On The Nolocal Problem In The Field Of Eligible Types

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Issue I.

By Ω we define the area $\left\{(x,y): x^2+4(m+2)^{-2}y^{m+2}<1,\ y>0\right\}$. In this area

$$y^m u_{xx}+u_{yy}=0,\ \ m=const>0 \tag{1}$$

look at the equation.

(1) – Ω refers to the elliptical type in the field. For this equation, we will learn this issue:

Issue 1. Find the function that is $u(x,y)\in C(\overline{\Omega})\cap C^2(\Omega)$ in this area Ω Equation (1) and

$$u_{xx}(x,y),\ u_y(x,y)\in C(\Omega\cup l_0) \tag{2}$$

condition, within its limits

$$u(x,y)\big|_{\sigma_0}=\varphi(x),\quad x\in[-1,1]; \tag{3}$$

$$u_{xx}(x,0)+a(x)u_x(x,0)+b(x)u(x,0)+u_y(x,0)=c(x),\ x\in(-1,1) \tag{4}$$

satisfy boundary conditions, $\varphi(x),\ a(x),\ b(x),\ c(x)-$ continuous functions given here,

$$\sigma_0=\left\{(x,y): x^2+4(m+2)^{-2}y^{m+2}=1,\ \ y\ge 0\right\},\quad l_0=\left\{(x,y): -1<x<1,\ y=0\right\}.$$

We will prove that the problem is solved in the same way.

In this

$$\left.\begin{aligned} u(x,0)=\tau(x),\ -1\le x\le 1; \\ u_y(x,0)=\nu(x),\ -1<x<1 \end{aligned}\right\} \qquad (5)$$

we use markings.

II. Unity of solution. For this purpose $b(x)<0$, $x\in(-1,1)$ Assuming that the problem of homogeneity is equation (1), (2),

$$u(x,y)\big|_{\sigma_0}=0, \qquad (3')$$

$$\tau''(x)+a(x)\tau'(x)+b(x)\tau(x)+\nu(x)=0,\ \ -1<x<1 \qquad (4')$$

prove that the solution satisfying the conditions is only a trivial function.

Let's say the opposite $u(x,y)\not\equiv 0,\ (x,y)\in\bar{\Omega}$ Let there be a solution. In it

$$\sup_{\bar{\Omega}} u(x,y)=\left|u(x_0,y_0)\right|>0,\ (x_0,y_0)\in\bar{\Omega}$$

will be Based on the principle of extremity for elliptic type equations [2] (x_0,y_0) the point Ω is non-sectoral. $u|_{\sigma_0}=0$ And (x_0,y_0) the point σ_0 does not lie above the surface.

So, $(x_0,y_0)=(x_0,0)\in l_0$. At that point, x_0 $\tau(x)$ the function reaches a positive maximum or a negative minimum. If $\tau(x_0)>0(<0)$ at this point

$$\tau''(x_0)\le 0(\ge 0),\ \tau'(x_0)=0$$

relationships are appropriate, $(4')$ from equality

$$\nu(x_0)=\left[-\tau''(x_0)-a(x_0)\tau'(x_0)-b(x_0)\tau(x_0)\right]>0\ (<0)$$

resulting inequality. This is contrary to the Zarembo-Jiro principle [2].

This implies that the opposite $\bar{\Omega}$ the assumption $u(x,y)\not\equiv 0$ is false. I mean, $u(x,y)\equiv 0,\ (x,y)\in\bar{\Omega}$..

From this, if there is a solution to problem 1, it follows that the following is the only theorem:

Theorem 1. If $b(x)<0,\ x\in(-1,1)$ there is only one solution to the problem.

Proof. Imagine having issues and solutions. In it $u(x,y)=u_1(x,y)-u_2(x,y)$ function is homogeneous $\{(1),(2),(3'),(4')\}$ satisfies the issue. It's just a matter of time $u(x,y)\equiv 0,(x,y)\in\overline{\Omega}$ have a solution. In it $u_1(x,y)\equiv u_2(x,y),(x,y)\in\overline{\Omega}$.

Theorem 1 proved.

III. Availability of a solution. We look for the solution of the problem as the solution of the Dirixle problem in the field Ω for equation (1) as follows:

$$u(x,y)=k_2 y\int_{-1}^{1}\tau(t)\left\{\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}-\right.$$

$$\left.-\left[(1-xt)^2+\frac{4}{(m+2)^2}t^2y^{m+2}\right]^{\beta-1}\right\}dt+g(x,y), \qquad (6)$$

бу ерда $\beta=m\setminus(2m+4);\ k_2=\frac{1}{4\pi}\left(\frac{4}{m+2}\right)^{2-2\beta}\frac{\tilde{A}^2(1-\beta)}{\tilde{A}(2-2\beta)}$.

$$g(x,y)=-2k_2(1-\beta)(1-2\beta)y\left(1-R^2\right)\times\int_{-1}^{1}\varphi(\xi)\left(r_1^2\right)^{\beta-2}F\left(1-\beta,2-\beta,2-2\beta;1-\frac{r^2}{r_1^2}\right)d\xi,\quad (\xi,\eta)\in\sigma.$$

$$R^2=x^2+\frac{4}{(m+2)^2}y^2,\qquad \left.\begin{matrix}r^2\\ r_1^2\end{matrix}\right\}=(x-\xi)+\frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}}\mp\eta^{\frac{m+2}{2}}\right)^2.$$

Different from y and in this formula $y\neq 0$:

$$\frac{\partial}{\partial y}=k_2\int_{-1}^{1}\tau(t)\frac{\partial}{\partial y}\left\{y\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}dt-$$

$$-k_2\int_{-1}^{1}\tau(t)\frac{\partial}{\partial y}\left\{y\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}dt+\frac{\partial}{\partial y}g(x,y). \quad (7)$$

Indirectly we can make sure that,

$$\frac{\partial}{\partial y}\left\{y\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}=\frac{1}{2\beta-1}\frac{\partial}{\partial x}\left\{(x-t)\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}, \quad (8)$$

$$\frac{\partial}{\partial y}\left\{y\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}=\frac{1}{2\beta-1}\frac{\partial}{\partial x}\left\{\left(x-\frac{1}{t}\right)\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}. \quad (9)$$

Taking into account equations (8) and (9), (7) is written as follows:

$$\frac{\partial u}{\partial y}=\frac{k_2}{2\beta-1}\int_{-1}^{1}\tau(t)\frac{\partial}{\partial x}\left\{(x-t)\left[(x-t)^2+\frac{4}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}dt-$$

$$-\frac{k_2}{2\beta-1}\int_{-1}^{1}\tau(t)\frac{\partial}{\partial x}\left\{\left(x-\frac{1}{t}\right)\left[(1-xt)^2+\frac{4t^2}{(m+2)^2}y^{m+2}\right]^{\beta-1}\right\}dt+g'_y(x,y).$$

Here we set y to zero. In it $\lim_{y\to 0}u(y)=\nu(x)$ considering the note

$$\nu(x)=\frac{k_2}{2\beta-1}\int_{-1}^{1}\tau(t)\frac{\partial}{\partial x}\left\{(x-t)|x-t|^{2(\beta-1)}\right\}dt-$$

$$-\frac{k_2}{2\beta-1}\int_{-1}^{1}\tau(t)\frac{\partial}{\partial x}\left[\left(x-\frac{1}{t}\right)(1-xt)^{2(\beta-1)}\right]dt+g'_y(x,0)=$$

$$=\frac{k_2}{2\beta-1}\frac{d}{dx}\int_{-1}^{1}\tau(t)(x-t)|x-t|^{2\beta-2}dt-k_2\int_{-1}^{1}\tau(t)(1-xt)^{2\beta-2}dt,$$

that is

$$\nu(x)=\frac{k_2}{2\beta-1}\frac{d}{dx}\int_{-1}^{1}\tau(t)(x-t)|x-t|^{2(\beta-1)}dt-k_2\int_{-1}^{1}\tau(t)(1-xt)^{2\beta-2}dt+f(x) \quad (10)$$

we get equality, here

$$f(x)=\lim_{y\to 0}\frac{\partial}{\partial y}g(x,y)=2k_2(\beta-1)(1-2\beta)\times$$

$$\times\lim_{y\to 0}\frac{\partial}{\partial y}\left\{y\left(1-R^2\right)\int_{-1}^{1}\varphi(\xi)\left(r_1^2\right)^{\beta-2}F\left(1-\beta,2-\beta,2-2\beta;1-r^2/r_1^2\right)d\xi\right\}. \quad (11)$$

On the other hand, in accordance with (4) and (3) from conditions (5)

$$\tau''(x)+a(x)\tau'(x)+b(x)\tau(x)+\nu(x)=c(x), \quad x\in(-1,1); \quad (12)$$

$$\tau(-1)=\varphi(-1), \quad \tau(1)=\varphi(1) \quad (13)$$

equations arise.

So, issue 1 is about having a solution $\{(10),(12),(13)\}$ equals the issue. So we will continue to work on this. To this end $\nu(x)$ equation (1) of the function (10) is equivalent to problem 1 by equating (12)

$$\tau''(x)+a(x)\tau'(x)+b(x)\tau(x)+\frac{k_2}{2\beta-1}\frac{d}{dx}\int_{-1}^{1}\tau(t)(x-t)|x-t|^{2(\beta-1)}dt-$$

$$-k_2\int_{-1}^{1}\tau(t)(1-xt)^{2\beta-2}dt=f_1(x), \quad x\in(-1,1) \quad (14)$$

We find the solution to the equation (13) satisfying the conditions, here

$$f_1(x)=c(x)-f(x).$$

To solve this problem, use equation (14)

$$\tau''(x)=f_2(x), \quad x\in(-1,1) \quad (15)$$

We'll record it here

$$f_2(x)=f_1(x)-a(x)\tau'(x)-b(x)\tau(x)-$$

$$-\frac{k_2}{2\beta-1}\frac{d}{dx}\int_{-1}^{1}\tau(t)(x-t)|x-t|^{2\beta-2}dt+k_2\int_{-1}^{1}\tau(t)(1-xt)^{2\beta-2}dt.$$

If we consider the right-hand side of equation (15) as a function of time, $\{(15),(13)\}$ we will have an issue. To solve this problem

$$\tau(x)=z(x)+\varphi(-1)+\frac{1}{2}[\varphi(1)-\varphi(-1)](x+1)$$

Replace. As a result $z(x)$ in relation to function

$$z''(x)=f_2(x),\ x\in(-1,1);\quad z(-1)=0,\quad z(1)=0 \tag{16}$$

we will have an issue.

There is a Green function for this issue and it looks like this:

$$G(x,s)=\begin{cases}\frac{1}{2}(x-1)(s+1), & s\le x,\\ \frac{1}{2}(x+1)(s-1), & s\ge x.\end{cases}$$

Then according to Gilbert's theorem (16) is the solution to the problem can be identified by the

$$z(x)=\int_{-1}^{1} f_2(s)G(x,s)ds,\quad x\in(-1,1) \tag{17}$$

If we put this $z(x)$ expression $f_2(x)$ functions in this equation.

$$\tau(x)=\varphi(-1)+\frac{1}{2}[\varphi(1)-\varphi(-1)](x+1)+\int_{-1}^{1}G(x,s)\,f_1(s)ds-$$

$$-\int_{-1}^{1}G(x,s)\{a(s)\tau'(s)+b(s)\tau(s)+$$

$$+\frac{k_2}{2\beta-1}\frac{d}{ds}\int_{-1}^{1}\tau(t)(s-t)|s-t|^{2\beta-2}dt-k_2\int_{-1}^{1}\tau(t)(1-st)^{2\beta-1}dt\Big\}ds$$

Will be this equation.

Using the formula for the final integral $G(x,-1)=G(x,1)=0$ taking into account the equations.

$$\tau(x)-\int_{-1}^{1}\Big\{\frac{\partial}{\partial s}[G(x,s)a(s)]\tau(s)+b(s)\tau(s)+\frac{k_2G_s'(x,s)}{2\beta-1}\int_{-1}^{1}\tau(t)(s-t)|s-t|^{2\beta-2}dt-$$

$$-k_2G(x,s)\int_{-1}^{1}\tau(t)(1-st)^{2\beta-2}dt\Big\}ds=f_3(x)$$

we are coming to equality, here

$$f_3(x)=\varphi(-1)+\frac{1}{2}\left[\varphi(1)-\varphi(-1)\right](x+1)+\int_{-1}^{1}G(x,s)\,f_1(s)\,ds.$$

By rearranging the integral in the second integral in the last equation and

$$K(x,t)=\frac{\partial}{\partial t}\left[G(x,t)a(t)\right]+b(t)G(x,t)+$$

$$+\frac{k_2}{2\beta-1}\int_{-1}^{1}G'_s(x,t)(s-t)|s-t|^{2\beta-2}\,ds-k_2\int_{-1}^{1}G(x,s)(1-st)^{2\beta-2}\,ds$$

enter an attribute with respect to an unknown function $\tau(x)$

$$\tau(x)-\int_{-1}^{1}\tau(t)K(x,t)\,dt=f_3(x),\qquad x\in(-1,1) \tag{18}$$

we get the integral equation.

We will study the nucleus $K(x,t)$ and the right side $f_3(x)$ of this equation. First we will study the core $K(x,t)$. Let's say $x\geq t$. Then depending on the description of the module and the function of the function.

1)

$$\frac{k_2}{2\beta-1}\int_{-1}^{1}(s-t)|s-t|^{2\beta-2}\frac{\partial}{\partial s}G(x,s)\,ds=\frac{k_2}{2\beta-1}\left[-\int_{-1}^{t}(t-s)^{2\beta-1}\frac{\partial}{\partial s}\left[\frac{1}{2}(x+1)(s-1)\right]ds+\right.$$

$$\left.+\int_{t}^{x}(s-t)^{2\beta-1}\frac{\partial}{\partial s}\left[\frac{1}{2}(x+1)(s-1)\right]ds+\int_{x}^{1}(s-t)^{2\beta-1}\frac{\partial}{\partial s}\left[\frac{1}{2}(x-1)(s+1)ds\right]\right]=$$

$$=\frac{k_2}{4\beta(2\beta-1)}\left[2(x-t)^{2\beta}-(1+x)(1+t)^{2\beta}-(1-x)(1-t)^{2\beta}\right].$$

2)

$$\int_{-1}^{1}G(x,s)(1-st)^{2\beta-2}\,ds=\frac{1}{2}\int_{-1}^{x}(x-1)(s+1)(1-st)^{2\beta-2}\,ds+\frac{1}{2}\int_{x}^{1}(x+1)(s-1)(1-st)^{2\beta-2}\,ds.$$

Then there are the following inequalities:

$$\left|\int_{-1}^{1} G(x,s)(1-st)^{2\beta-2}\,ds\right| \le \frac{1}{2}\int_{-1}^{x}(1-x)\frac{1+s}{1-st}(1-st)^{2\beta-1}\,ds +$$

$$+\frac{1}{2}\int_{x}^{1}(1+x)\frac{1-s}{1-st}(1-st)^{2\beta-1}\,ds = I_1 + I_2.$$

Let's look at the integral I_1. $s=t=-1$ even though they are special. There is an inequality around this point $(1+s)/(1-st)$. In it

$$I_1 \le \frac{1}{2}(1-x)\int_{-1}^{x}(1-st)^{2\beta-1}\,ds \le \frac{1}{2}(1-x)\int_{-1}^{x}(1+s)^{2\beta-1}\,ds =$$

$$= \frac{1}{2}(1-x)\frac{(1+s)^{2\beta}}{2\beta}\Bigg|_{s=-1}^{s=x} = \frac{1}{4\beta}(1-x)(1+x)^{2\beta}.$$

Now, look the integral I_2. This integral is special in this point $s=t=1$. Around this point, $(1-s)<(1-st)$ inequality is appropriate. In it

$$I_2 \le \frac{1}{2}(1+x)\int_{x}^{1}(1-st)^{2\beta-1}\,ds \le \frac{1}{2}(1+x)\int_{x}^{1}(1-s)^{2\beta-1}\,ds =$$

$$= -\frac{1}{4\beta}(1+x)(1-s)^{2\beta}\Bigg|_{s=x}^{s=1} = \frac{1}{4\beta}(1+x)(1-x)^{2\beta}.$$

As can be seen from the above, $K(x,t)$ the last two compounds of the nucleus are constrained functions.

In it $a(x)\in C[-1,1]$ $b(x)\in C[-1,1]$ when $K(x,t)-$ limited nuclei $\{(x,t):\ -1\le x,t\le 1,\ t\ne x\}$ is continuous, $t=x$ the first round has a break. It means that $K(x,t)-$ Fredgolm`s nucleus.

Now $f_3(x)$ let's check the function. First we look at the function $f(x)$:

$$f(x)=-2k_2(\beta-1)(1-2\beta)\lim_{y\to 0}\frac{\partial}{\partial y}\left\{y(1-R^2)\times\int_{-1}^{1}\varphi(\xi)(r_1^2)^{\beta-2}F(1-\beta,2-\beta,2-2\beta;1-r^2/r_1^2)d\xi\right\}=$$

$$=-2k_2(\beta-1)(1-2\beta)\lim_{y\to 0}\left\{(1-R^2)\times\int_{-1}^{1}\varphi(\xi)(r_1^2)^{\beta-2}F(1-\beta,2-\beta,2-2\beta;1-r^2/r_1^2)d\xi\right\}=$$

$$=-2k_2(\beta-1)(1-2\beta)\lim_{y\to 0}y\frac{\partial}{\partial y}\{(1-R^2)\times$$

$$\times\int_{-1}^{1}\varphi(\xi)(r_1^2)\beta-2F(1-\beta,2-\beta,2-2\beta;1-r^2/r_1^2)d\xi\right\}=$$

$$=k_3(1-x^2)\int_{-1}^{1}\varphi(\xi)(x^2-2x\xi+1)^{\beta-2}d\xi,$$

here $k_3=-2k_2(\beta-1)(1-2\beta)$.

Given $\varphi(\xi)$ function

$$\varphi(\xi)=(1-\xi^2)^{\varepsilon}\overline{\varphi}(\xi) \qquad (*)$$

Let's assume, here it is $\varepsilon>\frac{1}{2}-\beta,\ \overline{\varphi}(\xi)\in C[-1,1]$.

$f(x)$ set the integral $(*)$ and then apply the mean value theorem:

$$f(x)=k_3(1-x^2)\overline{\varphi}(\xi_1)\int_{-1}^{1}(1-\xi^2)^{\varepsilon}(x^2-2x\xi+1)^{\beta-2}d\xi,$$

here $\xi_1-[-1,1]$ Any number in between. Here's the replacement in the integral: $\xi=-1+2t$

$$f(x)=2k_3(1-x^2)\overline{\varphi}(\xi_1)\int_{0}^{1}[4t(1-t)]^{\varepsilon}\left[(1+x)^2-4xt\right]^{\beta-2}dt=$$

$$=2^{1+2\varepsilon}k_3\overline{\varphi}(\xi_1)(1-x^2)\int_{0}^{1}t^{\varepsilon}(1-t)^{\varepsilon}\left[(1+x)^2-4xt\right]^{\beta-2}dt.$$

This integral $x=1$ and $x=-1$ behaves uniformly around the dots. Therefore, we check it by point: $x=1$

$$f(x)=2^{1+2\varepsilon}k_3\bar{\varphi}(\xi_1)(1-x^2)(1+x)^{2\beta-4}\int_0^1 t^{\varepsilon}(1-t)^{\varepsilon}\left(1-\frac{4x}{(1+x)^2}t\right)^{\beta-2}dt.$$

To the last integral

$$\int_0^1 x^{a-1}(1-x)^{c-a-1}(1-xt)^{-b}\,dx=\frac{\tilde{A}(a)\tilde{A}(c-a)}{\tilde{A}(c)}F(a,b,c;t)$$

we use the formula.

Here, $a=1+\varepsilon,\ b=2-\beta,\ c-a=1+\varepsilon,\ c=2+2\varepsilon$. So,

$$f(x)=2^{1+2\varepsilon}k_3\bar{\varphi}(\xi_1)(1-x^2)(1+x)^{2\beta-4}\frac{\Gamma(1+\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(2+2\varepsilon)}\times F\left[1+\varepsilon,2-\beta,2+2\varepsilon;\frac{4x}{(1+x)^2}\right]=$$

$$=2^{1+2\varepsilon}k_3\frac{\tilde{A}^2(1+\varepsilon)}{\tilde{A}(2+2\varepsilon)}\bar{\varphi}(\xi_1)(1-x)(1+x)^{2\beta-3}\left[1-\frac{4x}{(1+x)^2}\right]^{\varepsilon+\beta-1}\times$$

$$\times F\left(1+\varepsilon,2\varepsilon+\beta,2+2\varepsilon;\frac{4x}{(1+x)^2}\right)=c_1(1+x)^{2\varepsilon-1}(1-x)^{2\varepsilon+2\beta-1}$$

Here $c_1=2^{1+2\varepsilon}k_3\tilde{A}^2(1+\varepsilon)\tilde{A}^{-1}(2+2\varepsilon)\bar{\varphi}(\xi_1)$. If $\varepsilon>\frac{1}{2}-\beta$ given inequality, we can conclude that the latter is $|f(1)|<+\infty$

$f(x)$ the function $x=-1$ is also checked around the point. So, $|f(x)|<+\infty$ and $f(x)\in C[-1,1]$. This and $c(x)\in C[-1,1]$ given that, $f_1(x)\in C[-1,1]$ The reason for this is that. This will be $f_3(x)\in C[-1,1]$.

Hence, (18)– $\tau(x)$ the second type with respect to the unknown function (18) is the Fredholm integral equation. It is the equivalent of 1 issue under study.

Homogeneous in (18)

$$\tau(x)-\int_{-1}^{1}\tau(t)K(x,t)dt=0,\quad x\in[-1,1] \qquad (18')$$

the integral equation is equivalent to problem 1of same-sex. Since the last problem has only the trivial solution, the homogeneous integral equation has only the trivial solution. Therefore, based on the alternative of Fredholm (18), the solution of the integral equation exists and is unique.

After the function $\tau(x)$ is found in the integral (18) equation, the solution of problem 1 is found by the formula (6). Thus, the following theorem was proved:

2 − theorem. If $a(x)\in C^1[-1,1];\ b(x)\in C[-1,1],\ b(x)<0,\ x\in(-1,1);$ $\varphi(x)=\left(1-x^2\right)^{\varepsilon}\overline{\varphi}(x),\ \varepsilon>(1/2)-\beta,\ \overline{\varphi}(x)\in C[-1,1]$ if the conditions are met, the issue 1 is the only solution.

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