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Spreading dynamics of a well-known brand recognition model on scale-free networks

Youming Xue, Tao Li*
School of Electronics and Information, Yangtze University, Jingzhou 434023, PR China

Abstract:

In order to study the influence of the comment mechanism and the heterogeneity of underlying networks on the spreading of well-known brands' recognition, we present a new SFAD (Susceptible-Favorite-Amnesic-Disliked) well-known brands' recognition spreading model based on scale-free network. By using the mean-field theory, the spreading dynamics of the model is analyzed in detail. Then, the basic reproductive number R_0 and equilibriums are derived. The relationships among the basic reproduction number R_0, favorite rate, dislike rate are studied. Furthermore, the global stability of the information-elimination equilibrium the permanence of well-known brands' recognition spreading are proved in detail. Numerical simulations are performed to confirm the analytical results.

Keywords

Brand spreading, Scale-free networks, SFAD model, Heterogeneity, permanence

1. Introduction

With the rapid development of the information age, the Internet has penetrated into every aspect of people's daily life. Many information [1-2] on the Internet will affect people's perceptions and evaluations of some things. For example, many Internet celebrities can use their influence to greatly increase the sales of some branded items through the network. Conversely, if some brands gain a bad reputation on the Internet, they will greatly reduce the sales of the branded items. Network users are no longer a brand media audience that simply or passively receives information, but can voice themselves and become the evaluation party of brand information. Therefore, it is important to study the impact of brand reputation on the impact of brand communication on brand sales.

Researchers have made a lot of achievements in brand spreading. Regina Virvilaite and Dovile Tumasonyte studied the particularity of the influence of word-of-mouth communication on brand equity, confirming the positive correlation between word-of-mouth communication and brand equity dimension [3]. A (PWOM) mechanism was found that it can greatly enhance brand reputation through social networks [4]. Jing Yang and Hai Lin studied in the small world [5] and the scale-free network [6] in 2010 that the priority of brand information dissemination is widespread in social networks, and it is believed that priority does affect the communication process. They also studied the brand communication model on the network and proposed a critical threshold [7]. Christina A. Kuchmaner and Jennifer Wiggins proposed that in addition to the relationship between brand and consumption, there are many consumers communicating with each other on the Internet [8]. The study explored the role of network embedding, especially network centrality and network density, as well as consumer psychological ownership of brand violations. In the three studies of the virtual brand community and the offline brand community, the inherent conflicts of people occupying a central position in the brand community were recorded [9]. It is known to all that the brand is a kind of social information. Some researchers pay close attention to how to improve the efficiency of communication by adopting special strategies in order to apply which to systems such as broadcasting. There is also a large amount of literature research on brand communication. The article studies brand communication information on the network from various aspects such as loyalty, brand behavior, and priority [10-13], which ultimately explains that brand’s reputation in the people’s minds has a major role in brand’s final achievement.

From the above research work, the influence of brand word-of-mouth information dissemination on consumers on social networks has not been considered. Based on the results of previous researchers, this paper proposes different research directions to study the extent to which people's evaluation of brands on social networks can influence brand recognition in modern society. In fact, with the progress of the times, people pay more and more attention to the brand's reputation. The reputation of the brand in people's minds is largely derived from the word of mouth. Therefore, this paper attempts to study the influence of brand reputation on the brand on the basis of scale-free

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network, and proposes a new type of *SFAD* (Susceptible-Favorite-Amnesic-Disliked) brand communication dynamics model. As is known to all, the scale-free network feature is one of the important features of social networks. In order to further understand the dynamics of brand information in the network, the scale-free characteristics of the network need to be considered. In this network, nodes represent customers and boundaries represent relationships between customers. Regarding the dynamics of brand information, the relationship is that nodes forward brand information through mutual contact. Considering the heterogeneity of the underlying network and the attraction of brand information to customers, this paper studies a new type of brand information communication dynamics model and carries out detailed verification.

The remaining part of the paper has been arranged as follows: Section 2 presents the *SFAD* Word of Mouth Communication on Brand spreading model on scale-free networks. In Section 3, the basic reproductive number and equilibriums are obtained. Section4 presents the results of our numerical simulation. Finally, we conclude the paper in Section 5

2. Model formulation

In order to study the influence conditions of the spreading of well-known brand recognition in the crowd and the influence of network heterogeneity on the spreading of well-known brand recognition in the network, we propose a new brand spreading dynamics model *SFAD* (Susceptible-Favorite-Amnesic-Disliked). The spreading flow chart is shown in Fig.1.

We divide the crowd into four categories: Susceptible (denoted by S), Favorited (denoted by F), Amnesiac (denoted by A), Dislike (denoted by D). S stands for people who have never heard of the brand. F stands for brand enthusiasts, who very much agree with the brand, and will actively spread the brand information. A stands for former brand enthusiasts who, as the environment changes, now forget the brand. D stands for people who dislike this brand and don't spread the brand information.

In the process of spreading the recognition of well-known brands, some brand enthusiasts have α probability of changing into Amnesiac because of the passing of time or the decline of brand quality. There will also be some people who have ε probability of changing into Dislike because the users of the brand's products have some uncomfortable experience. There will also be β_1 probability that the susceptible will recognize the brand and become brand enthusiastists through the word-of-mouth spreading of the brand enthusiasts. Amnesiac will also have β_2 probability of recognizing the brand as favorite because of re-contact with brand enthusiasts. At the same time, we consider the immigration and outmigration of population in the network, and assume that the immigration and outmigration rates of population are equal as l. All the new immigrants in the network are susceptible.

Assuming that the parameters are normal, and the transformation relationship between various states is shown in the figure.

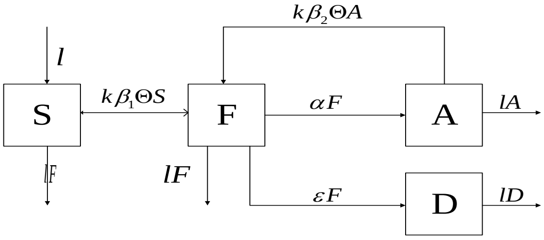

Figure 1.The flow diagram of the SAFD model

In scale-free networks, the degree of nodes is power-law distribution and the disturbance amplitude is large, so the heterogeneity of node degree must be considered. $S_k(t)$, $A_k(t)$, $F_k(t)$ and $D_k(t)$ which are all at k degrees respectively denote the densities of the four types of nodes at t time.

According to the equilibrium field theory, the following equations of propagation dynamics are obtained:

$$\begin{cases} \dfrac{dS_k(t)}{dt} = l - k\beta_1\Theta S_k(t) - lS_k(t) \\ \dfrac{dF_k(t)}{dt} = k\beta_1\Theta S_k(t) + k\beta_2\Theta A_k(t) - (\varepsilon+\alpha+l)\, F_k(t) \\ \dfrac{dA_k(t)}{dt} = \alpha F_k(t) - k\beta_2\Theta A_k(t) - lA_k(t) \\ \dfrac{dD_k(t)}{dt} = \varepsilon F_k(t) - lD_k(t) \end{cases}$$

(1)

Where $\Theta(t)$ represents the probability of random contact with brand enthusiasts at t time, which satisfies the relation:

$$\Theta(t) = \frac{1}{\langle k \rangle} \sum\nolimits_k kP(k)F_k(t)$$

(2)

$\langle k \rangle$ denotes the average of nodes in a network, $P(k)$ denotes the degree of distribution, $F(t) = \sum\nolimits_k P(k)F_k(t)$ represents the density of all favorite.

Thinking about the standard conditions, we can get:

$$S_k(t) + F_k(t) + A_k(t) + D_k(t) = 1 \qquad (3)$$

3. Basic Reproduction Number and Equilibrium Point

Theorem 1. Define the basic reproduction number

$$R_0 = \frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\beta_1}{\alpha + l + \varepsilon}$$

And there is always an equilibrium point of information disappearance $E_0(1,0,0,0)$ in the system (1). When $R_0 > 1$,System (1) has a unique information popularity equilibrium point $E_+(S_k^\infty, F_k^\infty, A_k^\infty, D_k^\infty)$.

Proof. In order to solve the information popular equilibrium point, let the equation set (1) equal to zero, we can obtain:

$$\begin{cases} l - k\beta_1 \Theta^\infty S_k^\infty - lS_k^\infty = 0 \\ k\beta_1 \Theta^\infty S_k^\infty + k\beta_2 \Theta^\infty A_k^\infty - (\varepsilon + \alpha + l) F_k^\infty = 0 \\ \alpha F_k^\infty - (k\beta_2 \Theta^\infty + l) A_k^\infty = 0 \\ \varepsilon F_k^\infty - lD_k^\infty = 0 \end{cases} \tag{4}$$

Therefore, we can get

$$A_k(t) = \frac{\alpha}{l + k\beta_2 \Theta} F_k(t) \qquad D_k(t) = \frac{l}{\varepsilon} F_k(t) \tag{5}$$

Since Θ^∞ is satisfied with the self-compatibility condition, the stable solution of the above equation is obtained.

$$\Theta^\infty = \frac{\frac{1}{\langle k \rangle} \sum_{k=1}^{n} k^2 P(k) l \beta_1 \Theta}{(\varepsilon + \alpha + l)(k\beta_1 \Theta + l)\left(1 - \frac{\alpha \beta_2 \Theta k}{(\alpha + l + \varepsilon)(k\beta_2 \Theta + l)}\right)} = f(\Theta^\infty) \tag{6}$$

It's obvious that Θ^∞ is a trivial solution to the equation, where the system has a unique equilibrium point, the disease-free equilibrium point as $E(1,0,0,0)$; If the equation is to have a non-trivial solution $\Theta^\infty \neq 0$, the following conditions must be satisfied

$$\frac{df(\Theta_1^\infty)}{d\Theta_1^\infty}\Big|_{\Theta_1^\infty = 0} > 1 \text{ and } f(1) \le 1$$

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$$\frac{df(\Theta^\infty)}{d\Theta}\Big|_{\Theta^\infty = 0} = \frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\beta_1}{\alpha + l + \varepsilon} > 1 \tag{7}$$

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$$\frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\beta_1}{\alpha + l + \varepsilon} > 1$$

Thus, the basic reproductive number of system (1) can be obtained:

$$R_0 = \frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\beta_1}{\alpha + l + \varepsilon} > 1$$

Therefore, only when $R_0 > 1$, system (1) has a unique non-trivial solution. If you put the nontrivial solution in equation (6) into equation (4), we can get F_k^∞ . By formula (5), we can easily get:

$$0 < S_k^\infty < 1, 0 < F_k^\infty < 1, 0 < A_k^\infty < 1, 0 < D_k^\infty < 1$$

At this point, well-known brand recognition equilibrium point $E_+(S_k^\infty, F_k^\infty, A_k^\infty, D_k^\infty)$ is obvious. Therefore, when the basic reproductive number $R_0 > 1$, there is only one well-known brand recognition equilibrium point $E_+(S_k^\infty, F_k^\infty, A_k^\infty, D_k^\infty)$. And that leads to the conclusion.

Theorem 2. The well-known brand recognition equilibrium point E_0 of *SFAD* system (4) is globally asymptotically stable when $R_0 < 1$, and it is unstable when $R_0 > 1$.

Proof. For $S_k(t)+F_k(t)+A_k(t)+D_k(t)=1$, if the values of $S_k(t)$, $F_k(t)$ and $A_k(t)$ are fixed there is only one corresponding $D_k(t)$, we will discuss the first three equations of (1)

$$\begin{cases} \frac{dS_k(t)}{dt} = l - k\beta_1 \Theta S_k(t) - lS_k(t) \\ \frac{dF_k(t)}{dt} = k\beta_1 \Theta S_k(t) + k\beta_2 \Theta A_k(t) - \varepsilon F_k(t) - \alpha F_k(t) - lF_k(t) \\ \frac{dA_k(t)}{dt} = \alpha F_k(t) - k\beta_2 \Theta A_k(t) - lA_k(t) \end{cases} \tag{8}$$

For simplicity, let $P_i = \frac{iP(i)}{\langle k \rangle}$ and $n = k_{\max}$. then, P_i is a function of degree k , k=1,2…, n . The Jacobian matrix of system (8) at the well-known brand recognition equilibrium is a $3n*3n$ matrix as follows:

$$J = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{pmatrix},$$

Where

$$A_{11} = \begin{pmatrix} -l & -\beta_1 P_1 & 0 \\ 0 & \beta_1 P_1 - (\alpha + \varepsilon + l) & 0 \\ 0 & \alpha & -l \end{pmatrix},$$

$$A_{1n} = \begin{pmatrix} 0 & -\beta_1 P_n & 0 \\ 0 & \beta_1 P_n & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_{n1}=\begin{pmatrix}0 & -\beta_1 nP_n & 0\\ 0 & \beta_1 nP_n-(\alpha+\varepsilon+l) & 0\\ 0 & \alpha & 0\end{pmatrix},$$

$$A_{nn}=\begin{pmatrix}-l & -\beta_1 nP_n & 0\\ 0 & \beta_1 nP_n-(\alpha+\varepsilon+l) & 0\\ 0 & \alpha & -l\end{pmatrix}.$$

The characteristic equation of the well-known brand recognition equilibrium is

$$(\lambda+l)^{n-1}(\lambda+l)^{n-1}(\lambda^2+b\lambda+c)=0$$

Where $b=2l-\beta_1\sum_{k=1}^{n}k^2P\langle k\rangle/\langle k\rangle$ and $c=l(\alpha+\varepsilon+l)-l\beta_1\sum_{k=1}^{n}k^2P\langle k\rangle/\langle k\rangle$.

Obviously,

$$\frac{\langle k^2\rangle}{k}=\frac{\sum_{i=1}^{n}i^2P(i)}{\langle k\rangle}=\frac{\sum_{i=1}^{n}iP_i(i)}{\langle k\rangle}=\sum_{i=1}^{n}P\,i,$$

if $R_0<1$, we can obtain

$$R_0=\frac{\beta_1}{(\alpha+\varepsilon+l)}\frac{\langle k^2\rangle}{\langle k\rangle}<1$$

This is to say,

$$(\alpha+\varepsilon+l)>\beta_1\sum_{i=1}^{n}iP_i$$

Which is equivalent to $c>0$. and $R_0<1$ also means

$$\frac{l(\alpha+\varepsilon+l)-l\beta_1\sum_{i=1}^{n}iP_i}{l}-\beta_1\sum_{i=1}^{n}iP_i>0$$

And we can get

$$\frac{l(\alpha+\varepsilon+l)-l\beta_1\sum_{i=1}^{n}iP_i}{l}<l(l+\alpha+\varepsilon)$$

This is, $b>0$. From the above analysis, we can draw that if $R_0<1$, the real eigenvalues λ of J are all negative, otherwise, there is a unique positive eigenvalues λ of J if and only if $R_0>1$. On the basis of the Perron-Frobenius theorem, this indicates that the maximal real part of all eigenvalues of J is positive if and only if. Thus, we can find the results of this is in accord with a theorem of Lajmanovich and York [14]. The proof of this theorem is completed now.

4. Numerical simulation

In this section, we validate the theoretical analysis results by numerical simulation. System (1) is considered to be in a scale-free network with a degree distribution of $P(k)=\omega k^{-3}$, where the parameter ω satisfies $\sum_{k=1}^{n}\omega k^{-3}=1$, n=1000.

In Fig.2, select $l=0.76,\varepsilon=0.2,\alpha=0.1,\beta_1=0.4$, and calculate the number of basic regeneration directly $R_0=0.5698<1$; In Fig.3, the parameter value is $l=0.76,\varepsilon=0.1,\alpha=0.1,\beta_1=0.65$, and calculate the basic reproductive number directly $R_0=0.9260<1$. Both figures describe that when the basic reproductive number $R_0<1$ occurs, the people who spread the brand information will eventually disappear, and the smaller the basic reproductive number , the more rapidly the brand information will disappear.

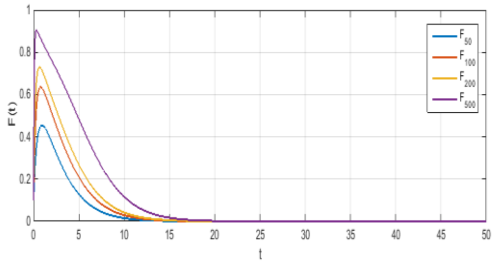

Figure 2. The time series and orbits of the favorited with $R_0=0.5698$ and initial value $F_k(0)$=0.1, $k=50,100,200,500$.

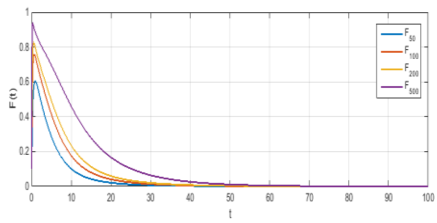

Figure 3. The time series and orbits of the favorited with $R_0=0.9260$ and initial value $F_k(0)$=0.1, $k=50,100,200,500$.

In Fig.4, we choose $l=0.76,\varepsilon=0.2,\alpha=0.1$, $\beta_1=0.9$ and calculate the basic reproductive number directly $R_0=1.1612>1$; In Fig.5, the parameter value is $l=0.76,\varepsilon=0.2,\alpha=0.1,\beta_1=0.9$, and calculate the basic reproductive number directly $R_0=3.0771>1$. Both figures describe that when the basic reproductive number $R_0>1$ occurs, the people who spread brand recognition information will always exist in the network and converge to a normal number. It can be found that the greater the degree of spreading of brand recognition information, the more people will eventually spread the recognition information, and thus more people will be affected to recognize the brand.

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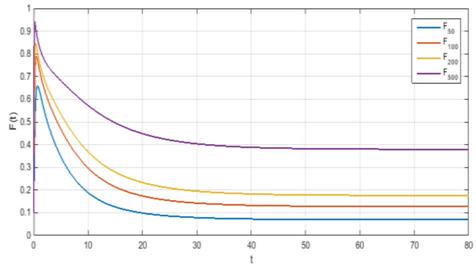

Figure 4. The time series and orbits of the favorited with $R_0 = 1.1612$ and initial value $F_k(0)$ =0.1, $k = 50,100,200,500$

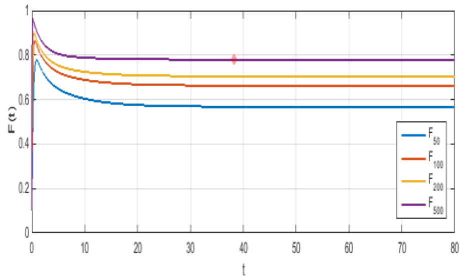

Figure 5. The time series and orbits of the favorited with $R_0 = 3.0771$ and initial value $F_k(0)$ =0.1, $k = 50,100,200,500$.

5. Conclusion

To study the spreading of well-known brand recognition on scale-free networks, a new *SFAD* model has been proposed. We determined the spreading dynamics of the model based on the basic reproductive number. If $R_0 < 1$, the equilibrium point is globally asymptotically stable, i.e. both the information of brand recognition will eventually disappear. If $R_0 > 1$, the favorited will persist and converge to a uniquely prevailing equilibrium level, respective. Through further research, it shows that if there are more brand lovers in the society, this will help to enhance the brand's popularity in social networks. If the number of brand enthusiasts increases, it will be more able to increase the brand's visibility in the community. Therefore, this study plays an important role in promoting brand recognition, this is, how to improve the social recognition of well-known brands is also of great significance to improve the social recognition of well-known brands is also of great significance to improve the social recognition of famous brands. The more brand favorited people in a scale-free network, the easier it is to increase brand awareness.

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