Modes Of Fluids And Their Stability

Dr. Vivek Parkash

Asstt. Prof. of Mathematics , Dyal Singh College , Karnal

E.mail.: lethal007@hotmail.com

Abstract:

Thermal instability of viscous horizontal fluid layer under adverse temperature gradient (heated form below) is the simplest problem in the Stability Theory and is known as Benard's problem. In this, the buoyancy force acts upwards which is opposed by the viscous force and the equilibrium is reached when these opposing forces are equal in magnitude, giving the marginal state in the Benard's problem.

Keywords: Non-oscillatory modes, dispersion relation,number density,unstable modes, stratification,temperature gradient.

Assuming that non-oscillatory modes which exist i.e. $\sigma_r \neq 0, \sigma_i = 0$. Then from the dispersion relation

$$Y(Y+P_r\sigma)[\sigma^2(f'+M_1\sigma)+(1+M_1\sigma)Qa_x^2]$$
$$-[R_1\beta a^2(Y+P_r\sigma)(1+M_1\sigma)+\sigma(1+M_1\sigma)Ra^2]=0$$

Where $Y=\left(\frac{L\pi}{d}\right)^2+a^2$

We have:

$$A\sigma_r^4+B\sigma_r^3+C\sigma_r^2+D\sigma_r+E=0 \qquad (1)$$

where,

$$A = YP_rM_1,$$

$$B = Y^2M_1 + YP_r f'$$

$$C = (f'Y^2 - Ra^2M) + (Qa_x^2Y - R_1\beta a^2)M$$

$$D = (Qa_x^2Y - R_1\beta a^2)(M_1Y + P_r) - Ra^2$$

$$E = Y(YQa_x^2 - R_1\beta a^2)$$

If $E > 0$ i.e. $Qa_n^2 > \frac{R_1\beta a^2}{Y}$, which implies that the product of four roots is positive and sum of roots is negative, that is, there is at least one real root. Hence, non-oscillatory modes are unstable.

we now analyze the unstable modes with respect of suspended particles number density and magnetic field and we examine the nature of $d\sigma_r/df'$ and $d\sigma_r/dQ$ respectively. Equation (1) yields

$$4\sigma_r^3YP_rM_1\frac{d\sigma_r}{df'} + 3\sigma_r^2\frac{d\sigma_r}{df'}(Y^2M_1 + YP_r f') + \sigma_r^3YP_r$$
$$+ 2\sigma_r\frac{d\sigma_r}{df'}(Y^2f' + YP_rM_1Qa_x^2 + P_rM_1 - Ra^2) + \sigma_r^2Y^2$$
$$+ \frac{d\sigma_r}{df'}(Y^2M_1Qa_x^2 + YP_rQa_x^2 - R_1Ba^2(M_1Y + P_r) - Ra^2M_1) = 0$$

or $$\frac{d\sigma_r}{df'}(4A\sigma_r^3 + 3B\sigma_r^2 + 2(\sigma_r + D) = -\sigma_r^2Y(Y + \sigma_rP_r)$$

or $$\frac{d\sigma_r}{df'} = -\frac{U_1}{U}$$

where, $U_1 = \sigma_r^2Y(Y + \sigma_rP_r)$

Similarly, we have

$$\frac{d\sigma_r}{dQ} = -\frac{U_2}{U},$$

where,

$$U_2 = \sigma_r Y a_x^2 (\sigma_r P_r M_1 + Y M_1 + P_r)$$

and $U = 4A\sigma_r^3 + 3B\sigma_r^2 + 2C\sigma_r + D$

A, B, C, D are prescribed above the numerator will be positive and $d\sigma_r/df'$, $d\sigma_r/dQ$ may be positive or negative according to the denominator which should be either negative or positive. Therefore, the growth rate of unstable modes increase as well as decrease with the increase in number density of particles and magnetic field respectively.

If $\beta < 0$ (stable stratification) then the sufficient conditions for stability is

$$R < \max.\frac{1}{a^2}\left[\frac{f'Y^2}{M_1}, \frac{Qa_x^2 Y - R_1\beta a^2}{M_1 Y + P_r}\right]:$$

For stable stratification i.e. $\beta < 0$. Then the term $(Qa_x^2 Y - R_1\beta a^2)$ is definite positive and if $f'Y^2 - Ra^2 M_1 > 0$ as well as $D > 0$

which implies that

$$R < \max\frac{1}{a^2}\left\{\frac{f'Y^2}{M_1}, \frac{Qa_x^2 Y - R_1\beta a^2}{M_1 Y + P_r}\right\}$$

Therefore, equation (1) does not permit any positive root of σ, since all the coefficients of this equation have definite positive value. Hence, the above inequality is the sufficient condition for the stability of the system.

If $\beta > 0$ (unstable stratification), $E > 0$ and under the condition of above theorem, system is again stable:

Assuming that $E > 0$ i.e. $Qa_x^2Y - R_1\beta a^2 > 0$ and $\beta > 0$ then under the condition.

$$R < \max \frac{1}{a^2}\left[\frac{f'y^2}{M_1}, \frac{Qa_x^2Y - R_1\beta a^2}{M_1Y + P_r}\right]$$

the configuration is stable.

If $\beta > 0, E < 0$, the system is unstable:

For $\beta > 0$ (unstable stratifications) and assuming that $E < 0$ i.e. $Qa_x^2Y - R_1\beta a^2 < 0$. It is clear that the product of four roots of equation (1) is negative while the sum of roots is positive, therefore equation (1) contain at least one positive root. Hence the system is stable.

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