Non-linear oscillation of inextensible cable-connected satellites system at the equilibrium position near the main resonance for small eccentricity e of the orbit with oblateness of the earth and air resistance as perturbative forces

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ABSTRACT

This paper is devoted to study the non-linear main resonant oscillation of the system about stable position of equilibrium where it oscillates like a dumb-bell satellite in the phase plane (a,θ), B.K.M. method has been exploited to get the general solution valid at and near the main resonance n=1.

Key word: Satellite, B.K.M. Method, parametric resonance, oblateness.

INTRODUCTION

The Russian mathematics Belestky; V. V. (1960) made significant studies on the effect of perturbative forces on cable-connected satellites system. Similar problems have been studied by Singh, R.B. and Sharma, B; B.R.A.Bihar University, Muzaffarpur, India.

Non-linear oscillation of the system about stable position of equilibrium for small eccentricity near the main resonance n=1.

The differential equation of motion of inextensible cable-connected satellites system in the central gravitational field of oblate earth with air resistance in polar form is obtained in the form :

$$(1+\mathrm{ecosv})\Psi''+2e\Psi'\sin\mathrm{v}+3\,sin\Psi\,\cos\Psi$$
$$=2esinv+5B\,(1+e\,cosv)^2\sin\Psi\,\cos\Psi-\mathrm{f}(1+\mathrm{ecosv})^{-2}\cos\Psi$$
$$+fe\,(1-3e\,cosv)sinv\,sin\Psi \quad (1)$$

Where
B = oblateness force parameter
f = Air resistance force parameter
e = eccentricity of orbit
v = True anomaly of the centre of mass of the system (2)

Here dashes denote differentiation w.r. to v.

The Equilibrium position are given by

$$\phi=\phi_o\ and\ \Psi=\Psi_0=sin^{-1}\left(\frac{-f}{3-5B}\right)=A_o \quad (3)$$

The equation of small oscillation about the position of equilibrium is obtained by putting $\Psi=\Psi_0+\delta$ in (1) and considering expansion only up to third order infinitesimals in the form:

$$\delta'' + n^2\delta = e\Big[(2 + A_0 f)\sin v + 2\delta' \sin v - \delta'' \cos v + 10A_0 B\sqrt{\sqrt{1 - A_0^2}}\cos v + 10B(1 - 2A_0^2)\delta \cos v - 20BA_0\sqrt{1 - A_0^2}\,\delta^2 \cos v$$
$$- \frac{20}{3}B(1 - 2A_0^2)\delta^3 \cos v + 2f\sqrt{1 - A_0^2}\cos v - 2A_0 f\delta \cos v - f\sqrt{1 - A_0^2}\,\delta^2 \cos v + f\sqrt{1 - A_0^2}\,\delta \sin v$$
$$- \frac{A_0}{2} f\delta^2 \sin v - \sqrt{1 - A_0^2} f\delta^3 \sin v + \frac{A_0}{3} f\delta^3 \cos v\Big] \quad (4)$$

Where $n^2 = (3 - 5B)(1 - 2A_0^2) - A_0 f$ (5)

Now let us construct the general solution of the oscillation system based on B.K.M. method which will be valid at and near the main resonance n=1. Assuming e to be a small parameter, the solution in the first approximation can be sought in the form :

$\delta = a\cos k$ where k=v+θ

Here the amplitude a and phase θ must satisfy the system of ordinary differential equations

$$\left.\begin{aligned} \frac{da}{dv} &= e\,A_1(a,\theta) \\ \frac{d\theta}{dv} &= n - 1 + e\,B_1(a,\theta) \end{aligned}\right\} \quad (7)$$

Where $A_1(a,\theta)$ and $B_1(a,\theta)$ are the periodic solutions periodic with respect to θ of the system of partial differential equations:

$$\left.\begin{aligned} (n-1)\frac{\partial A_1}{\partial\theta} - 2\,a\,nB_1 &= \frac{1}{\pi}\int_0^{2\pi} f_0(v,\delta,\delta',\delta'')\cos k\,dk \\ \text{and}\quad a(n-1)\frac{\partial B_1}{\partial\theta} + 2\,nA_1 &= -\frac{1}{\pi}\int_0^{2\pi} f_0(v,\delta,\delta',\delta'')\sin k\,dk \end{aligned}\right\} \quad (7)$$

where $f_0(v,\delta,\delta',\delta'')$ can be easily obtained in the form :

$$f_0(v,\delta,\delta',\delta'') = (2 + A_0 f)\sin v - 2\,a\,n\sin v\sin k + a\,n^2\cos v\cos k$$
$$+\left(10A_0 B\sqrt{1 - A_0^2} + 2f\sqrt{1 - A_0^2}\right)\cos v$$
$$+[10B(1 - 2A_0^2) - 2A_0 f]\,a\cos k\cos v$$
$$-\left[2B\,A_0\sqrt{1 - A_0^2} + f\sqrt{1 - A_0^2}\right]a^2\cos^2 k\cos v$$
$$+\left[\frac{A_0 f}{3} - \frac{20}{3}B(1 - 2A_0^2)\right]a^3\cos^3 k\cos v$$
$$+f\sqrt{1 - A_0^2}\,a\cos k\sin v - \frac{A_0}{2} f\,a^2\cos^3 k\sin v$$
$$-\sqrt{1 - A_0^2} f\,a^3\cos^3 k\sin v \quad (9)$$

Now, substituting the value of $f_0(v,\delta,\delta',\delta'')$ from (9) in (8), we get on integrating.

$$(n-1)\frac{\partial A_1}{\partial \theta} - 2\,a\,nB_1 = \mu cos\theta - \nu\ sin\theta \qquad (10)$$

and $a(n-1)\frac{\partial B_1}{\partial \theta} + 2\,nA_1 = -\nu cos\theta - \mu \sin\theta \qquad (11)$

where $\mu = \left\{(10A_0B + 2f) - \frac{1}{2}(20A_0B + f)a^2\right\}\sqrt{1 - A_0^2} \qquad (12)$

and $\nu = (2 + A_0f) - \frac{1}{4}a^2 f$

The particular solution periodic with respect to θ of the system of equations (10) and (11) can be easily obtained as

$$\left.\begin{aligned} A_1 &= \frac{1}{n+1}(-\nu\, cos\theta - \mu \sin\theta) \\ B_1 &= \frac{1}{a(n+1)}(\nu\, sin\theta - \mu \cos\theta) \end{aligned}\right\} \qquad (13)$$

Putting the values of A_1 and B_1 from (13) in (7), we get

$$\left.\begin{aligned} \frac{da}{dv} &= -\frac{e}{n+1}(\mu \sin\theta + \upsilon \cos\theta) \\ \frac{d\theta}{dv} &= (n-1) - \frac{e}{a(n+1)}(\mu \cos\theta - \upsilon \sin\theta) \end{aligned}\right\} \qquad (14)$$

The system of equations (14) may be written as

$$\left.\begin{aligned} \frac{da}{dv} &= \frac{1}{a}\frac{\partial \phi}{\partial \theta} \\ \frac{d\theta}{dv} &= -\frac{1}{a}\frac{\partial \phi}{\partial a} \end{aligned}\right\} \qquad (15)$$

Where $\phi = \frac{ae}{n+1}(\mu\, cos\theta - \upsilon sin\theta) - \frac{(n-1)}{2}a^2 \qquad (16)$

obviously, the system of equations (15) are in canonical form and hence admits a first integral of the form:

$$\phi = c_0^1 \qquad (17)$$

Where c_0^1 is the constant of integration In order to examine the stability, the integral curve (16) in the phase plane (a,θ) have been plotted with the equation.

$$(n^2 - 1)a^2 - 2\,a\,e\,(\mu\, cos\theta - \nu\ sin\theta) + c_0 = 0 \qquad (18)$$

Where $C_0 = 2(n+1)C_0^1$= constant

Integral curves plotted in fig1 and fig 2 for n =0.957 and n=1.29 respectively for different values of parameters involved. Since both curves are closed curves and so we get the stability.

Conclusions: We conclude that the non-linear oscillation of satellites system about the equilibrium position is $\emptyset = \emptyset_0 and\ \Psi = \Psi_0 = sin^{-1}\left(\frac{-f}{3-5B}\right) = A_o$. It can be used to study the equilibrium position and stability of oscillatory satellites system in other perturbative forces.

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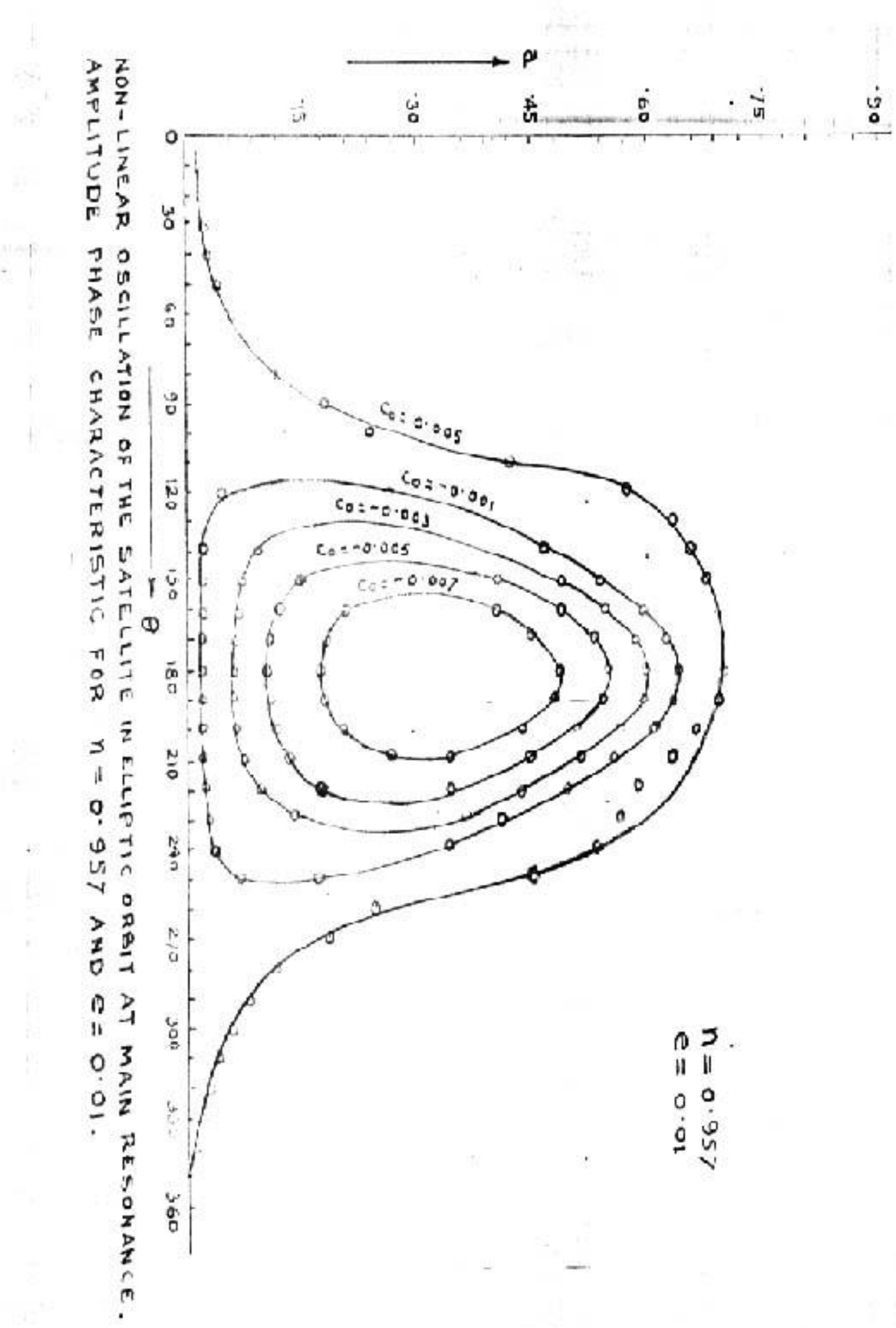

Fig – 1

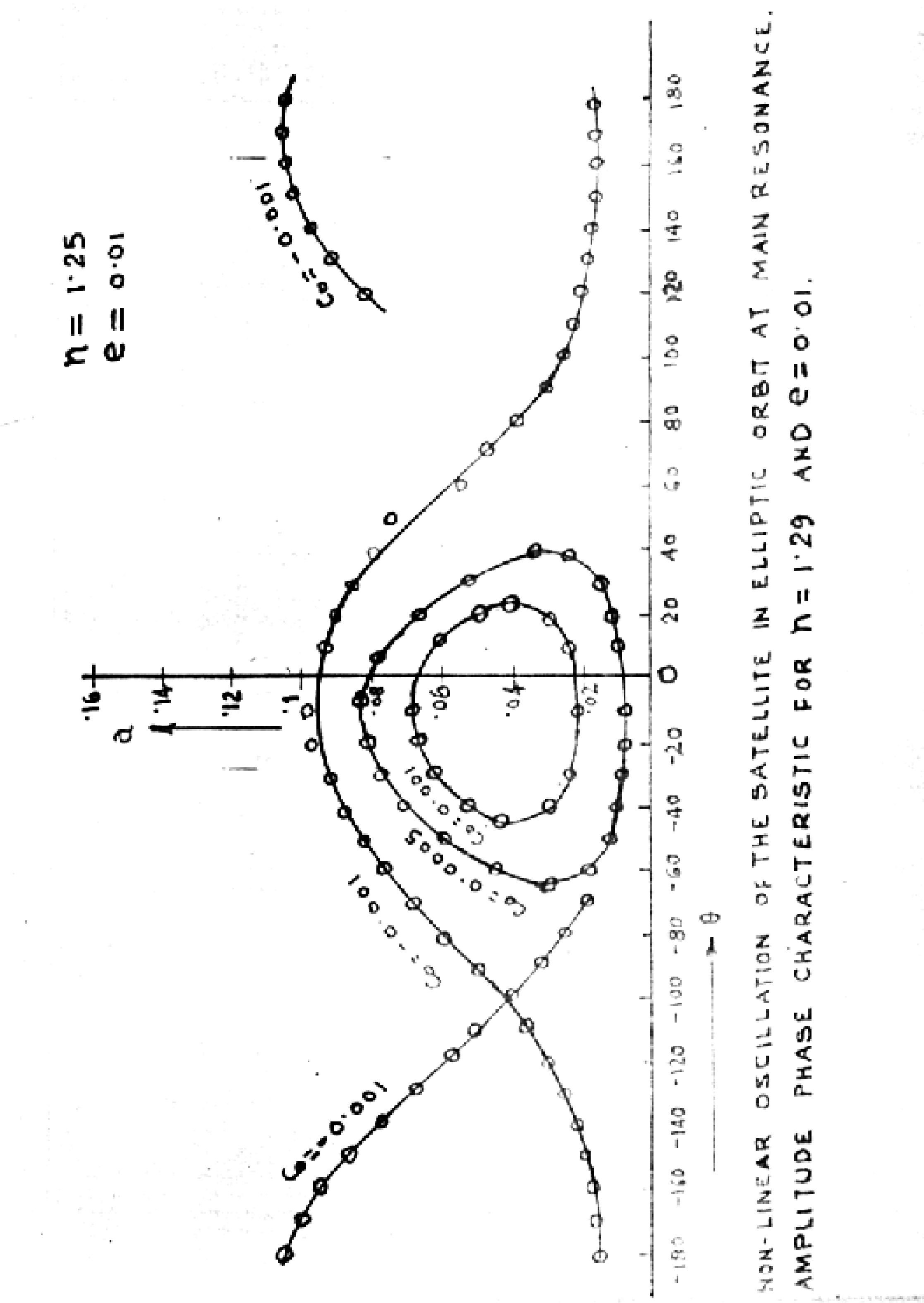
n = 1·25
e = 0·01
NON-LINEAR OSCILLATION OF THE SATELLITE IN ELLIPTIC ORBIT AT MAIN RESONANCE.
AMPLITUDE PHASE CHARACTERISTIC FOR n = 1·29 AND e = 0·01.

Fig -2

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