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Laminary Fluid Flow In A Pipe And Dimensional Number Of Reynolds

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Abstract: The article provides information on the mathematical modeling of the incompressible viscous fluid in the pipe. The study shows laminar and turbulent regimes of fluid motion, as well as the physical meaning of these regimes. Consider a straight round pipe with a diameter constant along the entire length. The flow rate on the walls of the pipe due to adhesion is zero, in the middle of the pipe, it has the greatest value. A cylinder with a characteristic length and a characteristic radius inside the liquid whose axis coincides with the axis of the pipe is considered and the flow of the liquid through the cylinder is studied. The calculation formulas for calculating the maximum flow velocity in the cylinder, the volume of liquid passing through the cross-section of the pipe, the coefficient of resistance to friction in the pipe along the flow length, and the maximum value of the tangential stress are derived. The results of comparison of empirical and semi-empirical formulas for calculating the coefficient of resistance to friction are presented.

Keywords: Reynolds number, laminar flow, turbulent flow, parabolic flow, the friction force is the integral coordinate of the pipe, viscosity, density, bulk flow velocity, average speed, maximum speed, radius, Gegen, Poisal, Darcy-Weisbach, the volume of fluid resistance coefficient.

The flow of real fluids in many cases differs sharply from laminar flow. They have a special property called turbulence. In real flows, which occurs in pipes, channels, and in the boundary layer with increasing values of the Reynolds number, the laminar form of the flow becomes turbulent. Such a transition from laminar to turbulent flow is called the occurrence of turbulence, and they play a fundamental role throughout hydrodynamics. Initially, such a transition was detected in flows occurring in straight pipes and channels.

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In a straight pipe with a smooth wall and a constant cross section, each particle of liquid at small Reynolds numbers moves along a straight path. Due to the presence of viscosity, the particles of the liquid close to the wall move more slowly than away from the wall. The flow moves in the form of ordered layers moving relative to each other. However, observations show that for large Reynolds numbers, the flow goes into an unordered state or goes into a turbulent flow. There is a strong mixing in the liquid, this can be seen by introducing paint into the liquid moving in the pipe.

In 1883, Osborne Reynolds, studying the movement of water in a circular pipe, found that with an increase in the flow velocity, the steady laminar nature of the movement is disturbed. Perturbations appear that are expressed in the fact that the previously rectilinear movement of fluid particles, laminar in some areas becomes erratic, while maintaining the general direction of motion. A further increase in speed leads to chaotic motion throughout the flow. As is customary to say at the present time, the flow has turned from a steady-laminar into an unstable, disturbed-turbulent [2].

The presence of viscosity in liquids resists the movement of fluid layers relative to each other. In other words, in laminar (layered) flows, internal friction occurs due to viscosity; it is expressed by the number of tangential stresses at the boundaries of the layers, or is characterized by the number of tangential forces related to a unit area. Separate concentric layers of fluid relative to each other move in such a way that the velocity of the fluid will be directed in the direction of the main axis. The movement of this type of fluid is called laminar flow [1-12].

When the incompressible viscous fluid moves starting at the same value of the Reynolds number $\mathrm{Re} = \rho UL/\mu$, the laminar flow passes into a turbulent one, the same value of the Reynolds number is called the critical Reynolds number, where ρ - density, μ - viscosity of the liquid, U - the maximum velocity of the main flow, L - the characteristic scale of the length.

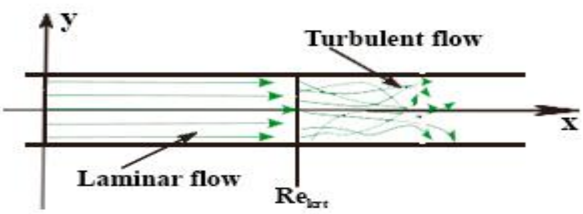

Fig. 1. The transition form laminar flow to turbulent

From Fig. 1. it is seen that at, $\mathrm{Re} < \mathrm{Re}_{krt}$, laminar flow, and $\mathrm{Re}_{krt} < \mathrm{Re}$ and the flow goes into turbulent mode.

In [2], information is given on the forces acting on flows in a cylindrical tube. We will consider the flow of a fluid in a straight circular pipe with a constant diameter over its entire length, inside of which are located a bundle of n tubes with a length L and a radius r.

In real liquids, the fluid adheres to the walls of the tube and transmits the shear stress to the surface of the streamlined fluid. Here the so-called internal friction force appears, in liquids this force is viscosity. Viscosity is such a property of gases and liquids, which is the resistance leading to the movement of liquids by external forces. The presence of tangential stresses and adhesion of liquids to solid walls leads to qualitative differences between real and ideal liquids. Now we study the movement of liquids in a pipe inside of which n tubes of the same length and radius are located. Taking into account the viscosity on the tube walls, the velocity is zero, reaches its maximum value in the middle of the tube. At a sufficiently remote distance from the tube entrance, the distribution of the flow velocity does not depend on the coordinate of the directional along the radius.

The movement of fluid in the pipe occurs under the action of a pressure drop in the direction of the pipe axis, but in each cross section perpendicular to the pipe axis, the pressure can be considered as constant. The movement of each fluid element is accelerated due to pressure drop and slows down due to shear stress caused by friction [2-12].

The pressure p is assumed to be constant, that is, it is assumed that over the section of the tube $p_0, p_l = const$ [3].

In the direction of the main axis, a pressure force $p_0 n\pi y^2$ and $p_l n\pi y^2$ applied to the inlet and outlet bases of the tube, respectively, as well as the tangential force $2\pi n y L\tau$ acting on the lateral surface of the cylinder, act on the tubes. It is required to determine the maximum flow velocity in the tube, the volume of fluid flowing through the cross-section of the tube, the coefficient of tube resistance to friction along the flow length, as well as the maximum value of the tangential stress.

Equating the forces acting fluid in the tube, we obtain as an equilibrium condition in the direction of motion the equation (Fig. 2.)

$$\sum_{i=1}^{n} p_0 \pi y^2 = \sum_{i=1}^{n} p_l \pi y^2 + \sum_{i=1}^{n} 2\pi y L \tau \cdot \qquad (1)$$

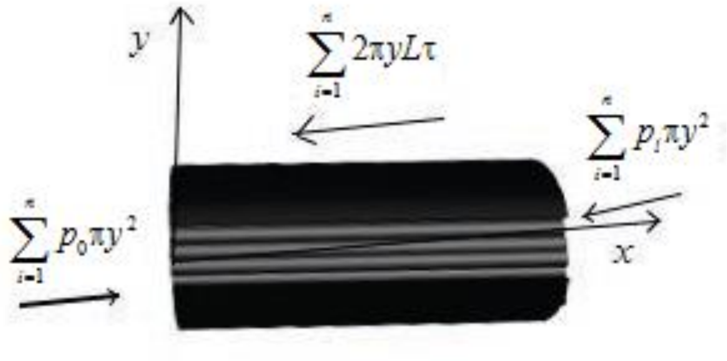

Fig. 2. In the tube is located a bunch of n tubes.

The projection of the internal friction force is taken with a plus sign, because the velocity gradient is negative (the velocity of the layer decreases with increasing radius r)

From the formula (1) we determine the tangent stress τ

$$\tau = \frac{p_0 - p_l}{L} \cdot \frac{y}{2}. \qquad (2)$$

In this case, the flow velocity u decreases with increasing coordinate y and is zero at $y = r$. Therefore, on the basis of the law of friction $\tau = \mu \frac{du}{dy}$ Hooke should take that $\tau = -\mu \frac{du}{dy}$. Substituting this expression in (2), we obtain

$$-\mu \frac{du}{dy} = \frac{p_0 - p_l}{L} \cdot \frac{y}{2},$$

from here, you can see that

$$\frac{du}{dy} = -\frac{p_0 - p_l}{\mu L} \cdot \frac{y}{2}. \qquad (3)$$

Now, given that $y = r$ with velocity $u(y) = 0$ and integrating equation (3) with this initial condition we have

$$u(y) = -\frac{p_0 - p_l}{4\mu L} y^2 + C, \qquad (4)$$

to determine the constant C of equation (4), use the condition $u(r)=0$ at $y=r$, or

$$u(r) = -\frac{p_0 - p_l}{4\mu L} r^2 + C = 0,$$

from here you can see that

$$C = \frac{p_0 - p_l}{4\mu L} r^2. \qquad (5)$$

Substituting the value of the constant C from (5) to equation (4) we have

$$u(y) = -\frac{p_0 - p_l}{4\mu L} y^2 + \frac{p_0 - p_l}{4\mu L} r^2,$$

And in turn, we obtain an equation to determine the flow rate of the following formula

$$u(y) = \frac{p_0 - p_l}{4\mu L}(r^2 - y^2). \qquad (6)$$

Thus, we have a parabolic velocity distribution along the radius of the pipe (Fig. 3.). The greatest value of speed is in the middle of the pipe ($y=0$), where it is

$$u_{\max} = \frac{p_1 - p_2}{4\mu L} r^2. \qquad (7)$$

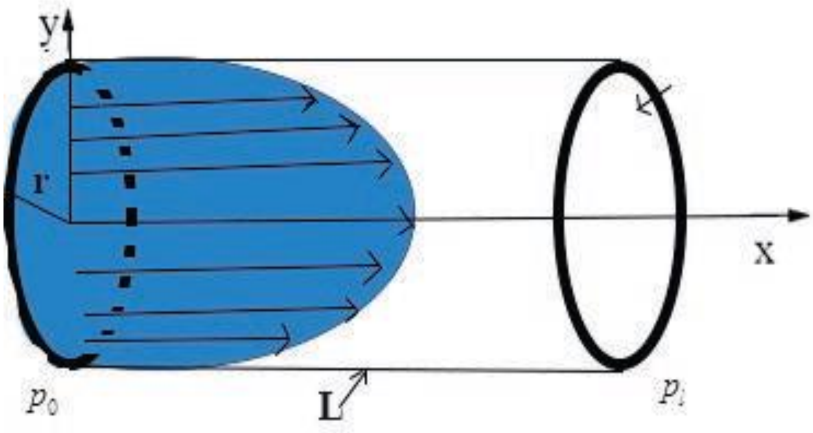

Fig. 3. Fluid flow rate for one tube

The total amount Q of liquid flowing through the pipe section (fluid flow) is defined as the volume of the paraboloid of rotation (Fig.3.) and acreage is defined as follows.

Equation (6) is rewritten as follows:

$$u(y) = \frac{p_0 - p_l}{4\mu L}(r^2 - y^2),$$

from here, you can see that

$$u(y)=\frac{p_0-p_l}{4\mu L}r^2(\frac{r^2-y^2}{r^2})=u_{\max}\left(1-\frac{y^2}{r^2}\right). \qquad (8)$$

The total liquid flow through a tube with a circular cross section on the basis of the Gagen-Poiseuille formula is determined as follows [1,3,7,8,11,12]

$$Q=\int_0^r u(y)2\pi y dy=2\pi u_{\max}\int_0^r\left(y-\frac{y^3}{r^2}\right)dy==2\pi u_{\max}\left[\frac{y^2}{2}-\frac{y^4}{4r^2}\right]_0^r,$$

or given the formula (7), for the flow of liquid have the formula

$$Q=2\pi\cdot\frac{p_0-p_l}{4\mu L}\cdot r^2\cdot\frac{r^2}{4}=\frac{\pi(p_0-p_l)r^4}{8\mu L}. \qquad (9)$$

Enter the average flow rate, the values of which are determined by the cross section of the tube as follows:

$$\bar{u}=\frac{Q}{\pi r^2}. \qquad (10)$$

Equation (10) with the formula (9) is written as

$$\bar{u}=\frac{(p_0-p_l)r^2}{8\mu L},$$

by comparing the function $\bar{u}(y)$ with the maximum speed $u_{\max}$ determined by the formula (7) it can be seen that $\bar{u}(y)=\frac{1}{2}u_{\max}$ or the average speed of the laminar flow in the tube is half the maximum speed (Fig. 4).

Determine the pressure difference (p_0-p_l)

$$p_0-p_l=\frac{8\mu L\bar{u}}{r^2},$$

from here we have

$$p_0-p_l=\frac{8\mu L\bar{u}}{r^2}=\frac{32\mu\bar{u}}{2r}\cdot\frac{L}{2r}=\frac{32\mu\bar{u}}{D}\cdot(\frac{L}{D}), \qquad (11)$$

here $D=2r$ is the diameter of the tube.

The pressure loss along the flow length is determined by Darcy-Weisbach equation

$$p_0-p_l=\sum_{i=1}^{n}\frac{\lambda_i}{2}\rho\bar{u}^2(\frac{L}{D}), \qquad (12)$$

here, λ - is the hydraulic loss ratio along the length of the pipe or the resistance coefficient of the pipe. From the last equation we have

$$\lambda = \frac{(p_1 - p_2)}{\frac{1}{2}\rho\bar{u}^2}\cdot\left(\frac{D}{L}\right) \qquad (13)$$

Substituting $p_0 - p_l$ the value of the formula (11) in the equation (13) we obtain, for the resistance coefficient of the pipe following formula

$$\lambda = \frac{32\mu\bar{u}}{D}\cdot\left(\frac{L}{D}\right)\cdot\frac{2}{n\rho\bar{u}^2}\cdot\left(\frac{D}{L}\right) = \frac{64\mu}{n\rho\bar{u}D}$$

or from here you can see that

$$\lambda = \frac{64}{n\,\mathrm{Re}}, \qquad (14)$$

Here is $\mathrm{Re} = \frac{\rho\bar{u}D}{\mu}$ - Reynolds number.

According to formula (14) to calculate the resistance coefficient, we present the results of calculations for various numbers of tube bundles n. (4 fig.).

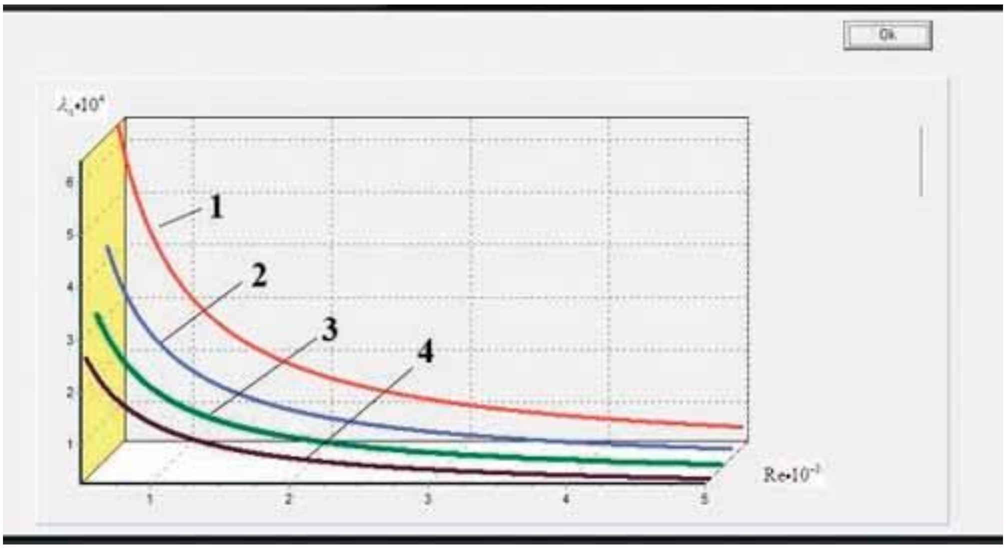

Fig. 4. The dependence of the resistance coefficient for smooth tubes on the number of tubes n and the Reynolds number Re: 1) n = 200, 2) n = 300, 3) n = 400, 4) n = 500.

Figure 4 for n smooth tubes shows the results illustrating the dependence of the resistance coefficient λ_n on the Reynolds number Re.

A comparison of the obtained results shows that for all values of the Reynolds number the theoretical formula (14) holds. In computational experiments, the following range of variation of the characteristic parameters Re

and λ_n: $\mathrm{Re} = 500 \div 5000$, $\lambda_n = 0.0001 \div 0.0007$ was considered. From Fig. 4 it is seen that as the number of tube n increases, the resistance coefficient decreases.

Thus, it is shown that the motion of incompressible viscous flows in channels, pipes and in the boundary layer can be laminar and turbulent, and the physical meaning of the appearance of these modes is indicated. For the fluid flowing through the tube n inside the tube, the formulas for calculating the maximum velocity of the fluid volume flowing through the cross section of the tube, the coefficient of tube resistance to friction along the length of the flow, are derived.

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