Nolocal Problems In Half-Selected Half For Equipment

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Annotation: This article explores the non-zero conditional problem for the elliptic type equation. The uniqueness of the solution to the problem is proved by the principle of Zarembo-Jiro, and the existence of a solution is proved by the use of a Drixle problem.

Keywords: the decomposition equation, the zero problem, the Zarembo-Jiro principle, the Drixle problem.

I. Issue. Let us look at this perturbed differential equation of the elliptical type in the sector

\[ \Omega = \{(x,y): -\infty < x < +\infty, y > 0\} \]

(1)

\[ \Omega \]

of the sector \( S = \{(x,y): -\infty < x < +\infty, y = 0\} \) subdivide three he border

\[ l_0 = \{(x,y): -1 < x < 1, y = 0\}, \quad l_1 = \{(x,y): -\infty < x < -1, y = 0\}, \quad l_2 = \{(x,y): 1 < x < +\infty, y = 0\} \]

Now, make notes. I mean, \( S = l_1 \cup l_0 \cup l_2 \).

The issue. Thus \( u(x,y) \in C(\Omega \cup S) \cap C^2(\Omega) \) find the function, it \( \Omega \) in the sector (1) is the regular solution of the equation,

\[ u_{xx}(x,y), u_y(x,y) \in C(\Omega \cup l_0); \]

(2)

\[ u_{xx}(x,y) + a(x)u_x(x,y) + b(x)u(x,y) + u_y(x,y) = c(x), \quad (x,y) \in l_0; \]

(3)

\[ u(x,y) = \varphi_1(x), \quad (x,y) \in l_1; \quad u(x,y) = \varphi_2(x), \quad (x,y) \in l_2; \]

(4)

\[ \lim_{R \to \infty} u(x,y) = 0, \quad y \geq 0 \]

(5)

satisfy the conditions, here \( \varphi_1(x), \varphi_2(x), a(x), b(x), c(x) \) – given continuous functions, \( \lim_{x \to -\infty} \varphi_1(x) = 0, \lim_{x \to -\infty} \varphi_2(x) = 0, \quad b(x) < 0; \quad R = \sqrt{x^2 + y^2}. \)

We will prove that the solution to this problem is unique. In this

\[ u(x,0) = \tau(x), \quad -1 \leq x \leq 1; \quad u_y(x,0) = v(x), \quad -1 < x < 1 \]

(6)

we use markings.
II. Unity of solution. Imagine that \( u_1(x, y) \) and \( u_2(x, y) \) have solutions. In that case \( u_1(x, y) - u_2(x, y) = u(x, y) \) function \( \Omega \) in the field (1) is the regular solution of the equation, (2), (5) and
\[
\begin{align*}
u''(x) + a(x)\nu'(x) + b(x)\nu(x) + \nu(x) &= 0, \quad x \in (-1,1) \\
u'(x) &= 0, \quad (x, y) \in I_1 \cup I_2;
\end{align*}
\]
satisfies the conditions.

Imagine that, \( u(x, y) \neq 0, \ x \in \Omega \cup S \). Then it is
\[
\Omega_r = \{(x, y) : |x^2 + y^2| \leq r^2, \ y > 0\}
\]
There is a sector (here \( r = \text{const} > 0 \)) that
\[
u(x, y) \neq 0 \quad \text{that is why}
\]
\[
\sup_{\overline{\Omega}_r} \|\nu(x, y)\| = \|\nu(x_0, y_0)\| > 0, \ (x_0, y_0) \in \overline{\Omega}_r.
\]

Based on the principle of extremity for elliptic type equations \( (x_0, y_0) \notin \Omega_r \).
\[
(4') \quad \text{subject to condition } (x_0, y_0) \notin I_1 \cup I_2. \quad \text{Accordingly, } (x_0, y_0) \in I_0, \ \text{namely}
\]
\[
(x_0, y_0) = (x_0, 0), \quad -1 < x < 1.
\]
In that \( x_0 \) at the point \( \tau(x) \) the function reaches a positive maximum or a negative maximum. That is why \( \tau(x_0) > 0(<0) \) when \( \tau''(x_0) \leq 0 (\geq 0), \ \tau'(x_0) = 0 \quad \text{that's it,}
\]
\[
\tau''(x_0) + a(x_0)\tau'(x_0) + b(x_0)\tau(x_0) + \nu(x) = 0
\]
from equality \( \nu(x_0) > 0(<0) \) resulting inequality. This is contrary to the Zarembo-Jiro principle. So \( (x_0, y_0) \notin I_0 \).

In that case, that is
\[
\sup_{\overline{\Omega_r}} \|\nu(x, y)\| = \sup_{\bar{S}_r} \|\nu(x_0, y_0)\| > 0. \quad \text{(7)}
\]
Now \( \forall R > r \) get the numbers and consider the above \( \Omega_R \) in the field,
\[
\sup_{\overline{\Omega_R}} \|\nu(x, y)\| = \sup_{\bar{S}_r} \|\nu(x, y)\| > 0 \quad \text{(8)}
\]

We have a relationship. \( \overline{\Omega_R} \supset \overline{\Omega_r} \) because of this
\[
\sup_{\overline{\Omega_R}} \|\nu(x, y)\| \geq \sup_{\overline{\Omega_r}} \|\nu(x, y)\|.
\]
In that case (7) and (8) based on inequalities
\[
\sup_{\bar{S}_r} \|\nu(x, y)\| \geq \sup_{\bar{S}_y} \|\nu(x, y)\| > 0
\]
the inequalities are relevant. This is contrary to condition (5). This contradiction leads to the conclusion that our hypothesis is wrong. So, \( u_1(x, y) \equiv u_2(x, y), (x, y) \in \Omega \cup S \).

**III. Availability of a solution.** The solution of the problem \( \Omega \) in the sphere (1) as the solution of the Dirichle problem for equation

\[
u(x, y) = k_2 y \int_{-\infty}^{+\infty} \left[ \frac{4}{(m+2)^2} y^{m+2} \right]^{\beta-1} dt (9)
\]

Search like. (9) da \( x \in (-1,1) \) va \( y > 0 \) that's it \( y \) differentiate. In this

\[
\frac{\partial}{\partial y} \left\{ y \left[ (x-t)^2 + \frac{4}{(m+2)^2} y^{m+2} \right]^{\beta-1} \right\} =
\]

\[
= \frac{1}{\beta-1} \frac{\partial}{\partial x} \left\{ (x-t) \left[ (x-t)^2 + \frac{4}{(m+2)^2} y^{m+2} \right]^{\beta-1} \right\} dx
\]

Taking into account the equality, then zero and (6)

\[
\nu(x) = \frac{k_2}{2\beta-1} \left[ \frac{d}{dx} \int_{-\infty}^{+\infty} (x-t)^{2\beta-2} u(t,0) dt \right]
\]

based on equality, or (4) terms and (6)

\[
\nu(x) = \frac{k_2}{2\beta-1} \frac{d}{dx} \int_{-1}^{1} (x-t)^{2\beta-2} \tau(t) dt + f(x) (10)
\]

Here we go equality

\[
f(x) = \frac{k_2}{2\beta-1} \frac{d}{dx} \int_{-\infty}^{-1} (x-t)^{2\beta-1} \varphi_1(t) dt - \frac{k_2}{2\beta-1} \frac{d}{dx} \int_{1}^{+\infty} (t-x)^{2\beta-1} \varphi_2(t) dt =
\]

\[
= k_2 \int_{-\infty}^{-1} (x-t)^{2\beta-2} \varphi_1(t) dt + k_2 \int_{1}^{+\infty} (t-x)^{2\beta-2} \varphi_2(t) dt (11)
\]

Using the fractional order differential sign in Eq

\[
\nu(x) = -k_2 \bar{A} (1-2\beta) \left[ D_{-1x}^{1-2\beta} \tau(x) + D_{x1}^{1-2\beta} \tau(x) \right] + f(x) (12)
\]

written in the form.
(11) equation, \( f(x) \) so that the function is continuous \( \varphi_1(x) = (1 + x)^{1-2\beta+\varepsilon} \), \( \varphi_2(x) = (1 - x)^{1-2\beta+\varepsilon} \). It should be, here \( \varepsilon > 0 \) and (11) subject to equation (11)

\[
\tau''(x) = a(x)\tau'(x) + b(x)\tau(x) - k_2\bar{A}(1-2\beta) \times \\
\times \left[ D_{1x}^{-1} \tau(x) + D_{1x}^{-1} \tau(x) \right] = c(x) - f(x) 
\] (13)

differential equation.

(4) and from the boundary conditions

\[
\tau(-1) = 0, \quad \tau(1) = 0 
\] (14)
equations arise.

Ergo, \( \tau(x) \) – function \( \{13\},\{14\} \) as the solution. \( \tau(x) \) Once the problem is found, the solution to this problem is determined by (1).

References:

