[
\mu = \left\{ (10a_0 B + 2f) - \frac{1}{2} (20a_0 B + f)a^2 \right\} \sqrt{1 - A_0^2} \tag{12}
\]

and

\[
\nu = (2 + A_0 f) - \frac{1}{4} a^2 f
\]

The particular solution periodic with respect to \( \theta \) of the system of equations (10) and (11) can be easily obtained as

\[
A_1 = \frac{1}{n+1} (-\nu \cos \theta - \mu \sin \theta) \tag{13}
\]

\[
B_1 = \frac{1}{\alpha(n+1)} (\nu \sin \theta - \mu \cos \theta)
\]

Putting the values of \( A_1 \) and \( B_1 \) from (13) in (7), we get

\[
\frac{da}{dv} = -\frac{e}{n+1} (\mu \sin \theta + \nu \cos \theta) \tag{14}
\]

The system of equations (14) may be written as

\[
\frac{da}{dv} = -\frac{1}{\alpha} \frac{\partial \phi}{\partial \theta} \quad \frac{dv}{dv} = -\frac{1}{\alpha} \frac{\partial \phi}{\partial v} \tag{15}
\]

Where

\[
\phi = \frac{a\varepsilon}{n+1} (\mu \cos \theta - \nu \sin \theta) - \frac{(n-1)}{2} a^2 \tag{16}
\]

obviously, the system of equations (15) are in canonical form and hence admits a first integral of the form:

\[
\phi = C_0^1 \tag{17}
\]

Where \( C_0^1 \) is the constant of integration In order to examine the stability, the integral curve (16) in the phase plane \((a, \theta)\) have been plotted with the equation.

\[
(n^2 - 1)a^2 - 2a \varepsilon (\mu \cos \theta - \nu \sin \theta) + c_0 = 0 \tag{18}
\]

Where \( C_0 = 2(n + 1)C_0^1 \) is constant

Integral curves plotted in fig1 and fig 2 for \( n = 0.957 \) and \( n=1.29 \) respectively for different values of parameters involved. Since both curves are closed curves and so we get the stability.

Conclusions: We conclude that the non-linear oscillation of satellites system about the equilibrium position is \( \Phi = \Phi_0 \) and \( \Psi = \Psi_0 = \sin^{-1} \left( \frac{-f}{3-55} \right) = A_0 \). It can be used to study the equilibrium position and stability of oscillatory satellites system in other perturbative forces.
Fig – 1

NON-LINEAR OSCILLATION OF THE SATELLITE IN ELLIPTIC ORBIT AT MAIN RESONANCE FOR \( n = 0.957 \) AND \( e = 0.01 \).
Fig -2
References:

1. Beletsky VV: "Motion of Artificial Satellite Relative to the Centre of Mass of the system", Nauka, 1965 (Russian)


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