

## Plates Bending Analysis Using Finite Element Method

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### ABSTRACT:

Advent of approximate Numerical methods for the analysis of complicated problems has provided platform to the researchers and engineers. Towards this, advent of digital computer has increased exponentially the useful application of approximate methods. Using Finite Element Method, an approximate method, the analysis over the bending of plate has been done using four noded isoperimetric elements in this thesis. The element properties and some essential criterions like convergence, compatibility and geometric invariance has been described. Numerical difficulties like locking phenomenon and stress smoothing technique has also been overcome with Hinton & Campbell technique [1974]. Thin plate theory i.e. Kirchoff's and Mindlin theory along with Finite Element Formulation has been described. Variation of deflection and moments at various points on the plates with the variation of percentage error in respect of each variable have been shown in graphical form with the variation of number of elements. Effect of mesh fineness and element aspect ratio on the deflection as well as the stress resultants have been studied for various sizes of plates using Lisa/Ansys.

**Keywords:-Bending Analysis, Finite Element Method, Mindlin's Theory**

### INTRODUCTION

The present study is devoted to analyze plate with various boundary

conditions. A finite element based software package has been developed which can solve plates of any arbitrary shape. During last three decades finite element procedures have also been successfully employed to solve these problems. As a numerical solution to the problem a number of finite element models have been proposed. Of these, the present study will use four-noded isoperimetric plate elements. Detailed mathematical formulation has been presented in the following sections. The numerical difficulties like locking phenomenon, local stress smoothing has also been described. Vast literature based on classical plate theory exists for solving these commonly used structural elements. But the classical theory is very cumbersome for the arbitrary shape of plates and cannot be formulated. The three methods that are used are as

1. Functional approximation
2. Finite difference method
3. Finite element method

Out of these three methods Finite Element Method is mostly used because of its accuracy and simplicity than that of other two methods.

In functional approximation, a set of independent functions satisfying the boundary conditions is chosen and a linear combination of a finite number of them is taken to approximately specify the field variable at any point.

In the finite difference method the system is discretised by a mesh of nodal points. The

field variable is represented by the discrete value of the variables at the nodes.

In the finite element method the body is divided into a number of smaller elements which are called finite elements. For each element equilibrium equation is formulated and then combined for whole structure and the simultaneous equations are solved for unknowns.

### OBJECTIVES

The present study envisages fulfilling the following objectives:

1. To use four-noded isoperimetric plate element to analyze plates.
2. To develop a finite element based software package for analyzing plate bending problems.
3. Various possible boundary conditions for various shapes of plates will be investigated.
4. Results from analysis using above two plate element will be compared to see their relative performance with respect to classical solution.
5. To study the effect of mesh fineness and element aspect ratio on finite element solution.

### FINITE ELEMENT METHOD

Finite element analysis is a method for numerical solution of field problems. A field problem requires that determination of spatial distribution of one or more independent variables. Mathematically, a field problem is described by differential equations or by integral expressions.

The word finite distinguishes these pieces from infinitesimal elements used in calculus. In each finite element a field quantity is allowed to have only a simple spatial variation. The actual variation in the region spanned by an element is more complicated. So, FEA provides an approximate solution. The particular arrangement of an element is called mesh. Numerically an FEA mesh is represented by a system of algebraic equations to be solved for unknowns at the nodes. The solution for

the nodal quantities when combined with the assumed field in any given element, completely determines the spatial variations of the field in that element.

### ADVANTAGES OF FEA

1. FEA is applicable to all types of problem.
2. There is no geometric restriction.
3. Boundary conditions and loading are not restricted.
4. Material properties are not restricted to isotropy and may change from one element to other or even within an element.
5. The components have different mathematical behaviour and different mathematical descriptions, can be combined.

The FEA also suffers from mainly two types of errors, i.e. modeling error and discretization error. The modeling error can be reduced by improving the models and discretization error can be reduced by increasing elements.

### THEORY OF ELEMENT PROPERTIES

#### Isoperimetric element

For the element description, shape functions are used to interpolate both the displacement field and element geometry. That is, displacement of a point within an element can be expressed in terms of nodal degree of freedom and shape function  $[N]$ , which are the functions of reference coordinates. Similarly the global coordinates of a point within the element can be expressed in terms of global nodal positions and shape functions  $[\bar{N}]$ , which are also the functions of reference coordinates.

Symbolically

1. Nodal degree of freedom  $\{d\}$  defines displacements  $[u, v, w]$  of a point within the element, i.e.  $[u, v, w]^T = [N]\{d\}$ .

2. Nodal coordinates  $\{c\}$  defines coordinates  $[x, y, z]$  of a point within the element; i.e.  $[x, y, z]^T = [\bar{N}]\{c\}$ .

The shape function matrices  $[N]$  and  $[\bar{N}]$  are the functions of  $r, s$  and  $t$ . An element is called isoperimetric, if  $[N]$  and  $[\bar{N}]$  are identical. If  $[\bar{N}]$  is of lower degree than  $[N]$ , the element is called subparametric and if  $[\bar{N}]$  is of higher degree than  $[N]$ , the element is called super parametric.

### Bilinear quadrilateral (Q4)

This plane element is a generalisation of rectangular Q4 element that removes the restriction to rectangular shape. The locking effect also appears in this element. In this, in physical space, reference coordinates  $r$  &  $s$  need not be orthogonal unlike rectangular Q4 element and need not be parallel to Cartesian coordinates  $x$  and  $y$ . Element sides having coordinates  $r = \pm 1$  and  $s = \pm 1$  are bisected by axes  $r$  &  $s$  regardless of the shape or physical size of the element and regardless of its orientation in Cartesian coordinates. The point  $r = s = 0$  is normally the element centre, but in general it is not the centroid of the physical element. The displacements are directed parallel to Cartesian coordinates, not parallel to local coordinates  $r$  &  $s$ .

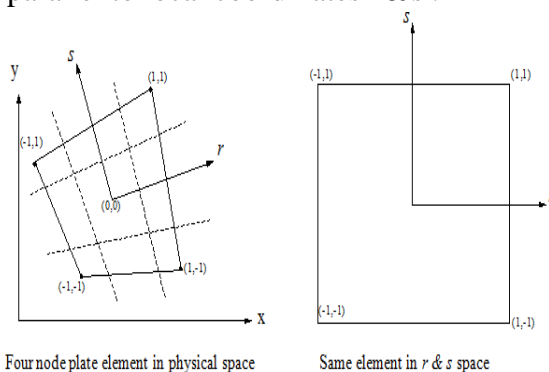


Fig. 3.1 An element in Natural Coordinate System

### STATIC CONDENSATION

The internal nodes of any element do not connect with any node of adjoining

elements in the assemblage. So, the degrees of freedom of such nodes do not appear in the compatibility conditions that are used to formulate the overall equations for the structure. The internal degrees of freedom can be eliminated from the equilibrium equations of each element so that these extra unknowns do not increase the number of overall equations.

### SOME ESSENTIAL CRITERIA

#### Convergence requirements

The finite element method provides a numerical solution to a complex problem. It is expected that the solution must converge to exact solution under certain circumstances. Hence as the mesh is made finer, the solution should converge to the correct result and this would be achieved if the three conditions are satisfied by assumed displacement functions.

1. The displacement function must be continuous within the element and it can be satisfied by choosing polynomials for the displacement model.
2. The element displacement function must be capable of representing rigid body displacements of the elements. That is when nodes are given such displacements corresponding to a rigid body motion, the element should not experience any strain.
3. The element displacement function must be capable of representing constant strain states within element. For one, two or three dimensional elasticity problems the linear terms present in the polynomial satisfy the requirement. However, in the case of beam, plate and shell elements, this condition will be referred to as 'constant curvature' instead of 'constant strains'.

#### Geometric invariance

Besides the convergence and compatibility requirements, another important consideration is choosing proper terms in polynomials expansion is that the element should have no preferred direction. This isotropy is known as geometric

isotropy or geometric invariance. Geometric invariance is achieved if the polynomial includes all the terms i.e. polynomial is complete one. Invariance may also be achieved if polynomial is balanced, in case all the terms can not be included.

For example

$$u = a_1 + a_2x + a_3y + a_4xy \quad (3.1)$$

It can be achieved from the Pascal triangle for two dimensional elements.

### NUMERICAL DIFFICULTIES

#### Locking phenomenon (URI & SRI)

A locking phenomenon is a well-known event in finite element applications. It may be due to shear locking or membrane locking, which results when proper order of integration is used to find out the stiffness matrix. When Q4 element bends, it displays shear strain as well as bending strain. When a Q4 element is bent, its top and bottom sides remain straight, and each node has only horizontal displacement of certain magnitude.

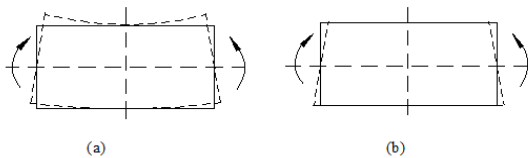


Fig. 3.2 (a) Deformation mode of rectangular block of material in pure bending. (b) Deformation mode of the Q4 element under bending load

Inclusion of transverse shear strains, in the equations presents computational difficulties when span to thickness ratio of plate is large.

For thin plates, the transverse shear strains are negligible and consequently the element stiffness matrix becomes stiff and yields erroneous results for the generalized displacements. This phenomenon is known as shear locking, and it can be interpreted as being caused by the inclusion of the following constraint in the variational form (Averill & Reddy, 1992)

$$\phi_x + \frac{\partial w}{\partial x} = 0 \quad \text{and} \quad \phi_y + \frac{\partial w}{\partial y} = 0 \quad (3.2)$$

If the plate is thick, the above condition does not satisfied and locking does not

occur. For thin plate the above constraints are valid but not satisfied in the numerical model. To avoid this locking phenomenon URI (uniformly reduced integration) or SRI (selective reduced integration) technique are used. When four noded element is used one point gauss rule should be used for shear energy terms while two point gauss rule should be used for all other terms.

#### Stress smoothing technique

Using displacement method, it was reported in the literature that the stresses obtained from finite element solutions are discontinuous between elements.

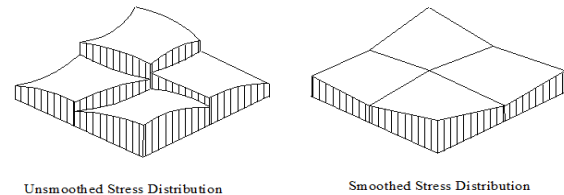


Fig. 3.3 Smoothed and Unsmoothed Stresses

Hinton & Campbell (1974) devised a local stress smoothing technique which is a natural method of sampling stresses in finite element using reduced integration. Here local smoothing is done by a bilinear extrapolation of the stress values computed at the Gauss points.

$$\begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_3 \\ \bar{\sigma}_4 \end{Bmatrix} = \begin{bmatrix} \left(1 + \frac{\sqrt{3}}{2}\right) & -\frac{1}{2} & \left(1 - \frac{\sqrt{3}}{2}\right) & -\frac{1}{2} \\ -\frac{1}{2} & \left(1 + \frac{\sqrt{3}}{2}\right) & -\frac{1}{2} & \left(1 - \frac{\sqrt{3}}{2}\right) \\ \left(1 - \frac{\sqrt{3}}{2}\right) & -\frac{1}{2} & \left(1 + \frac{\sqrt{3}}{2}\right) & -\frac{1}{2} \\ -\frac{1}{2} & \left(1 - \frac{\sqrt{3}}{2}\right) & -\frac{1}{2} & \left(1 + \frac{\sqrt{3}}{2}\right) \end{bmatrix} \begin{Bmatrix} \sigma_I \\ \sigma_{II} \\ \sigma_{III} \\ \sigma_{IV} \end{Bmatrix} \quad (3.3)$$

Where,  $\bar{\sigma}_1$  to  $\bar{\sigma}_4$  are the smoothed nodal values and  $\sigma_I$  to  $\sigma_{IV}$  are the stresses at the  $2 \times 2$  Gauss points.

### THIN PLATE THEORY

#### Love and Kirchhoff theory

The assumptions in classical plate theory, which is also known as Kirchhoff theory, are based upon the assumptions given by Bernoulli for beams. Love and Kirchhoff first applied these assumptions to the plate and shells. Assumptions are as follows:

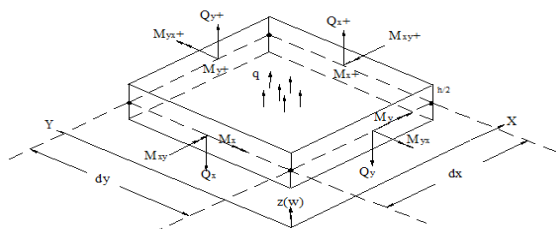
- After application of the external load, a lineal element of the plate normal to the mid surface
  - Undergoes at most a translation and rotation, which indicates that lineal element through the thickness does not elongate or contract and
  - Remains normal to the deformed mid-surface. It shows that shear strains  $\gamma_{xz}$  and  $\gamma_{yz}$  becomes zero.
- The stresses normal to the plate can be neglected, i.e.  $\sigma_z = 0$ .

### Basic Relationships

The stresses in thin plate vary linearly across the thickness and hence the stresses resultants can be computed as

$$\begin{aligned}
 M_x &= \int_{-h/2}^{+h/2} \sigma_x z dz \\
 M_y &= \int_{-h/2}^{+h/2} \sigma_y z dz \\
 M_{xy} = M_{yx} &= \int_{-h/2}^{+h/2} \tau_{xy} z dz \\
 Q_x &= \int_{-h/2}^{+h/2} \tau_{xz} dz \\
 Q_y &= \int_{-h/2}^{+h/2} \tau_{yz} dz
 \end{aligned} \tag{3.4}$$

and



Vectorial notation of external and internal forces on the element of the middle surface

Fig.3.4 Detailed illustration of various forces

if  $u$ ,  $v$  and  $w$  be the displacements at any point  $(x, y, z)$  then the displacement  $u$  and  $v$  across the thickness can be expressed in terms of the displacement  $w$  as

$$u = -z \frac{\partial w}{\partial x} \quad v = -z \frac{\partial w}{\partial y} \tag{3.5}$$

the strain distribution is given by

$$\begin{aligned}
 \epsilon_x &= \frac{\partial u}{\partial x} = -z \frac{\partial^2 w}{\partial x^2} = z k_x \\
 \epsilon_y &= \frac{\partial v}{\partial y} = -z \frac{\partial^2 w}{\partial y^2} = z k_y \\
 \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z \frac{\partial^2 w}{\partial x \partial y} = z k_{xy}
 \end{aligned} \tag{3.6}$$

and the shear strains  $\gamma_{xz} = \gamma_{yz} = 0$ . Thus problem reduced to plane stress problem.

The general constitutive law for plane stress is given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Conveniently in the case of plate,  $\{M\}$  can be written in place of  $\{\sigma\}$ .

Hence

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \tag{3.7}$$

i.e.  $\{M\} = [C_f] \{k_c\}$

In case of isotropic plates, the constitutive matrix is given by,

$$[C_f] = \frac{Eh^3}{12(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} \tag{3.8}$$

### Mindlin's Theory

This theory includes shear deformations in the plates, which was not considered in the Kirchhoff's theory. There are three assumptions in the Mindlin's theory of plates as given below:

- The deflections of plate,  $w$ , are small.
- Normal to the plate mid surface before deformation remains straight but is not necessarily normal to it after deformation.
- Stresses normal to the mid surface are negligible.

The average shear deformation,  $\phi_x$  and  $\phi_y$  are given by

$$\phi_x = \theta_y + \frac{\partial w}{\partial x} \quad \phi_y = -\theta_x + \frac{\partial w}{\partial y} \tag{3.9}$$

the strain energy for the thin plate due to shear be given as

$$U_s = \frac{1}{2} \alpha G A \iint_A [(\phi_x)^2 + (\phi_y)^2] dx dy \tag{3.10}$$

Where,  $G$  modulus of rigidity and is equal to  $E/2(1+\mu)$ .

Putting the value of  $\phi_x$  and  $\phi_y$  from equation (3.9) and  $G$  in equation (3.10) and simplifying, for an isotropic plate we



get:

$$U_s = \frac{Eh^3}{24(1-\mu^2)} \times \frac{6\alpha(1-\mu)}{h^2} \iint_A \left[ \left( \frac{\partial w}{\partial x} + \theta_y \right)^2 + \left( \frac{\partial w}{\partial y} - \theta_x \right)^2 \right] dx dy$$

The expression for the strain energy due to bending of an isotropic plate can be obtained as

$$U_b = \frac{Eh^3}{24(1-\mu^2)} \times \iint_A \left[ \left( \frac{\partial \theta_x}{\partial x} \right)^2 + 2\mu \left( \frac{\partial \theta_x}{\partial x} \right) \left( \frac{\partial \theta_y}{\partial y} \right) + \left( \frac{\partial \theta_y}{\partial y} \right)^2 + \frac{1-\mu}{2} \left( \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right)^2 \right] dx dy$$

The total strain energy is given by

$$U = U_b + U_s$$

Basic Relationships

The shear stresses  $\tau_{xz}$  &  $\tau_{yz}$  and shear deformations  $\phi_x$  &  $\phi_y$  are related as

$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} C_{44} & C_{45} \\ C_{54} & C_{55} \end{bmatrix} \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix}$$

the average shear deformations  $\phi_x$  &  $\phi_y$  are constant all over the thickness and allowing for warping of the x-section, the stress resultant  $Q_x$  &  $Q_y$  can be computed as

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \frac{Eh\alpha}{2(1+\mu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} \quad \text{or} \quad \{Q\} = [C_s] \{\phi\} \quad (3.11)$$

Where,  $\alpha$  is numerical correction factor used to represent the restraint of the x-section against warping. The value of  $\alpha$  commonly used is 5/6 but may be assumed between 2/3 for section having no restraint against warping and 1.0 for sections having complete restraint against warping.

The stress resultants  $\{M\}$  &  $\{Q\}$  can be combined and for homogeneous plate it can be expressed as:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} \frac{h^3}{12} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & h \begin{bmatrix} C_{44} & C_{45} \\ C_{54} & C_{55} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ \phi_x \\ \phi_y \end{Bmatrix}$$

For an isotropic plate, using specific values of coefficient for the consecutive matrix, we get

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} \frac{Eh^3}{12(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \frac{Eh}{2(1+\mu)} \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ \phi_x \\ \phi_y \end{Bmatrix}$$

In short the above relation can be written as

$$\begin{Bmatrix} \{M\} \\ \{Q\} \end{Bmatrix} = \begin{bmatrix} [C_f] & [O] \\ [O]^T & [C_s] \end{bmatrix} \begin{Bmatrix} \{k_c\} \\ \{\phi\} \end{Bmatrix}$$

FINITE ELEMENT FORMULATION FOR FOUR NODDED BILINEAR PLATE ELEMENT.

System of Coordinate Axes

Various coordinate system used in the present formulation are

- Natural coordinate system
- Local coordinate system
- Global coordinate system

In natural coordinate system the axes are taken as r and s and the coordinates of elements are taken as unity.

Geometry Representation

Element Geometry and Displacement Field: both the geometry of the element and displacement field is obtained using same polynomial functions (also called shape functions in FEM literature). Geometry is expressed by

$$\begin{aligned} x &= \sum_{i=1}^4 N_i x_i \\ y &= \sum_{i=1}^4 N_i y_i \end{aligned} \quad (3.13)$$

and displacement field is expressed as

$$\begin{aligned} w &= \sum_{i=1}^4 N_i w_i \\ \theta_x &= \sum_{i=1}^4 N_i \theta_{xi} \\ \theta_y &= \sum_{i=1}^4 N_i \theta_{yi} \end{aligned} \quad (3.14)$$

Where, the shape function for any  $i^{\text{th}}$  node is given by

$$N_i = \frac{1}{4} (1 + rr_i)(1 + ss_i)$$

### Displacement Vector

The typical nodal displacement vector for an element is given as

$$\{d\}^T = [w_1 \theta_{x1} \theta_{y1} w_2 \theta_{x2} \theta_{y2} w_3 \theta_{x3} \theta_{y3} w_4 \theta_{x4} \theta_{y4}] \quad (3.15)$$

The shape function used for describing the geometry of the element and displacement variation are expressed in natural coordinate (r,s). The relationship between two coordinate systems can be computed by using the chain rule of partial differentiation and is given below:

$$\begin{Bmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial s} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix} \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} = [J] \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix}$$

Where, [J] is Jacobian matrix. Hence, the derivatives with respect to Cartesian coordinate system can be given as

$$\begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial s} \end{Bmatrix}$$

Jacobian matrix for four noded element can be given as

$$[J] = \begin{bmatrix} \frac{\partial N_1}{\partial r} & \frac{\partial N_2}{\partial r} & \frac{\partial N_3}{\partial r} & \frac{\partial N_4}{\partial r} \\ \frac{\partial N_1}{\partial s} & \frac{\partial N_2}{\partial s} & \frac{\partial N_3}{\partial s} & \frac{\partial N_4}{\partial s} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} \quad (3.16)$$

$$\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial r} & \frac{\partial N_2}{\partial r} & \frac{\partial N_3}{\partial r} & \frac{\partial N_4}{\partial r} \\ \frac{\partial N_1}{\partial s} & \frac{\partial N_2}{\partial s} & \frac{\partial N_3}{\partial s} & \frac{\partial N_4}{\partial s} \end{bmatrix}$$

### 3.9.4 Strain displacement matrix [B]

The element curvature and shear deformation  $\{\varepsilon\}_p$  and nodal displacement  $\{d\}$  are related as

$$\{\varepsilon\}_p = [B]\{d\}$$

For getting the each terms of  $\{\varepsilon\}_p$  differentiate Eq. (3.14) w. r. t. x and y. we get

$$\begin{aligned} k_x &= \sum_{i=1}^4 \theta_{yi} \frac{\partial N_i}{\partial x} \\ k_{xy} &= \sum_{i=1}^4 \theta_{yi} \frac{\partial N_i}{\partial y} - \sum_{i=1}^4 \theta_{xi} \frac{\partial N_i}{\partial x} \\ \phi_x &= \sum_{i=1}^4 \omega_i \frac{\partial N_i}{\partial x} + \sum_{i=1}^4 \theta_{yi} N_i \\ \phi_y &= \sum_{i=1}^4 \omega_i \frac{\partial N_i}{\partial y} - \sum_{i=1}^4 \theta_{xi} N_i \end{aligned} \quad (3.17)$$

Where  $\frac{\partial N_i}{\partial x}$  and  $\frac{\partial N_i}{\partial y}$  are to be computed

from Eq. (3.16)

Eq. (3.17) shows that the elements of vector  $\{\varepsilon\}_p$  are expressed in terms of nodal displacements,  $w, \theta_{xi}$  and  $\theta_{yi}$ . Hence

$$\{\varepsilon\}_p = \begin{Bmatrix} k_x \\ k_y \\ \phi_x \\ \phi_y \end{Bmatrix} = [B] \begin{Bmatrix} \omega_1 \\ \theta_{x1} \\ \theta_{y1} \\ \dots \\ \theta_{x4} \\ \theta_{y4} \end{Bmatrix} \quad (3.18)$$

This equation can be expressed as

$$\{\varepsilon\}_p = \sum_{i=1}^4 [B_i] \{d_i\} \quad (3.19)$$

Where

$$[B_i] = \begin{bmatrix} 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & -\frac{\partial N_i}{\partial y} & 0 \\ 0 & \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial x} & 0 & N_i \\ \frac{\partial N_i}{\partial y} & -N_i & 0 \end{bmatrix}_{i=1,2,3,4} \quad (3.20)$$

and

$$\{d_i\} = \begin{Bmatrix} \omega_i \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix}$$

The stress resultant  $\{\sigma\}_p$  can be expressed in terms of nodal displacements as following

$$\begin{aligned} \{\sigma\}_p &= \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = [C]_p \{\varepsilon\}_p \\ &= [C]_p \sum_{i=1}^4 [B_i] \{d_i\} \end{aligned} \quad (3.21)$$

$$\text{or } \{\sigma\}_p = [C]_p [B] \{d\} \quad (3.22)$$

$$\text{Where, } [B] = \sum_{i=1}^4 [B_i] \quad (3.23)$$

### Element stiffness matrix

The element stiffness matrix  $[k]$  is given by

$$[k] = \iint_A [B]^T [C]_p [B] dx dy \quad (3.24)$$

The above expression in local coordinate is written as

$$[k] = \iint_A [B]^T [C]_p [B] |J| dr ds \quad (3.25)$$

The matrix  $[\bar{k}]$  can be written as sum of bending and shear contributions.

$$[\bar{k}] = [\bar{k}]_b + [\bar{k}]_s \quad (3.26)$$

Substituting  $[\bar{k}]$  for  $[B]^T [C]_p [B]$  in Eq.(3.25), the stiffness matrix is given by

$$[k] = \int_{-1}^{+1} \int_{-1}^{+1} [\bar{k}] |J| dr ds$$

$$[k] = \int_{-1}^{+1} \int_{-1}^{+1} \left[ [\bar{k}]_b + [\bar{k}]_s \right] |J| dr ds \quad (3.27)$$

Gauss quadrature rule is used to compute the stiffness matrix  $[k]$ .

### Element nodal load vector

The nodal load  $\{Q_i\}$  at the node  $i$  for a uniformly distributed load  $q$  is given by

$$\{Q_i\} = \begin{Bmatrix} F_z \\ M_x \\ M_y \end{Bmatrix} = \int_A N_i \begin{Bmatrix} q \\ 0 \\ 0 \end{Bmatrix} |J| dr ds \quad (3.28)$$

The element load vector  $\{Q\}$  can be calculated by combining the nodal load vectors  $\{Q_i\}$  as

$$\{Q\} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \end{Bmatrix} = q \sum_{i=1}^4 \sum_{j=1}^4 w_i w_j |J| \begin{Bmatrix} \tilde{N}_1 \\ \tilde{N}_2 \\ \tilde{N}_3 \\ \tilde{N}_4 \\ \tilde{N}_5 \\ \tilde{N}_6 \\ \tilde{N}_7 \\ \tilde{N}_8 \\ \tilde{N}_9 \\ \tilde{N}_{10} \\ \tilde{N}_{11} \\ \tilde{N}_{12} \end{Bmatrix} \quad (3.29)$$

Gauss quadrature of 2x2 sampling points is used to evaluate Eq. 3.29

### 3.9 RECTANGULAR AND CIRCULAR PLATE THEORY FOR ANALYTICAL SOLUTION

The equation for vertical deflection and moment resultants i.e.  $M_x$ ,  $M_y$ , and  $M_{xy}$  are given in the book 'Theory of Plates and Shells' by S. Timoshenko and S. W. Krieger second edition (1959). These equations for rectangular simply supported plates are given as

$$w = \frac{1}{\pi^4 D} \sum_{m=1,3,5,\dots} \sum_{n=1,3,5,\dots} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (3.30)$$

Where

$$a_{mn} = \frac{4q_0}{ab} \int_0^a \int_0^b \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dx dy = \frac{16q_0}{\pi^2 mn} \quad (3.31)$$

and  $m$  and  $n$  are odd integers. If  $m$  and  $n$  are both even numbers then  $a_{mn} = 0$ .

Moment  $M_x$  is represented as

$$M_x = \sum_{m=1,3,5,\dots} \sum_{n=1,3,5,\dots} \left( \frac{16q_0 \left( \frac{m^2 \pi^2}{a^2} + \mu \frac{n^2 \pi^2}{b^2} \right)}{m n \pi^2 \left( \frac{m^2 \pi^2}{a^2} + \frac{n^2 \pi^2}{b^2} \right)^2} \right) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (3.32)$$

$$M_y = \sum_{m=1,3,5,\dots} \sum_{n=1,3,5,\dots} \left( \frac{16q_0 \left( \mu \frac{m^2 \pi^2}{a^2} + \frac{n^2 \pi^2}{b^2} \right)}{m n \pi^2 \left( \frac{m^2 \pi^2}{a^2} + \frac{n^2 \pi^2}{b^2} \right)^2} \right) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

$$M_{xy} = \sum_{m=1,3,5,\dots} \sum_{n=1,3,5,\dots} \left( \frac{16q_0 (1-\mu)}{ab \left( \frac{m^2 \pi^2}{a^2} + \frac{n^2 \pi^2}{b^2} \right)^2} \right) \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \quad (3.34)$$

The deflection equation for simply supported circular plate is

$$w = \frac{q(a^2 - r^2)}{64D} \left( \frac{5 + \mu}{1 + \mu} a^2 - r^2 \right) \quad (3.35)$$

Radial moment  $M_r$  and  $M_t$  is given as

$$M_r = \frac{q}{16} (3 + \mu)(a^2 - r^2) \quad (3.36)$$

$$M_t = \frac{q}{16} [(3 + \mu)a^2 - r^2(1 + 3\mu)] \quad (3.37)$$

The deflection and moments for clamped circular plate are given as

$$w = \frac{q(a^2 - r^2)^2}{64D} \quad (3.38)$$

$$M_r = \frac{q}{16} [(1 + \mu)a^2 - r^2(3 + \mu)] \quad (3.39)$$

$$M_t = \frac{q}{16} [(1 + \mu)a^2 - r^2(1 + 3\mu)] \quad (3.40)$$

Where  $r$  is the radial distance of any point from centre of the plate and  $a$  is the radius of the circular plate.

### DESCRIPTION

#### GENERAL

The present study employs the Computer program PLATEFEM as provided by Krishnamoorthy (1994). The program is written in the standard FORTRAN language. Program presented here is only for the plate bending calculations, which includes deflections and stress resultants. The skyline concept has been used in this program for simplifying the cumbersome calculation procedure and associated storage



problems. An effective column storage scheme is adapted to store the global stiffness matrix elements as one dimensional array SK. This program is something different from a finite element program SAP in view of their capability to print the shear components, which is not included in SAP. The various subroutines are subroutine PASSIN, FELIB, COLUMH, CADNUM, PASSEM, PASOLV, PASLOD and DISP.

**MAIN PROGRAM**

The main program controls the various tasks of the program. It reads the control information namely the number of structure nodes (NSN), number of element types (NET), number of material property group (NMP) and number of load cases (NLC). It stores the dataset of all variables in the master array A and NA. The execution stops if the sum total of dimensions of all the variables i.e. floats or integer exceeds that of the master arrays A and NA. Three scratch files ISTRESS, NDARRAY and LEARRAY to store stress displacement matrix, degree of freedom and number of elements at particular nodes respectively. After reading the data execution is done in three segments according to the value of flag IND. The flow chart of main routine is given in Fig. 4.1

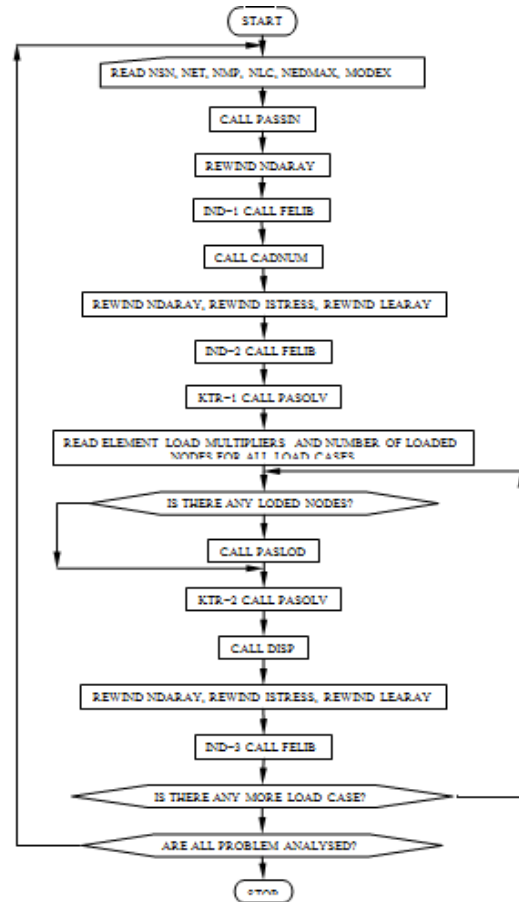


Fig. 4.1 Main Routine of the Program  
 Subroutine PASSIN

This subroutine is used to read, generate and print the nodal data. This subroutine is set up at maximum of six degree of freedom. Corresponding to each nodal point, it must then be identified which of these degrees of freedom shall actually be used in the finite element assemblage. The flow chart of this subroutine is given in Fig.2.

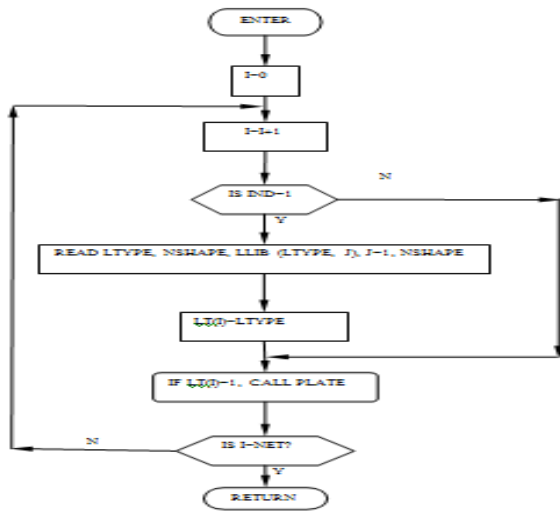


Fig. 3. Subroutine FELIB

### Subroutine COLUMNH

The subroutine COLUMNH computes the height of each column of the global stiffness matrix. The column height refers to the number of elements in each column below the sky line and above the leading diagonal excluding the diagonal element of the global stiffness matrix.

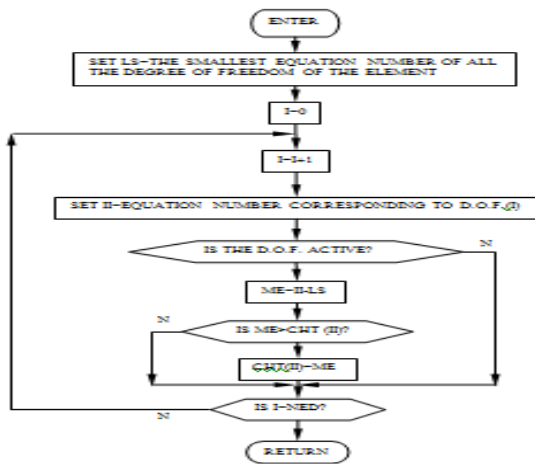


Fig. 4. SUBROUTINE COLUMNH

### Subroutine CADNUM

Subroutine CADNUM calculates the number of the diagonal element of the global stiffness matrix. The numbers of the diagonal elements of the global stiffness matrix are stored in a separate array called NDS. The number of diagonal element is obtained by adding the preceding column height one to its diagonal number, the

number of first diagonal element being taken as 1. The flow chart is given below.

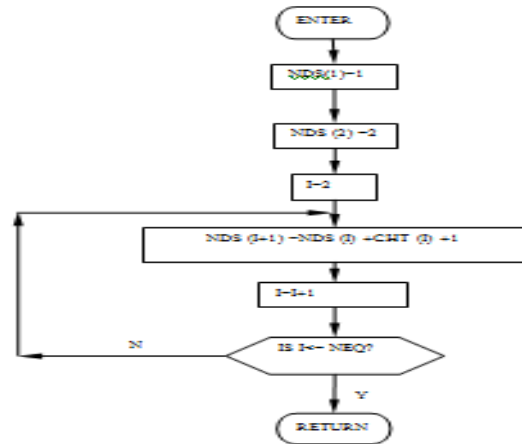


Fig. 5. Subroutine CADNUM

### Subroutine PASSEM

Subroutine PASSEM is called at the second stage i.e. at IND=2, by each element shape routine. It assembles the element stiffness matrix to global stiffness matrix in the form of one dimensional array SK. The flow chart of this subroutine is given below

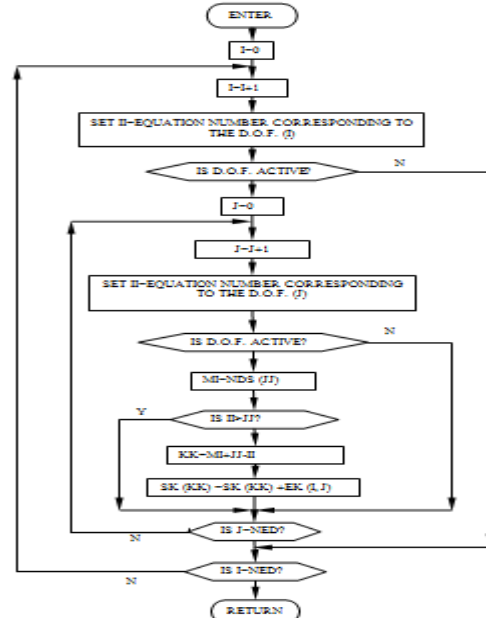


Fig.6. Subroutine PASSEM

### Subroutine PASOLV

Subroutine PASOLV is a direct solution routine, based on Gauss elimination technique. It solves the simultaneous equation for deflection of nodes along the free degree of freedom. The flow chart is given below in Fig.7.

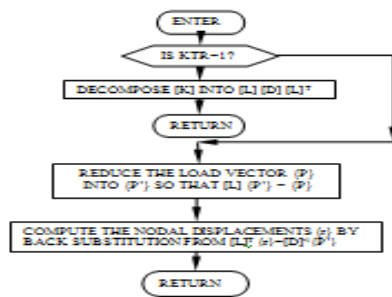


Fig 7. Subroutine PASOLV

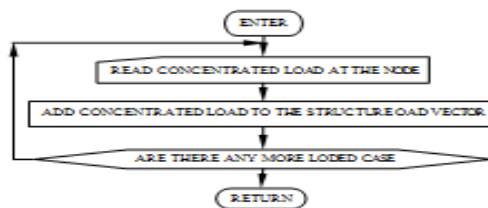


Fig 8. Subroutine PASLOD

### Subroutine PASLOD

The subroutine PASLOD is called by main routine for each load case to read and add the concentrated load in the direction of each degree of freedom of the node. These loads are added to the structure load vector from the element shapes.

### Subroutine DISP

Subroutine DISP prints out the displacements of each node. The displacement of each node is computed from the global displacement vector

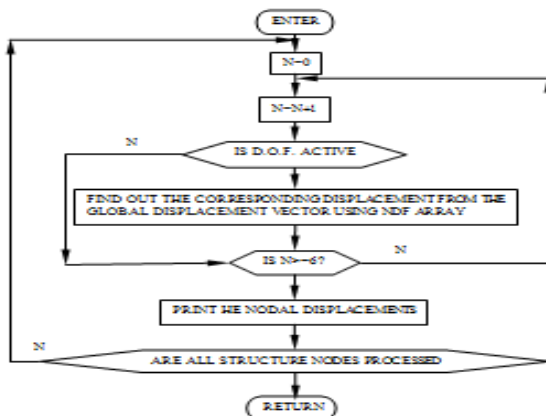


Fig 9. Subroutine DISP

### TYPICAL DETAILS OF INPUT DATA

This section describes a sample input for analysis of plates using computer program PLATEFEM. A typical input data has been presented in Appendix A. Herein

the brief description of variables along with their format is presented.

Read Statement No.	Description	Format
1.	Title	(20A4)
2.	NSN, NET, NMP, NLC, NEDMAX, MD OF, IPR where, NSN is Total Number of Nodes In Structure NET is Number of Element Types=1 for plate element NMP is Number of Materials Properties NLC is Number of Load Cases NEDMAX is Degree of Freedom At Each Node = 3 for NET = 1 IPR is counter to print out nodal data (if IPR = 0, print out; if ≠ 0, suppress	Unformatted
3.	NN, (JF(I), I=1,6), X(NN), Y(NN), Z(NN), NI, NYCL where, NN is Node Number JF(I), I=1,6 Six Boundary Conditions = 1, if d.o.f. is restrained = 0, if d.o.f. is not restrained X(NN) is X or r coordinate of node number Y(NN) is Y or θ coordinate of node number Z(NN) is Z coordinate of node number NI is node numbering increment between two nodes for automatic generation of nodal coordinates along a straight line. NYCL (if = 1, coordinates input are in cylindrical system)	Unformatted
4.	LE(I), PR(I), WD(I) PR(I) Poisson ratio of the material WD(I) Weight density of material	Unformatted
5.	NEL Total number of elements	Unformatted
6.	LNUM, MG, (MNC(J), J=1,4), TH, UDL, KI where, LNUM is Element Number MG is material group to which element belongs MNC (J), J=1,4 is Nodal Connectivity of the element nodes TH is Thickness of Element UDL is uniformly load applied in vertical direction KI is increment over each element for automatic data generation	Unformatted
7.	ELM(J), J=1, NLC where ELM is Element Load Multiplier, may be separate for each element.	Unformatted
8.	NLN (J), J=1, NLC where, NLN is the Loaded Node Number	Unformatted
9.	NODE, (CNL(I), I=1,6) where, NODE is Node Number CNL(I) is Concentrated Load at the Node Corresponding to six degrees of freedom i.e. F <sub>x</sub> , F <sub>y</sub> , F <sub>z</sub> , M <sub>x</sub> , M <sub>y</sub> , M <sub>z</sub>	Unformatted

### OUTPUT DETAILS

The output of program gives problem title, the controlling parameters like, number of structure nodes, element types, material groups and loading conditions. The nodal degrees of freedom

and the coordinates for each node is printed. The number of total equations and the total number of elements in global stiffness matrix are printed.

This program also prints the deflection at each node along three degrees of freedom i.e. one vertical deflection  $w$  and two rotations  $\theta_x$  and  $\theta_y$ . At each node the moments i.e.  $M_x$ ,  $M_y$  and  $M_{xy}$  are also printed. The constant shear force for each element is also printed in output file. A sample of output file is shown in Appendix A.

### **PROGRAM FOR ANALYTICAL SOLUTION**

For analytical solution a FORTRAN program has been written for simply supported rectangular plate and another program in FORTRAN is written in for simply supported and fixed circular plate. This program prints deflection and moments at any location as per requirements. The units of output file depend upon the units used in input data. These programs are shown in Appendix C.

### **RESULTS AND DISCUSSION**

In this chapter the problem for the validation of programme has been described. The results obtained from computer programme are compared with the solution obtained from analytical solution. The exact analytical solution procedure for plates of Timoshenko and Krieger (1959) has been used.

#### **PROBLEM FOR VALIDATION OF COMPUTER PROGRAM**

A simply supported square plate of 4.0m dimension is taken and analysed for the vertical deflection ( $w$ ) and moments  $M_x$ ,  $M_y$ , and  $M_{xy}$ . The thickness of plate is 0.12m and modulus of elasticity of material is taken as  $2 \times 10^7$  kN/m<sup>2</sup>. This result has been compared with the results obtained from the exact solution and it was found that computer programme is giving results close to those obtained from exact solution method. The discretisation scheme used is

shown in Fig. 5.1 and results for  $w$ ,  $M_x$  and  $M_{xy}$  are shown in Table 1(a) & (b).

#### **PLATE PROBLEMS ANALYSED**

For further analysis the rectangular plate of least dimension 4.0m with thickness 0.12m are taken with simply supported and fix boundary conditions. Other dimension of rectangular plate is decided on the basis of aspect ratios, viz., 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0. A set of circular plates of 4.0m diameter and thickness 0.12m are also analysed. The modulus of elasticity is taken as  $2.0 \times 10^7$  kN/m<sup>2</sup>.

#### **Plate boundaries and support conditions**

The different plate boundaries which are taken for the analysis are square, rectangular and circular, with all edges simply supported and all edges built-in. The plate boundaries are shown in Figures from Fig. 1 to Fig. 8. Support conditions for rectangular plate are shown in Fig. 5.1-5.4. Fig. 5.5 shows the one quadrant of the plate for both simply supported and fixed boundary. Fig. 6 shows the circular boundary for both simply supported and fixed boundary conditions.

#### **Discretization scheme**

The discretization schemes adopted for rectangular plate are 4x4, 6x6, 8x8 & 10x10. This means that the plate is divided into four parts in X and Y both directions for the discretisation of 4x4. Similarly for 6x6, 8x8 & 10x10 schemes the plate is divided into 6, 8 and 10 parts in both the directions. These different schemes are presented in Fig. 5.1 to Fig. 5.5 for rectangular plates and from Fig. 5.6 to Fig. 5.8 for circular plate. In Fig. 5.6, Fig. 5.7 and Fig. 5.8, the discretisation is shown for the one quadrant of circular plate. The actual boundary is shown in these figures in arc form, passing through outer nodes

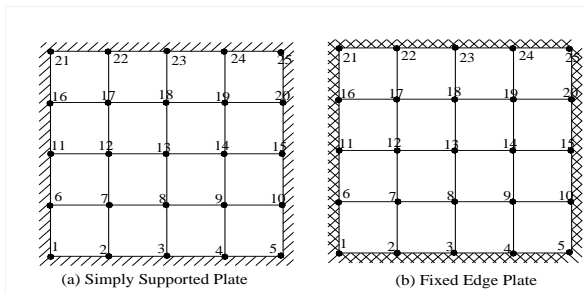


Fig. 5.1 Rectangular Plate (4x4 discretisation)

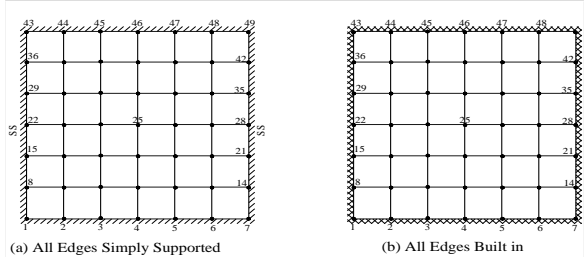


Fig. 5.2 Rectangular Plate (6x6 discretisation)

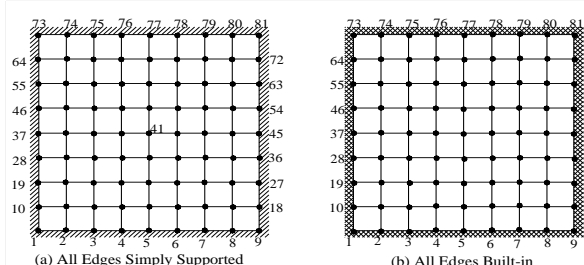


Fig. 5.3 Rectangular Plate (8x8 discretisation)

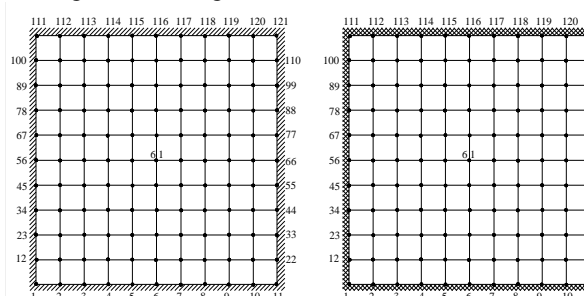


Fig. 5.4 Rectangular Plate (10x10 discretisation)

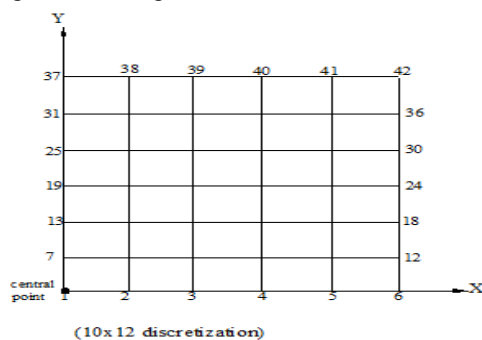


Fig. 5.5 Discretised figure of 4.0m x 4.8m size plate (one quadrant)

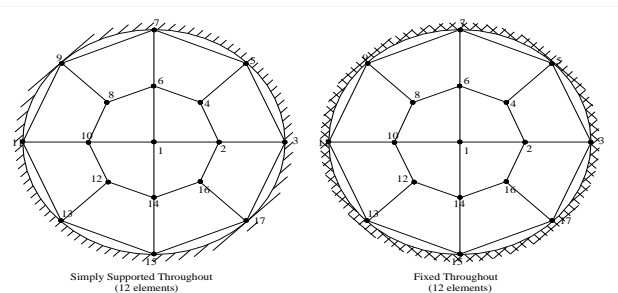


Fig. 5.6 Discretisation of circular plate of 4.0m dia.

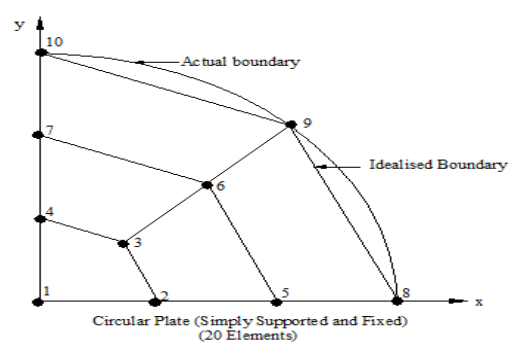


Fig. 5.7 Discretization of one quadrant of circular plate of 4.0m dia

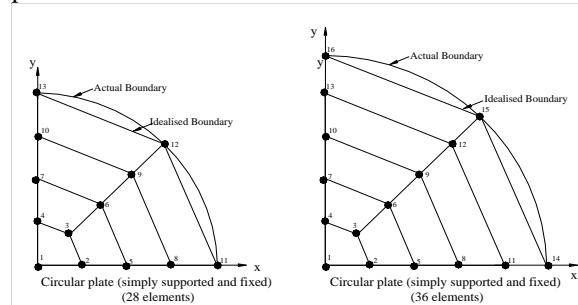


Fig. 5.8 Discretization of one quadrant of circular plate of 4.0m dia.

## RESULTS IN TABULAR FORM

The results obtained from the finite element analysis and analytical analysis for vertical deflections, moment resultants  $M_x$  and  $M_{xy}$  for simply supported and fixed edges rectangular plate are presented in tabular form in Table 1 to Table 13. Table 1 to Table 6 are the results for simply supported rectangular plate with the aspect ratio 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0. Table 7 presents the deflections and moments at various points in SS rectangular plate for plate aspect ratio 1.2 to 2.0, when elements aspect ratio is taken as 1.0. The results for fixed edges rectangular plate with the aspect ratios 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0 are



shown in Table 8 to Table 13. In Table 14 & 15 the vertical deflections and moment resultants  $M_x$  &  $M_y$  obtained from finite element solution and analytical solution are presented for circular plate with simply supported edges and fixed edges. The units used for vertical deflection ( $w$ ), moment resultants  $M_x$  and  $M_y$ , in case of rectangular plates and  $M_r$  and  $M_t$  in case of circular plates are millimetre (mm) and kN-m.

$M_y$  is not presented here because in square plate  $M_y$  is same as  $M_x$ . For rectangular plates the maximum value of  $M_y$  is same as the value of  $M_x$  of a square plate of dimension same as the shorter dimension of rectangular plates.

Table 5.1(a): Simply Supported plate with aspect ratio = 1.0,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	1.6	-3.10	0.00	1.20	-3.30	0.000	0.00	-3.70
0.67				-1.10	2.30	-2.80	-1.10	2.60	-2.90
1.00	-1.40	2.90	-2.10				-1.50	3.30	-2.30
1.34				-1.80	3.60	-1.50	-1.80	3.70	-1.60
2.00	-2.00	4.00	0.00	-2.10	4.00	0.00	-2.10	4.00	0.00

Table 5.1(b): Simply Supported plate with aspect ratio = 1.0,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	0.99	-3.50	0.00	0.83	-3.50	0.000	0.00	-3.70
0.40				-0.67	1.70	-3.30	-0.67	1.80	-3.40
0.50	-0.82	1.90	-3.10				-0.83	2.10	-3.20
0.80				-1.30	2.80	-2.60	-1.30	2.90	-2.70
1.00	-1.50	3.20	-2.20				-1.50	3.30	-2.30
1.20				-1.70	3.50	-1.80	-1.70	3.60	-1.80
1.50	-1.90	3.80	-1.20				-1.90	3.80	-1.20
1.60				-2.00	3.90	-0.93	-2.00	3.90	-0.94
2.00	-2.10	4.00	0.00	-2.10	4.00	0.00	-2.10	4.00	0.00

Table 5.2(a): Simply Supported plate with aspect ratio = 1.2,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	-2.20	-3.60	0.00	1.60	-3.90	0.00	0.00	-4.30
0.67				-1.50	3.00	-3.30	-1.50	3.30	-3.50
1.00	-2.00	3.80	-2.50				-2.10	4.30	-2.70
1.34				-2.50	4.80	-1.80	-2.50	4.90	-1.90
2.00	-2.80	5.30	0.00	-2.90	5.40	0.00	-2.90	5.40	0.00

Table 5.2(b): Simply Supported plate with aspect ratio = 1.2,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	1.30	-4.10	0.00	1.10	-4.10	0.00	0.00	-4.30
0.40				-0.92	2.10	-3.90	-0.92	2.30	-4.00
0.50	-1.10	2.50	-3.70				-1.10	2.70	-3.80
0.80				-1.70	3.70	-3.10	-1.70	3.80	-3.20
1.00	-2.10	4.20	-2.70				-2.10	4.30	-2.70
1.20				-2.40	4.70	-2.20	-2.40	4.70	-2.20
1.50	-2.70	5.10	-1.40				-2.70	5.10	-1.40
1.60				-2.80	5.20	-1.10	-2.70	5.20	-1.10
2.00	-2.90	5.40	0.00	-2.90	5.40	0.00	-2.90	5.40	0.00

Table 5.3(a): Simply Supported plate with aspect ratio = 1.4,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	2.70	-3.80	0.00	1.90	-4.20	0.00	0.00	-4.70
0.67				-1.80	3.60	-3.60	-1.70	4.00	-3.80
1.00	-2.50	4.70	-2.70				-2.60	5.20	-3.00
1.34				-3.10	5.90	-2.00	-3.10	6.00	-2.10
2.00	-3.50	6.50	0.00	-3.60	6.60	0.00	-3.60	6.70	0.00

Table 5.3(b): Simply Supported plate with aspect ratio = 1.4,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	1.50	-4.40	0.00	1.30	-4.50	0.00	0.00	-4.70
0.40				-1.10	2.50	-4.20	-1.10	2.70	-4.40
0.50	-1.40	3.00	-4.00				-1.40	3.20	-4.20
0.80				-2.20	4.40	-3.50	-2.20	4.50	-3.50
1.00	-2.60	5.10	-2.90				-2.60	5.20	-3.00
1.20				-3.00	5.70	-2.40	-2.90	5.80	-2.50
1.50	-3.30	6.30	-1.50				-3.30	6.30	-1.60
1.60				-3.40	6.40	-1.20	-3.40	6.40	-1.30
2.00	-3.60	6.60	0.00	-3.60	6.70	0.00	-3.60	6.70	0.00

Table 5.4(a): Simply Supported plate with aspect ratio = 1.6,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	3.10	-4.00	0.00	2.20	-4.40	0.00	0.00	-5.00
0.67				-2.10	4.10	-3.80	-2.20	4.50	-4.10
1.00	-3.00	5.40	-2.80				-3.00	6.00	-3.20
1.34				-3.70	6.80	-2.10	-3.70	7.00	-2.20
2.00	-4.20	7.50	0.00	-4.20	7.70	0.00	-4.20	7.70	0.00

Table 5.4(b): Simply Supported plate with aspect ratio = 1.6,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	1.80	-4.60	0.00	1.50	-4.80	0.00	0.00	-5.00
0.40				-1.30	2.80	-4.40	-1.30	3.00	-4.60
0.50	-1.60	3.40	-4.20				-1.70	3.60	-4.40
0.80				-2.50	5.10	-3.60	-2.50	5.20	-3.70
1.00	-3.00	5.80	-3.10				-3.00	6.00	-3.20
1.20				-3.50	6.60	-2.60	-3.50	6.60	-2.60
1.50	-3.90	7.20	-1.60				-3.90	7.30	-1.70
1.60				-4.00	7.40	-1.30	-4.00	7.50	-1.30
2.00	-4.20	7.70	0.00	-4.20	7.70	0.00	-4.20	7.70	0.00

Table 5.5(a): Simply Supported plate with aspect ratio = 1.8,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	3.50	-4.00	0.00	2.50	-4.50	0.00	0.00	-5.20
0.67				-2.40	4.60	-3.80	-2.40	5.00	-4.20
1.00	-3.30	6.00	-2.80				-3.40	6.60	-3.30
1.34				-4.10	7.50	-2.10	-4.10	7.70	-2.30
2.00	-4.70	8.40	0.00	-4.70	8.50	0.00	-4.70	8.60	0.00

Table 5.5(b): Simply Supported plate with aspect ratio = 1.8,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	1.90	-4.80	0.00	1.60	-4.90	0.00	0.00	-5.20
0.40				-1.50	3.10	-4.60	-1.50	3.30	-4.80
0.50	-1.80	3.70	-4.30				-1.90	3.90	-4.60
0.80				-2.80	5.60	-3.80	-2.80	5.70	-3.50
1.00	-3.40	6.40	-3.20				-3.40	6.60	-3.30
1.20				-3.90	7.30	-2.60	-3.90	7.30	-2.70
1.50	-4.40	8.00	-1.70				-4.40	8.10	-1.70
1.60				-4.50	8.30	-1.40	-4.50	8.30	-1.40
2.00	-4.70	8.60	0.00	-4.80	8.60	0.00	-4.70	8.60	0.00

Table 5.6(a): Simply Supported plate with aspect ratio = 2.0,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	$M_x$	$M_y$
	w	$M_x$	$M_y$	w	$M_x$	$M_y$			
0.00	0.00	3.80	-0.40	0.00	2.70	-4.60	0.00	0.00	-5.30
0.67				-2.60	4.90	-3.90	-2.60	5.30	-4.30
1.00	-3.60	6.50	-2.80				-3.70	7.10	-3.40
1.34				-4.50	8.10	-2.20	-4.50	8.30	-2.30
2.00	-5.10	9.10	0.00	-5.10	9.20	0.00	-5.20	9.30	0.00

Table 5.6(b): Simply Supported plate with aspect ratio =2.0,  $\mu = 0.15$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	2.10	-4.80	0.00	1.70	-4.90	0.00	0.00	-5.30
0.40				-1.60	3.30	-4.60	-1.60	3.50	-4.90
0.50	-2.00	4.00	-4.40				-2.00	4.20	-4.70
0.80				-3.10	6.00	-3.80	-3.10	6.10	-4.00
1.00	-3.70	6.90	-3.20				-3.70	7.10	-3.40
1.20				-4.20	7.80	-2.70	-4.20	7.90	-2.80
1.50	-4.80	8.70	-1.70				-4.80	8.80	-1.80
1.60				-4.90	8.90	-1.40	-4.90	9.00	-1.40
2.00	-5.20	9.20	0.00	-5.20	9.30	0.00	-5.20	9.30	0.00

Table 5.7(a): Simply Supported plate with aspect ratio =1.2 to 2.0,  $\mu = 0.15$

x(m)	10x12 discretization			10x14 discretization			10x16 discretization		
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>
0.00	0.00	1.10	4.20	0.00	1.30	4.60	0.00	1.50	4.80
0.40	-0.92	2.10	3.90	-1.10	2.50	4.30	-1.30	2.80	4.50
0.80	-1.70	3.70	3.10	-2.20	4.40	3.50	-2.50	5.00	3.70
1.20	-2.40	4.70	2.20	-2.90	5.70	2.40	-3.40	6.50	2.60
1.60	-2.80	5.20	1.10	-3.40	6.40	1.30	-4.00	7.40	1.30
2.00	-2.90	5.40	0.29	-3.60	6.60	0.32	-4.20	7.70	0.34

Table 5.7(b): Simply Supported plate with aspect ratio =1.2 to 2.0,  $\mu = 0.15$

x(m)	10x18 discretization			10x20 discretization		
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>
0.00	0.00	1.60	5.00	0.00	1.70	5.10
0.40	-1.50	3.10	4.70	-1.60	3.30	4.70
0.80	-2.80	5.50	3.80	-3.10	5.90	3.90
1.20	-3.90	7.20	2.70	-4.20	7.80	2.70
1.60	-4.50	8.20	1.40	-4.90	8.90	1.40
2.00	-4.70	8.50	0.35	-5.10	9.20	0.36

Table 5.8(a): Clamped plate with aspect ratio =1.0,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-1.80	0.00	0.00	-2.50	0.00	0.00	-4.83	
0.67				-0.18	-0.70	-0.28			
1.00	-0.30	0.23	-0.33						
1.34				-0.48	1.70	-0.19			
2.00	-0.59	2.30	0.00	-0.60	2.20	0.00	-0.644	2.17	

Table 5.8(b): Clamped plate with aspect ratio =1.0,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-2.90	0.00	0.00	-3.20	0.00	0.00	-4.83	
0.40				-0.086	-1.90	-0.18			
0.50	-0.12	-1.40	-0.22						
0.80				-0.27	0.27	-0.21			
1.00	-0.35	0.89	-0.22						
1.20				-0.44	1.40	-0.16			
1.50	-0.54	1.90	-0.12						
1.60				-0.57	2.00	-0.084			
2.00	-0.64	2.20	0.00	-0.61	2.20	0.00	-0.644	2.17	

Table 5.9(a): Clamped plate with aspect ratio =1.2,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-2.50	0.00	0.00	-3.33	0.00	0.00	-6.01	
0.67				-0.24	-1.10	-0.31			
1.00	-0.42	0.32	-0.37						
1.34				-0.65	2.00	-0.22			
2.00	-0.81	3.00	0.00	-0.82	2.90	0.00	-0.879	2.81	

Table 5.9(b): Clamped plate with aspect ratio =1.2,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-3.80	0.00	0.00	-4.20	0.00	0.00	-6.01	
0.40				-0.11	-2.60	-0.20			
0.50	-0.16	-1.90	-0.25						
0.80				-0.35	0.090	-0.25			
1.00	-0.47	0.94	-0.26						
1.20				-0.59	1.70	-0.19			
1.50	-0.73	2.40	-0.15						
1.60				-0.77	2.60	-0.10			
2.00	-0.82	2.80	0.00	-0.83	2.80	0.00	-0.879	2.81	

Table 5.10(a): Clamped plate with aspect ratio =1.4,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-2.90	0.00	0.00	-3.90	0.00	0.00	-6.83	
0.67				-0.28	-1.40	-0.33			
1.00	-0.49	0.24	-0.38						
1.34				-0.77	2.20	-0.24			
2.00	-0.96	3.40	0.00	-0.97	3.30	0.00	1.06	3.28	

Table 5.10(b): Clamped plate with aspect ratio =1.4,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-4.40	0.00	0.00	-4.90	0.00	0.00	-6.83	
0.40				-0.13	-3.10	-0.22			
0.50	-0.18	-2.40	-0.27						
0.80				-0.41	-0.079	-0.27			
1.00	-0.56	0.94	-0.28						
1.20				-0.70	1.90	-0.22			
1.50	-0.87	2.70	-0.16						
1.60				-0.91	3.00	-0.12			
2.00	-0.98	3.30	0.00	-0.99	3.30	0.00	1.06	3.28	

Table 5.11(a): Clamped plate with aspect ratio =1.6,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-3.30	0.00	0.00	-4.30	0.00	0.00	-7.33	
0.67				-0.31	-1.60	-0.34			
1.00	-0.54	0.20	-0.38						
1.34				-0.85	2.40	-0.25			
2.00	-1.10	3.70	0.00	-1.10	3.60	0.00	1.17	3.58	

Table 5.11(b): Clamped plate with aspect ratio =1.6,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-4.80	0.00	0.00	-5.30	0.00	0.00	-7.33	
0.40				-0.14	-3.40	-0.23			
0.50	-0.20	-2.70	-0.28						
0.80				-0.45	-0.22	-0.28			
1.00	-0.61	0.90	-0.29						
1.20				-0.78	1.90	-0.23			
1.50	-0.96	2.90	-0.17						
1.60				-1.10	3.20	-0.13			
2.00	-1.10	3.60	0.00	-1.10	3.60	0.00	1.17	3.58	

Table 5.12(a): Clamped plate with aspect ratio =1.8,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-3.60	0.00	0.00	-4.50	0.00	0.00	-7.64	
0.67				-0.32	-1.70	-0.34			
1.00	-0.58	0.17	-0.36						
1.34				-0.89	2.40	-0.25			
2.00	-1.20	3.90	0.00	-1.10	3.70	0.00	1.251	3.77	

Table 5.12(b): Clamped plate with aspect ratio =1.8,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-5.10	0.00	0.00	-5.60	0.00	0.00	-7.64	
0.40				-0.15	-3.60	-0.24			
0.50	-0.21	-2.80	-0.29						
0.80				-0.47	-0.32	-0.30			
1.00	-0.64	0.86	-0.30						
1.20				-0.82	1.90	-0.24			
1.50	-1.00	3.00	-0.18						
1.60				-1.10	3.30	-0.13			
2.00	-1.20	3.70	0.00	-1.20	3.80	0.00	1.251	3.77	

Table 5.13(a): Clamped plate with aspect ratio =2.0,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	4 x 4 discretisation			6 x 6 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-3.70	0.00	0.00	-4.60	0.00	0.00	-7.80	
0.67				-0.33	-1.80	-0.34			
1.00	-0.61	0.15	-0.34						
1.34				-0.91	2.40	-0.25			
2.00	-1.20	4.00	0.00	-1.20	3.80	0.00	1.298	3.88	

Table 5.13(b): Clamped plate with aspect ratio =2.0,  $\mu = 0.30$

x(m)	Finite Element Solution						Analytical solution		
	8 x 8 discretisation			10 x 10 discretisation			w	M <sub>x</sub>	M <sub>xy</sub>
	w	M <sub>x</sub>	M <sub>xy</sub>	w	M <sub>x</sub>	M <sub>xy</sub>			
0.00	0.00	-5.20	0.00	0.00	-5.70	0.00	0.00	-7.80	
0.40				-0.15	-3.80	-0.25			
0.50	-0.21	-2.90	-0.25						
0.80				-0.49	-0.39	-0.30			
1.00	-0.66	0.81	-0.31						
1.20				-0.84	2.00	-0.25			
1.50	-1.10	3.00	-0.18						
1.60				-1.10	3.40	-0.14			
2.00	-1.20	3.80	0.00	-1.20	3.80	0.00	1.298	3.88	

Table 5.14(a): Circular plate (simply supported), diameter=4.0m,  $\mu = 0.15$

R(m)	Finite Element Solution						Analytical		
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Table 5.14(b): Circular plate (simply supported) diameter=4.0m,  $\mu = 0.15$

R(m)	Finite Element Solution						Analytical solution		
	28 elements			36 elements			w	$M_x$	$M_y$
0.00	1.70	3.90	3.90	1.60	4.00	4.00	2.20	4.60	4.60
0.40				1.50	3.70	3.90	2.10	4.40	4.50
0.50	1.50	3.60	3.90				2.10	4.30	4.50
0.80				1.30	3.20	3.60	1.80	3.90	4.30
1.00	1.10	2.80	3.40				1.60	3.50	4.10
1.20				0.94	2.40	3.20	1.30	3.00	3.90

1.50	0.61	1.60	2.80				0.85	2.00	3.40
1.60				0.49	1.30	2.70	0.69	1.70	3.30
2.00	0.00	0.77	2.20	0.00	0.66	2.20	0.00	0.00	2.50

Table 5.15(a): Circular plate (clamped boundaries), diameter=4.0m,  $\mu = 0.15$

R(m)	Finite Element Solution						Analytical solution		
	12 elements			20 elements			w	$M_x$	$M_y$
0.00	0.43	1.40	1.40	0.40	1.50	1.50	0.50	1.70	1.70
0.67				0.28	0.94	1.30	0.39	1.20	1.50
1.00	0.18	0.17	1.10				0.28	0.53	1.20
1.34				0.10	-0.55	0.60	0.15	-0.37	0.74
2.00	0.00	-1.30	-0.28	0.00	-1.50	-0.27	0.00	-2.90	-0.44

Table 5.15(b): Circular plate (clamped boundaries), diameter=4.0m,  $\mu = 0.15$

R(m)	Finite Element Solution						Analytical solution		
	28 elements			36 elements			w	$M_x$	$M_y$
0.00	0.38	1.50	1.50	0.38	1.50	1.50	0.50	1.70	1.70
0.40				0.33	1.30	1.40	0.46	1.50	1.60
0.50	0.32	1.20	1.40				0.44	1.40	1.60
0.80				0.25	0.74	1.20	0.30	0.95	1.30
1.00	0.20	0.34	0.98				0.28	0.53	1.20
1.20				0.14	-0.056	0.77	0.20	0.024	0.92
1.50	0.064	-0.90	0.39				0.095	-0.91	0.49
1.60				0.044	-1.20	0.25	0.065	-1.30	0.35
2.00	0.00	-1.70	-0.28	0.00	-1.80	-0.29	0.00	-2.90	-0.44

## RESULTS IN FIGURES

The results tabulated are presented in graphical form for variation of deflections and moments along the central line of the square and circular plates only with simply supported and fix boundary conditions for the sake of convenience. Descriptions of Figures for deflection and moment patterns are given below from Fig. 5.9 to Fig. 5.19.

### Variation of deflection

Fig. 5.9 shows the variation of vertical deflection  $w$  along central line in SS rectangular plate with various discretisations. It shows very close variation of finite element solution with analytical solution. Fig. 5.12 shows the variation of displacements along central line in fixed square plate. Fig. 5.14 represents the variation of vertical deflection along radius of simply supported circular plate. Fig. 5.17 shows the variation of deflection along the radius of circular plate with fixed boundaries. This graph represents that the deflections are close to analytical values but not tend to converge with increasing the number of elements.

### Variation of Moment ( $M_x$ and $M_{xy}$ )

Fig. 5.10 shows the variation of  $M_x$  along central line in SS rectangular plate with different discretisation. This figure represents the convergence of FEM value to analytical value with increasing the number of elements. Fig 5.13 represents the variation of moments  $M_x$  in fixed rectangular plate. For positive moment the value from FEM is very close to analytical value but for support moment these values differ considerably. Fig. 5.11 represents the variation of  $M_{xy}$  along the SS edge. This represents that the results from FEM converges to analytical solution with increasing the number of elements.

### Variation of radial moment ( $M_r$ )

The variation of radial moment  $M_r$  along radius is presented in Fig 5.15. The results obtained from FEM are very near to analytical solution but to not fully converge with increasing the number of elements. Fig. 5.18 represents the variation of  $M_r$  along the radius of fix circular plate.

### Variation of tangential moment ( $M_t$ )

Fig. 5.16 shows the variation of tangential moment  $M_t$  along radius in SS circular plate. The FE solution follow the same pattern as exact solution but have approximately uniform difference between the FEM values and exact values. Variation of  $M_t$  along radius of fixed circular plate is presented in Fig. 5.19.

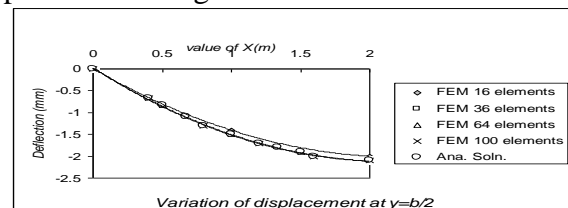


Fig. 5.9 Variation of displacement  $w$ , along mid-line (Simply supported rectangular plate)

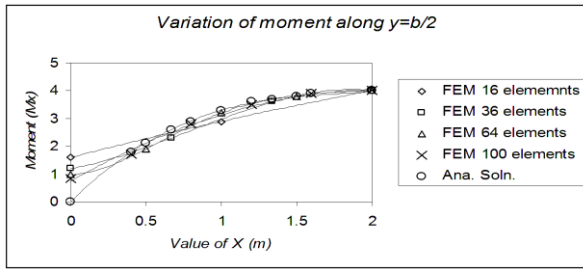


Fig. 5.10 Variation of  $M_x$  along mid-line (Simply supported rectangular plate)

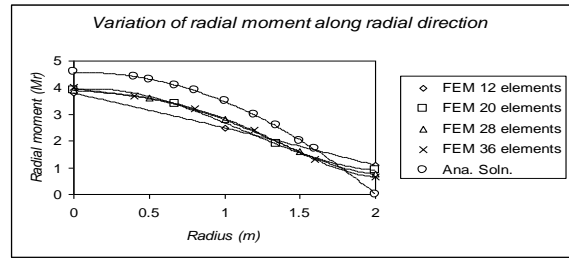


Fig. 5.15 Variation of radial moment  $M_r$  along radius (Simply supported circular plate)

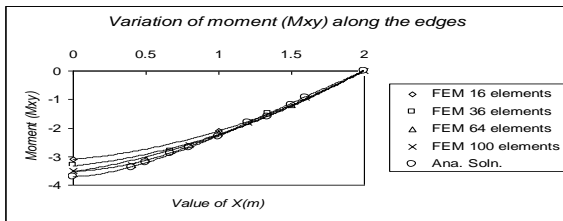


Fig. 5.11 Variation of moment about X-axis along edges (Simply supported rectangular plate)

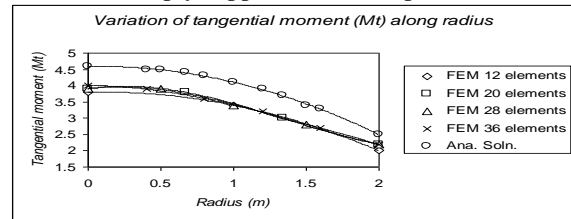


Fig. 5.16 Variation of tangential moment  $M_t$  along radius (Simply supported circular plate)

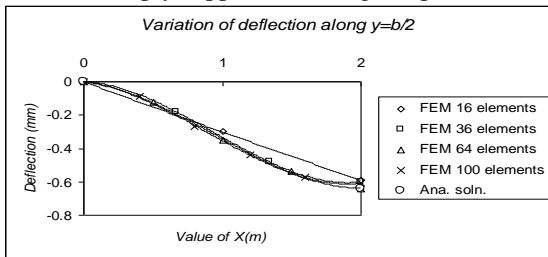


Fig. 5.12 Variation of  $w$  along mid-line (Clamped Rectangular plate)

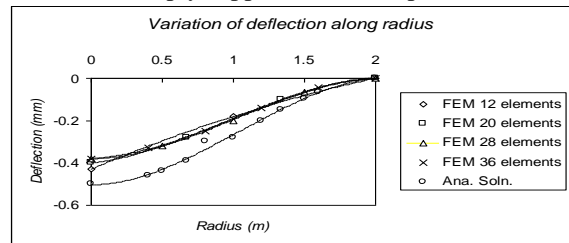


Fig. 5.17 Variation of vertical deflection  $w$  along radius (Fixed circular plate)

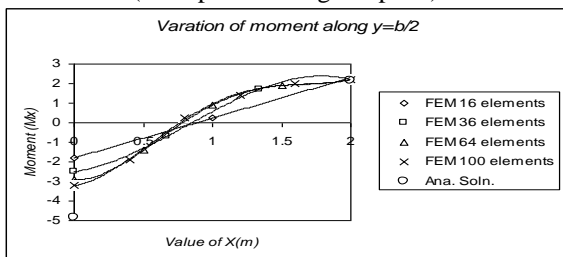


Fig. 5.13 Variation of  $M_x$  along mid-line (Clamped Rectangular plate)

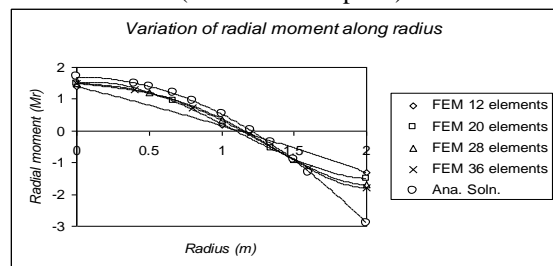


Fig. 5.18 Variation of radial moment  $M_r$  along radius (Fixed circular plate)

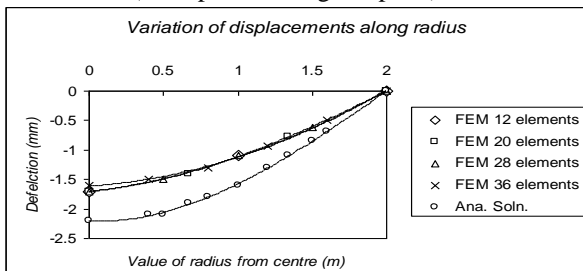


Fig. 5.14 Variation of vertical deflection  $w$  along radius (Simply supported circular plate)

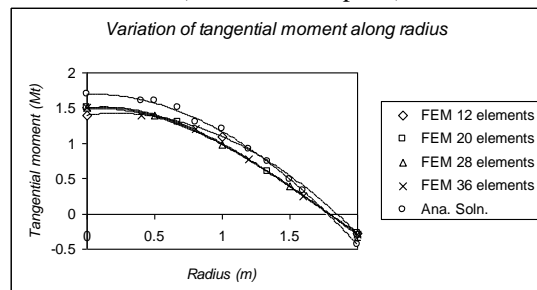


Fig. 5.19 Variation of tangential moment along radius (Fixed circular plate)



## VARIATION OF VALUES AT SELECTED POINTS

The description of figures from Fig. 20 to Fig. 31 for the values of deflections and moments at particular points are given below.

### Deflection variation

Fig. 5.20 shows the variation of deflection at the centre of SS rectangular plate with number of elements. This figure shows that the FEM values **converge to exact solution with increasing the number of elements**. Fig. 5.23 represents the values of deflection at the centre of fixed rectangular plate. This figure represents the rapid convergence of deflection values obtained by FEM to exact analysis solution. Both values differ but not converge with increasing the number of elements. Fig. 5.29 represents the variation of vertical deflection at centre of the clamped circular plate. But here results are out of trends. The FEM results differ increasingly from exact value with increasing the number of elements. This may be due to approximate geometry and mesh discretization technique adopted near the central zone.

### Bending and torsion moments variation

Fig. 5.21 represents that the value of  $M_x$  at the centre of SS rectangular plate fully converge with exact solution, for increasing the number of elements from 16 to 100. Fig. 5.24 represents the values of  $M_x$  at the centre of fixed rectangular plate and converges to exact value with increasing the number of elements. Fig. 5.25 represents the values of  $M_x$  at the support at central line in fix edges rectangular plate. The convergence of values is shown in the Figure. Fig. 5.22 represents the value of  $M_{xy}$  obtained from FEM at the corner node of SS rectangular plate converges with increasing the number of elements.

### Variation of radial moment $M_r$

Fig. 5.27 represents the variation of  $M_r$  at the centre of SS circular plate. These values converge slowly with increasing the number of elements. Fig. 5.30 shows the variation of

$M_r$  at the centre of clamped circular plate. Here the FEM results converge to exact value with increasing the number of elements.

### Variation of tangential moment $M_t$

Fig. 5.28 represents the variation of  $M_t$  at the supports with the number of elements. This figure shows the convergence of FEM value to exact value with increasing the number of elements from 12 to 36. Fig. 5.31 shows the variation of  $M_r$  at support of clamped circular plate. The FEM result converges to exact value on increasing the number of elements.

## CONCLUSIONS

On the basis of results obtained from analysis the following significant conclusions are drawn.

- The study of rectangular and circular shape plate with simply supported and fixed boundary conditions shows the potentiality of computer program developed especially to analyse the plates. This program is applicable to thin plates of any shape with all possible boundary conditions.
- The difference in the results obtained from FEM and exact solution, shows the effect of discretization and this type of error can be reduced by increasing the number of elements.
- The results confirm the shear locking effect in the Q4 element, which reduces with decreasing the size of elements. Because shear force is computed at the central Gauss point of the element and are constant throughout the element. Smaller elements show smaller shear forces and hence better results are obtained.
- The aspect ratio of the element does not show any effect on the nodal values. Numbers of element affect the nodal values.



- For fix boundaries the element shows their non usability for calculation of negative moments at supports.

#### FUTURE SCOPE OF STUDIES

- The same analysis can be done using eight noded, three noded and six noded isoparametric element, which confirm the geometry of curved boundary and quadratic interpolation. The relative performance of these elements can be compared and suitability of the elements can be checked for the bending analysis.
- Results obtained from analysis for various shapes of plates can be compared for their relative stability at different parameters.
- The best four noded element is square element, so the effect of the shape of four noded elements at centre of the circular plate can be analysed.

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