PSEUDO-DYNAMIC EVALUATION OF FAILURE SURFACE AND SEISMIC ACTIVE EARTH PRESSURE

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KEYWORDS

Seismic active earth pressure, pseudo-dynamic method, ϕ -backfill, coefficient of seismic earth pressure.

ABSTRACT

Retaining wall is a structure used for supporting the soil mass laterally so that the soil can be retained at different levels on the two sides of it. Design of retaining wall needs the complete knowledge of Earth Pressure for both Active and Passive conditions. In the present work, an effort has been made to develop a formulation for critical wedge angle and thus seismic active pressure behind a vertical retaining wall supporting ϕ -backfill using pseudo-dynamic method. The effect of various parameters viz. time, shear and primary waves, time period of earthquake ground motion, angle of internal friction (ϕ), angle of wall friction (δ), seismic acceleration co-efficients (k_h , k_v) are also taken into account to provide the variation of seismic active earth pressure.

INTRODUCTION

The concept of Earth Pressure under seismic loading condition is very much essential in designing a retaining wall as the failure of such structures may lead to catastrophic failure. Okabe (1926), Mononobe and Matsuo (1929) performed the analysis for earth pressure considering only ϕ backfill using pseudo-static method. These methods were later recognized as famous Mononobe-Okabe method. In the above stated method, the dynamic nature of earthquake loadings are considered in a very approximate way without taking any time effect. To overcome this, the time and phase difference due to finite shear wave propagation behind a retaining wall was considered using a simple and more realistic way of pseudo-dynamic method, proposed by Steedman and Zeng (1990). Choudhury and Nimbalkar (2005), Choudhury and Nimbalkar (2006) and Ghosh (2008) extended the Steedman and Zeng (1990) approach to find out the seismic passive earth pressure coefficients supporting ϕ backfill. After that Ghosh and Sharma (2012b) introduced seismic active response on the back of a battered retaining wall supporting inclined backfill. Again Ghosh and Sharma (2012a) introduced pseudo-dynamic analysis for passive earth pressure in case of c-ϕ backfill. In the pseudo-dynamic process here, an attempt is made to develop the formulation of critical wedge angle using which the value of coefficient of seismic earth pressure can be determined.

ANALYSIS FOR ACTIVE EARTH PRESSURE

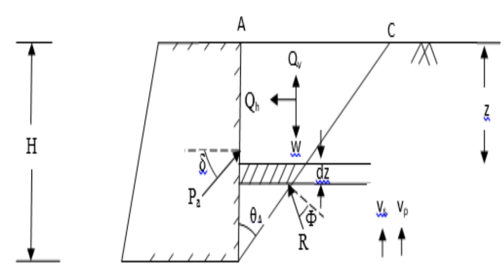


Figure 1. Model retaining wall considered for computation of pseudo dynamic active earth pressure

A rigid vertical, cantilever retaining wall of height H is considered with a dry, ϕ-horizontal backfill as shown in Fig.1.The wall face AB on the backfill side having wall friction angle δ. The objective is to develop formulation for the active earth pressure under seismic loading condition.

The mass of a thin element of wedge at depth z of thickness dz is given by

$$m(z)=\frac{\gamma}{g}(H-z)\tan\theta_A dz \quad (1)$$

The total weight of the failure wedge W is given by

$$W=\frac{\gamma H^2}{2}\tan\theta_A \quad (2)$$

For the triangular wedge ABC as shown in Fig.1, a sinusoidal base shaking subjected to both horizontal and vertical earthquake acceleration with amplitude $k_h g$ and $k_v g$, the horizontal and vertical acceleration at any depth z below the ground surface and time t respectively can be expressed as,

$$a_h(z,t)=k_h g\sin\omega(t-\frac{H-z}{V_s}) \quad (3)$$

$$a_v(z,t)=k_v g\sin\omega(t-\frac{H-z}{V_p}) \quad (4)$$

The horizontal inertia force exerted on the small element resulting from horizontal earthquake acceleration can be expressed as m (z).a_h (z, t). Therefore, the total horizontal inertia force Q_h(t) acting on the failure wedge can be expressed as,

$$Q_h(t)=\int_0^H m(z)a_h(z,t)dz$$

$$=\int_0^H k_h g\sin\omega(t-\frac{H-z}{V_s})\frac{\gamma}{g}(H-z)\tan\theta_A dz$$

$$=\frac{k_h\gamma\lambda H\tan\theta_A}{4\pi^2}$$

$$[2\pi cos(2\pi(\frac{t}{T}-\frac{H}{\lambda}))+\frac{\lambda}{H}\{\sin(2\pi(\frac{t}{T}-\frac{H}{\lambda}))-sin(2\pi(\frac{t}{T})\}]$$

Where λ = TV_s is the wave length of shear wave.

V_s=velocity of shear wave.

Similarly, the total vertical inertia force Q_v (t) acting on the failure wedge is given by

Q_v (t)

$$=\frac{k_v\gamma\eta H\tan\theta_A}{4\pi^2}$$

$$[2\pi cos(2\pi(\frac{t}{T}-\frac{H}{\eta}))+\frac{\eta}{H}\{\sin(2\pi(\frac{t}{T}-\frac{H}{\eta}))-sin(2\pi(\frac{t}{T})\}] \quad (6)$$

Where, η= TV_p =wavelength of the primary wave.

V_P= velocity of primary wave.

The total active force P_a (related to triangular wedge ABC) can be determined by taking the horizontal as well as vertical equilibrium of the failure wedge and is given by,

$$P_a = \frac{W\cos(\theta_A+\phi)+Q_h\sin(\theta_A+\phi)-Q_v\cos(\theta_A+\phi)}{\sin(\theta_A+A)}$$

$$=\frac{\gamma H^2}{2}\left[\frac{\tan\theta_A\cos(\theta_A+\phi)+B\tan\theta_A\sin(\theta_A+\phi)-C\tan\theta_A\cos(\theta_A+\phi)}{\sin(\theta_A+A)}\right]$$

(7)

Where,

$$A=\phi+\delta$$

(8)

$$B=\frac{k_h(\frac{\lambda}{H})}{2\pi^2}[2\pi\cos2\pi(\frac{t}{T}-\frac{H}{\lambda})-\frac{\lambda}{H}\{\sin2\pi\frac{t}{T}-\sin2\pi(\frac{t}{T}-\frac{H}{\lambda})\}]$$

(9)

$$C=\frac{k_v(\frac{\eta}{H})}{2\pi^2}[2\pi\cos2\pi(\frac{t}{T}-\frac{H}{\eta})-\frac{\eta}{H}\{\sin\frac{2\pi t}{T}-\sin2\pi(\frac{t}{T}-\frac{H}{\eta})\}]$$

(10)

Active earth pressure co-efficient K_{ae} can be defined as,

$$K_{ae}=\frac{2P_a}{\gamma H^2}$$

Therefore from equation (7) we can write

$$P_a=\frac{\gamma H^2}{2}K_{ae}$$

Where,

$$K_{ae}=\left[\frac{\tan\theta_A\cos(\theta_A+\phi)+B\tan\theta_A\sin(\theta_A+\phi)-C\tan\theta_A\cos(\theta_A+\phi)}{\sin(\theta_A+A)}\right]$$

$$=K_w+K_{qh}-K_{qv}$$

(11)

Where,

$$K_w=\frac{\tan\theta_A\cos(\theta_A+\phi)}{\sin(\theta_A+A)}$$

(12)

$$K_{qh}=\frac{B\tan\theta_A\sin(\theta_A+\phi)}{\sin(\theta_A+A)}$$

(13)

$$K_{qv}=\frac{C\tan\theta_A\cos(\theta_A+\phi)}{\sin(\theta_A+A)}$$

(14)

$$K_{ae}=\frac{\tan\theta_A[\cos(\theta_A+\phi)+B\sin(\theta_A+\phi)-C\cos(\theta_A+\phi)]}{\sin(\theta_A+A)}$$

$$= \frac{\tan\theta_A[D\cos(\theta_A+\phi)+B\sin(\theta_A+\phi)}{\sin(\theta_A+A)}$$

[In which D=1-C] (15)

From Equation (15) it is seen that K_{ae} is a function of θ_A and t/T. The optimum value of K_{ae} is obtained by optimizing K_{ae} with respect to θ_A and t/T.

$f_x = \frac{dK_{ae}}{d\theta_A} = 0$ Will give

$$\theta_A = \tan^{-1}\left(\frac{-G \pm \sqrt{G^2 - 4FH}}{2F}\right) \quad (16)$$

Where,

$$H = D\cos\phi\sin A + B\sin\phi\sin A \quad (17)$$

$$F = -D\sin\phi\cos A + B\cos\phi\cos A \quad (18)$$

$$G = D\cos(A+\phi) + B\sin(A+\phi) + E \quad (19)$$

$$D = 1 - C \quad (20)$$

$$E = -D\cos(A-\phi) + B\sin(A-\phi) \quad (21)$$

And,

$f_y = \frac{dK_{ae}}{d(\frac{t}{T})} = 0$ Will give,

$$\theta_A = \tan^{-1}\left(\frac{jg - je_0\tan\frac{2\pi t}{T}}{id\tan\frac{2\pi t}{T} - if}\right) - \phi \quad (22)$$

Where,

$$d = 2\pi\cos\frac{2\pi H}{\lambda} - \frac{\lambda}{H}\sin 2\pi\frac{H}{\lambda} \quad (23)$$

$$e_0 = 2\pi\cos 2\pi\frac{H}{\eta} - \frac{\eta}{H}\sin 2\pi\frac{H}{\eta} \quad (24)$$

$$f = 2\pi\sin 2\pi\frac{H}{\lambda} + \frac{\lambda}{H}\cos 2\pi\frac{H}{\lambda} - \frac{\lambda}{H} \quad (25)$$

$$g = 2\pi\sin 2\pi\frac{H}{\eta} + \frac{\eta}{H}\cos 2\pi\frac{H}{\eta} - \frac{\eta}{H} \quad (26)$$

i $= -\frac{\lambda}{H}.k_h$ and J$= \frac{\eta}{H}.k_v$

(27)

Equation number (16) and (22) are two linear equations having two variables θ_A and t/T. Therefore by solving these two equations we get the $(\theta_A)_{cr}$ and $(t/T)_{cr}$

Second Order partial derivative of K_{ae} with respect to θ_A

$$f_{xx} = \frac{d^2K_{ae}}{d\theta_A^2}$$

$$= \frac{\sin^2(\theta_A + A)(2F\tan\theta_A.\sec^2\theta_A + G.\sec^2\theta_A)}{\sin^4(\theta_A + A)}$$

$$-\frac{(F\tan^2\theta_A + G\tan\theta_A + H)(2\sin(\theta_A + A).\cos(\theta_A + A))}{\sin^4(\theta_A + A)}$$

(28)

Second order partial derivative of K_{ae} with respect to t/T

$$f_{yy} = \frac{d^2K_{ae}}{d(t/T)^2} = (2\pi)^2[x\cos(2\pi\frac{t}{T}) + y\sin(2\pi\frac{t}{T})]$$

(29)

Where,

$$x = b[2\pi\cos(2\pi\frac{H}{\lambda}) - \frac{\lambda}{H}\sin(2\pi\frac{H}{\lambda})] + c[2\pi\cos(2\pi\frac{H}{\eta}) - \frac{\eta}{H}\sin(2\pi\frac{H}{\eta})]$$

(30)

$$y = b(2\pi\sin(2\pi\frac{H}{\lambda}) + \frac{\lambda}{H}\cos(2\pi\frac{H}{\lambda}) - \frac{\lambda}{H})$$

$$+c(2\pi\sin(2\pi\frac{H}{\eta}) + \frac{\eta}{H}\cos(2\pi\frac{H}{\eta}) - \frac{\eta}{H})$$

(31)

$$b = -\frac{\lambda}{2\pi^2H}k_h\tan\theta\frac{\sin(\theta_A + \phi)}{\sin(\theta_A + A)}$$

(32)

$$c = \frac{\eta}{2\pi^2H}k_v\tan\theta\frac{\cos(\theta_A + \phi)}{\sin(\theta_A + A)}$$

(33)

Second order partial derivative of K_{ae} with respect to θ_A and t/T

$$f_{xy} = \frac{d^2K_{ae}}{d\theta_A d(t/T)}$$

$$= Q[\left\{-(2\pi)^2\sin 2\pi(\frac{t}{T} - \frac{H}{\eta})\right\} - \frac{\eta}{H}2\pi\left\{\cos(2\pi t/T) - \cos(2\pi(\frac{t}{T} - \frac{H}{\eta}))\right\}]$$

$$+P[\left\{-(2\pi)^2\sin(\frac{t}{T} - \frac{H}{\lambda})\right\} - \frac{\lambda}{H}(2\pi)\left\{\cos(2\pi t/T) - \cos(2\pi(\frac{t}{T} - \frac{H}{\lambda}))\right\}]$$

(34)

Where,

$$Q = L.\frac{k_v}{2\pi^2}.\frac{\eta}{H}$$

(35)

$$P = M.\frac{k_h}{2\pi^2}.\frac{\lambda}{H}$$

(36)

$$L = I\sin\phi\cos A - J\cos(A + \phi) - K\cos\phi\sin A + J\cos(A - \phi)$$

(37)

$$M = I\cos\phi\cos A + J\sin(A + \phi) + K\sin\phi\cos A + J\sin(A - \phi)$$

(38)

$$N = -I\sin\phi\cos A + J\cos(A + \phi) + K\cos\phi\sin A - J\cos(A - \phi)$$

(39)

$$I = \frac{\tan^2 \theta_A}{\sin^2(\theta_A + A)}$$

(40)

$$J = \frac{\tan(\theta_A)}{\sin^2(\theta_A + A)}$$

(41)

$$K = \frac{1}{\sin^2(\theta_A + A)}$$

(42)

Optimisation Criteria and Check for Optimisation:-

If f be a function with two variables with continuous second order partial derivatives f_{xx}, f_{yy} and f_{xy} at a critical point (a, b) and if

D.F = $f_{xx}(a,b)\, f_{yy}(a,b) - f_{xy}^2(a,b)$ then

a. If D.F > 0 , $f_{xx}(a,b) > 0$ and $f_{yy}(a,b) > 0$ f has a relative minimum at (a,b).

b. If D.F > 0, $f_{xx}(a,b) < 0$ and $f_{yy}(a,b) < 0$ then f has a relative maximum at (a,b).

c. If D.F < 0, then f has a saddle point at (a,b).

d. If D.F = 0, then no conclusion can be drawn.

In the present problem, K_{ae} is a function of two variables θ_A and t/Therefore, for finding out of critical value of θ_A and t/T i.e. for which values of θ_A and t/T, K_{ae} will have a relative maximum value, we need to satisfy the condition (a) as stated above.

Now, putting the values of θ_A and t/T which we got by solving the equations (16) and (22), we get,

D.F = $f_{xx}((\theta_A)_{cr}, (t/T)_{cr})\; f_{yy}(\theta_{cr}, (t/T)_{cr}) - f_{xy}^2((\theta_A)_{cr}, (t/T)_{cr}) > 0$

And $f_{xx}((\theta_A)_{cr} <0, (t/T)_{cr}) <0$

So, D.F > 0 and $f_{xx}((\theta_A)_{cr} <0, (t/T)_{cr}) < 0$, $f_{yy}((\theta_A)_{cr\ r}, (t/T)_{cr}) < 0$

Then K_{ae} has a relative maximum value at $((\theta_A)_{cr}, (t/T)_{cr}$

RESULTS

Results are given in tabular form in Table.1 for active condition

Table 1:- Pseudo-dynamic active earth pressure Coefficients K_{ae}

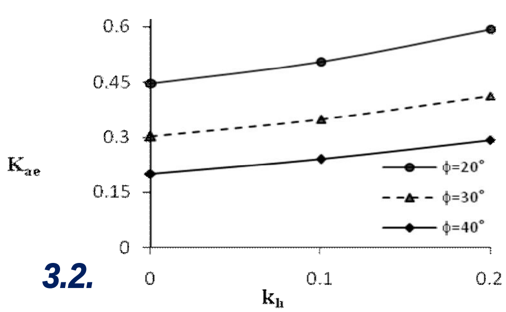
	$k_h = 0, k_v = 0$			$k_h = 0.1, k_v = 0.05$		
ϕ \ δ	0	$\phi/2$	ϕ	0	$\phi/2$	ϕ
20	0.49029	0.446742	0.426873	0.542344	0.505849	0.493724
30	0.333332	0.301416	0.297172	0.377502	0.349689	0.352877
40	0.217442	0.199404	0.210195	0.254799	0.239473	0.259097
	$k_h = 0.2, k_v = 0.1$			$k_h = 0.2, k_v = 0.2$		
20	0.615395	0.592571	0.595722	0.586222	0.566178	0.571785
30	0.433312	0.413569	0.430142	0.412817	0.334902	0.41234
40	0.300597	0.290782	0.324193	0.286738	0.278207	0.311437

PARAMETRIC STUDY

3.1. Variation of Active Earth Pressure for ϕ :-

Fig.2. shows the variations of seismic active earth pressure coefficient (K_{ae}) with k_h for different value of ϕ at k_v

= $k_h/2$ and δ = $\phi/2$. From the graph, it is seen that seismic active earth pressure coefficient is going to be decreased due to increase in ϕ but it is going to increase due to increase of k_h. For example, for k_h = 0.1, at ϕ = 20°, 30° and 40°,the magnitude of K_{ae} is 0.505, 0.349 and 0.239 respectively. Due to increase in ϕ, the self-retaining capacity of the backfill increases which resembles for the fact to decrease in the value of K_{ae}



3.2.

Figure 2. Variation of seismic active earth pressure coefficient (K_{ae}) with k_h for different value of ϕ when k_v= $k_h/2$, δ =ϕ/2

Variation of Active Earth Pressure for δ:-

Fig.3. shows the variations of seismic active earth pressure coefficient with k_h for different values of δ when ϕ =30° and k_v = $k_h/2$. From the graph, it is seen that the co-efficient of seismic active earth pressure K_{ae} is going to be decreased due to increase in δ at lower values of δ, but at higher values of δ it is going to be increased due to increase in values of δ. For example, at k_h = 0.2 and k_v = $k_h/2$, for ϕ =30° due to increase in δ from 0 to $\phi/2$, the coefficient K_{ae} is decreased by 4.55% but due to increase in δ from $\phi/2$ to ϕ K_{ae} is increased by 4%

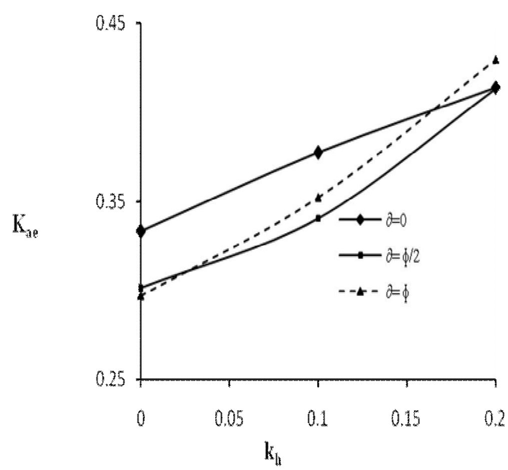


Figure 3. Variation of seismic active earth pressure coefficient (K_{ae}) with k_h for different value of δ when k_v= $k_h/2$, ϕ=30°, H=10 m, γ=18 KN/m^3

COMPARISON

Table.2 shows the comparison of K_{ae} values obtained from present analysis and available theories in seismic case for ϕ=30, δ=ϕ/2

k_h	k_v	Mononobe-Okabe Method	Sharma and Ghosh (2012)	Present analysis
0	0	0.3010	0.301	0.301416
0.1	0.05	0.3386	0.343	0.349689
0.2	0.1	0.4130	0.413	0.413569
0.2	0.2	0.4030	0.378	0.334902

CONCLUSION

Formulations are developed for wedge angle and t/T at collapse considering active state of equilibrium using pseudo-dynamic criteria of failure. On the basis of formulated wedge angle and t/T at collapse, active earth pressure co-efficient are determined. A detailed parametric study shows that the coefficient of active earth pressure is going to be reduced due to increase in ϕ , δ but it is going to be increased due to increase in seismic accelerations. A detailed comparison of the results as obtained from present analysis with Mononobe-Okabe methodology justifies the acceptability of the results as obtained from this solutions.

NOTATIONS

a_h,a_v=amplitude of horizontal and vertical seismic acceleration respectively.

g = acceleration due to gravity.

H = height of the retaining wall

K_{ae}, = pseudo dynamic seismic active earth pressure coefficient

k_h , k_v =seismic acceleration coefficient in the horizontal and vertical direction respectively.

P_a= pseudo-dynamic active thrust

W = weight of failure wedge.

Q_h, Q_v = horizontal and vertical inertia force due to seismic accelerations respectively.

t, T = time (seconds) and period of lateral shaking (seconds).

V_s,V_p = shear and compression wave velocity respectively.

θ = angle of inclination of the failure surface with the vertical.

ϕ = friction angle of the backfill soil.

δ = soil- wall interface friction angle.

γ = unit weight of the soil.

λ = TV_s = wave length of shear wave.

η = TV_p = wave length of compression wave.

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