Determination of Time to Recruitment and Recruitment Time in a Manpower System with Two Groups

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Abstract

When the Manpower system of an organization is exposed to Cumulative Shortage Process due to attritions that cause manpower loss, breakdown occurs at threshold level. In this paper, I consider the Manpower System with two groups A and B. Group A consists of man- power other than top management level executives. Group B consists of top level management executives. The shortages of group A occur in accordance with the Modified Erlang process and group B has shortage process with varying shortage rates. Recruitment is done to fill all the shortages of the two groups. We find the expected time to recruit and recruitment time. Numerical illustrations are presented.

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Keywords:

Manpower System, Attrition, Shortage, Cumulative Shortage Process, Extended Exponential distribution.

1. INTRODUCTION

The total flow out of the Manpower System (MPS) of an organization due to resignation, dismissal and death is called shortage. This shortage, due to the manpower loss, should be compensated by recruitment. But recruitment involves huge cost and hence cannot be made frequently to match the attritions. Hence the MPS is allowed to undergo Cumulative Shortage Process (CSP). The accumulated random amount of shortages due to successive attritions leads to the breakdown of the MPS when the total shortage crosses a random threshold level. The breakdown point or threshold is that point at which immediate recruitment becomes necessary.

The shortage of MPS due to manpower loss depends on many factors. Such models have been discussed by Grinold and Marshall [5], Bartholomew and Forbes [7] and Vajda [6]. Statistical approach in manpower planning has been discussed by Bartholomew [1]. Markovian models are designed for shortage and promotion in MPS by Vassiliou [4]. Subramanian. V. [9] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models. Lesson [8] has given methods to compute shortages and promotion intensities which produce the proportions corresponding to some desired planning proposals.

Esary et al. [3] have discussed that any component or device, when ex- posed to shocks which cause damage to the device, is likely to fail when the total accumulated damage exceeds a level called threshold. Gaver. D.P. [2] has discussed point process problems in Reliability Stochastic point processes. S. Mythili and R. Ramanarayanan have done probabilistic analysis of time to recruit and recruitment time in manpower planning [13]. They have also analysed the same in MPS with two groups [14]. Sathiyamoorthi. R. and Parthasarathy. S. [11] have found the expected time to recruit when threshold distribution has SCBZ property. The shortage rate changes after an exponential time from one rate to another. In [ ], Mythili and Ramanarayanan constructed a model by applying a new concept that is slightly modified upon the concept introduced as Setting the Clock Back to Zero (SCBZ) by Raja Rao [10] and studied by S. Murthy and R. Ramanarayanan [12]

In this paper, I consider MPS with two groups A and B. Group A consists of manpower other than top management level executives, group B consists of top management level executives. Group A is exposed to shortage process which is Modified Erlang. The shortage process of group B is considered extended exponential distribution The time to recruit T is given by $T = \min\{T_1, T_2\}$ where T_1 and T_2 are the times to breakdown of groups A and B respectively. Assuming that the recruitment time R of a shortage is independent of the shortage magnitude, I find the joint Laplace-Stieltjes transform of time to recruit and recruitment time.

2.ASSUMPTIONS

1. Group A is given at the most k observation times each with exponential distribution with parameter 'λ' before recruitment. On completion of the first exponential observation time, recruitment is done with probability α, or, the second observation starts with probability β, where $\alpha + \beta = 1$. The process is repeated upto i observations for $1 \le i \le k - 1$. On completion of the k^{th} observation, recruitment is done with probability $= 1$. If T_1 is the time to recruit due to group A and X_1 , X_2, . . . are the shortages caused by manpower loss in group A, then,

$$T_1 = \sum_{j=1}^{i} X_j \text{ with probability } \alpha\beta^{i-1} \text{ for } 1 \le i \le k-1,$$

or, $T_1 \quad \sum_{j=1}^{k} X_j$ with probability β^{k-1}.

2. Group B has shortage process with extended exponential distribution..

3. Let T_2 be the time at which breakdown of group B occurs necessitating immediate recruitment.

4. Recruitment for MPS starts if either of the groups A or B has a break- down. All the shortages due to manpower loss are compensated by recruitment.

5. When recruitment is done due to breakdown of group A, recruitment time corresponding to the i^{th} observation is R_i, $1 \le i \le k$. When the breakdown occurs due to group B, recruitment is done for

shortages in group B and also for shortages in group A for the number of observations completed. All the recruitment times are independent and identically distributed random variables with distribution function R(y) such that

$\int_0^y y dR(y) < \infty$. All the recruitments are done one by one.

3.ANALYSIS

Based on the assumptions, recruitment starts at time T = min{T_1, T_2}. Identifying the exponential phase time of the modified Erlangian, the pdf of time T_1 is given by

$$f(x)=\alpha\, e^{-\lambda x} \lambda \sum_{i=0}^{k-2} \frac{(\lambda x)^i}{i} \beta_i \div \beta^{k-1} \lambda \frac{(\lambda x)^{k-1}}{(k-1)} e^{-\lambda x} \quad (1)$$

The p.d.f of time T_2 is given by

$$H(x)=(1 - e^{-\theta x})^2 \quad (2)$$

The recruitment time Rt is given by $Rt = \sum_{j=0}^{i} R_j$ when breakdown occurs due to group A for $1 \le i \le k$. $Rt = \sum_{j=0}^{i} R_j + R$ when breakdown occurs due to group B and i observations are completed for group A.

The pdf of T and Rt is given by

$$\frac{\partial^2}{\partial x \partial y} P(T \le x, Rt \le y) = (1 - H(x)) \left[\lambda e^{-\lambda x} \alpha r^*(y) + \lambda \frac{(\lambda x)}{1!} e^{-\lambda x} \alpha \beta r^{*2}(y) + \lambda \frac{(\lambda x)^2}{2!} e^{-\lambda x} \alpha \beta^2 r^{*3}(y) + \ldots\ldots + \lambda \frac{(\lambda x)^{k-1}}{(k-1)!} e^{-\lambda x} \beta^{k-1} r^{*k}(y) \right] + h(x) \sum_{i=0}^{k-1} e^{-\lambda x} \frac{(\lambda x)^i}{i!} \beta^i r^*(i+1)(y)$$

$$= \left(2e^{-\theta x} - e^{-2\theta x}\right) \sum_{i=0}^{k-2} \alpha e^{-\lambda x} \lambda \frac{(\lambda x)^i}{i!} \beta^i r^{*(i+1)}(y) + \left(2e^{-\theta x} - e^{-2\theta x}\right) \lambda \frac{(\lambda x)^{k-1}}{(k-1)!} e^{-\lambda x} \beta^{k-1} r^{*k}(y) + \left(2\theta e^{-\theta x} - 2\theta e^{-2\theta x}\right) \sum_{i=0}^{k-1} e^{-\lambda x} \frac{(\lambda x)^i}{i!} \beta^i r^{*(i+1)}(y) \quad (3)$$

Using double Laplace tansform we get

$$E\left(e^{-\in T}e^{-\eta Rt}\right) =$$

$$\left\{\frac{2r^*(\eta)}{\lambda+\in+\theta-\lambda\beta r^*(\eta)}\right\}\times\left[\left(\theta+\alpha\lambda-\alpha\lambda\left(\frac{\lambda\beta r^*(\eta)}{\lambda+\in+\theta}\right)^{k-1}\right)-\theta\left(\frac{\lambda\beta r^*(\eta)}{\lambda+\in+2}\right)^{k}\right]-$$

$$\left\{\frac{r^*(\eta)}{\lambda+\in+2\theta-\lambda\beta r^*(\eta)}\right\}\times\left[\left(2\theta+\alpha\lambda-\alpha\lambda\left(\frac{\lambda\beta r^*(\eta)}{\lambda+\in+2\theta}\right)^{k-1}\right)-2\theta\left(\frac{\lambda\beta r^*(\eta)}{\lambda+\in+2\theta}\right)^{k}\right]+$$

$$2\beta^{k-1}\left(\frac{\lambda r^*(\eta)}{\lambda+\in+2}\right)^{k}-\beta^{k-1}\left(\frac{\lambda r^*(\eta)}{\lambda+\in+2\theta}\right)^{k} \qquad (4)$$

When $\eta = 0 and \in = 0$ we have

$$E\left(e^{-\in T}\right)=\frac{2\lambda\alpha\left[1-\left(\frac{\lambda\beta}{\lambda+\in+\theta}\right)^{k-1}\right]}{(\in+\theta+\lambda\alpha)}-\frac{\lambda\alpha\left[1-\left(\frac{\lambda\beta}{\lambda+\in+2\theta}\right)^{k-1}\right]}{(\in+2\theta+\lambda\alpha)}$$

$$=2\beta^{k-1}\left(\frac{\lambda}{\lambda+\in+\theta}\right)^{k}-\beta^{k-1}\left(\frac{\lambda}{\lambda+\in+2\theta}\right)^{k}+\frac{2\theta\left[1-\left(\frac{\lambda\beta}{\lambda+\in+\theta}\right)^{k}\right]}{(\in+\theta+\lambda\alpha)}-$$

$$\frac{2\theta\left[1-\left(\frac{\lambda\beta}{\lambda+\in+2\theta}\right)^{k}\right]}{(\in+2\theta+\lambda\alpha)} \qquad (5)$$

$$E\left(e^{-\eta Rt}\right)=$$

$$\left\{\frac{2r^{*}(\eta)}{\lambda+\theta-\lambda\beta r^{*}(\eta)}\right\}\times\left[\left(\theta+\alpha\lambda-\alpha\lambda\left(\frac{\lambda\beta r^{*}(\eta)}{\lambda+2}\right)^{k-1}\right)-2\left(\frac{\lambda\beta r^{*}(\eta)}{\lambda+2}\right)^{k}\right]-$$

$$\left\{\frac{r^{*}(\eta)}{\lambda+2\theta-\lambda\beta r^{*}(\eta)}\right\}\times\left[\left(2\theta+\alpha\lambda-\alpha\lambda\left(\frac{\lambda\beta r^{*}(\eta)}{\lambda+2\theta}\right)^{k-1}\right)-2\theta\left(\frac{\lambda\beta r^{*}(\eta)}{\lambda+2\theta}\right)^{k}\right]+$$

$$2\beta^{k-1}\left(\frac{\lambda r^{*}(\eta)}{\lambda+2}\right)^{k}-\beta^{k-1}\left(\frac{\lambda r^{*}(\eta)}{\lambda+2\theta}\right)^{k} \qquad (6)$$

From (5) and (6), the expected time to recruitment and expected recruitment time are obtained as below

$$E(T)=\frac{2\left[1-\left(\frac{\lambda\beta}{\lambda+\theta}\right)^{k}\right]}{(\theta+\lambda\alpha)}-\frac{\left[1-\left(\frac{\lambda\beta}{\lambda+2\theta}\right)^{k}\right]}{(2\theta+\lambda\alpha)}$$

$$E(Rt)=E(R)\left\{1+\frac{2\beta\lambda\left[1-\left(\frac{\lambda\beta}{\lambda+\theta}\right)^{k-1}\right]}{(\theta+\lambda\alpha)}-\frac{\beta\lambda\left[1-\left(\frac{\beta\lambda}{2\theta+\lambda}\right)^{k-1}\right]}{(2\theta+\alpha\lambda)}\right\}$$

4. NUMERICAL ILLUSTRATION

The values of the mean and variance of the time to recruitment can be determined numerically using the above expressions when the values of the various parameters are given. The impact of the nodal parameters λ ***and a*** on these measures is given as findings.

Table 1 : Effect of λ on the Expected time to recruitment and Expected Recruitment time

$$\left(\alpha=0.2,\ \beta=0.8,\quad \theta=0.2,\ \ k=2 \quad E(R)=3\right)$$

λ	r1 = E(T)	r2 = E(Rt)
1	1.3444	5
2	0.8009	5.2400
3	0.5633	5.3143
4	0.4325	5.3467
5	0.3503	5.3636
6	0.2941	5.3736
7	0.2533	5.3800
8	0.2223	5.3843
9	0.1981	5.3874
10	0.1786	5.3896

Figure 1 & 2 : Effect of λ on the Expected time to recruitment and Expected Recruitment time $(\alpha = 0.2,\ \beta = 0.8,\quad k = 2\quad E(R) = 3)$

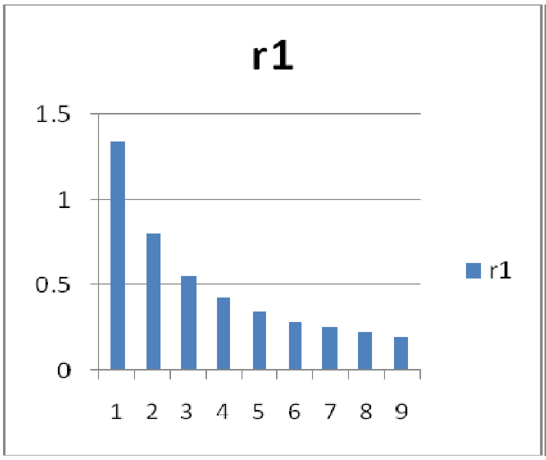


Figure 1

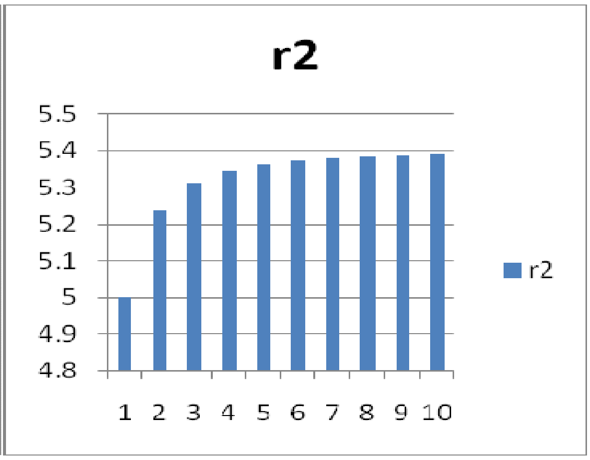
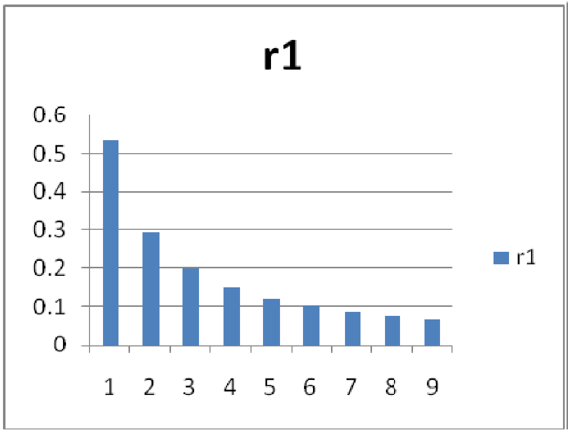


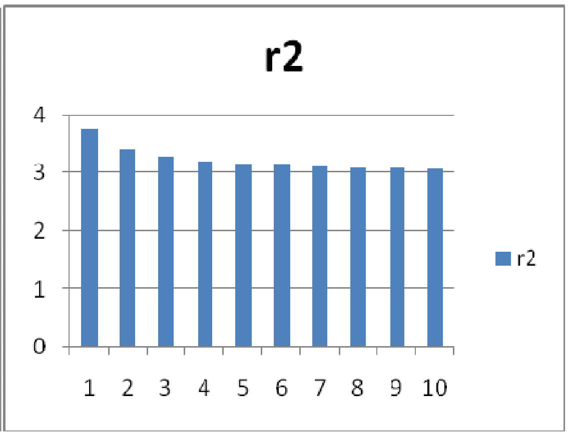
Figure 2

Table 2 : Effect of α on the Expected time to recruitment and Expected Recruitment time $(\lambda = 0.2,\ \beta = 0.8,\quad k = 2\quad E(R) = 3)$

α	r1 = E(T)	r2 = E(Rt)
1	0.5341	3.7733
2	0.2924	3.4160
3	0.1989	3.2800
4	0.1500	3.2098
5	0.1202	3.1673
6	0.1002	3.1389
7	0.0858	3.1187
8	0.0750	3.1035
9	0.0666	3.0918
10	0.0599	3.0824

Figure 3 & 4 : Effect of α on the Expected time to recruitment and Expected Recruitment time $(\lambda = 0.2,\ \beta = 0.8,\quad k = 2\quad E(R) = 3)$





5.CONCLUSION

From table 1 (also from Figures 1& 2) we observe the behaviour of E(T) and E(Rt) i.e., mean Time to recruit and mean Recruitment time for fixed values of

$$\alpha = 0.2,\ \beta = 0.8,\quad \theta = 0.2,\ k = 2\quad E(R) = 3$$

When the parameter λ increases, the value of E(T) increases and E(Rt) decreases.

Table 2 and also figures 3 & 4) show if α increases, both E(T) and E(Rt) decrease.

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