

Dynamic Modeling of Induction Motor with Rotor Field Orientation

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Abstract

Induction motor is the most commonly used in industrial applications because it has a remarkably simple, cheap, highly reliable and robust construction. The induction motor control by varying its input according to a mathematical model of specific synchronous reference frame in a complex vector is called vector control. In vector control, a complex current is synthesized from two quadrature components, one of which to determine the flux level in the motor, and another which controls the motor torque. In order to comprehend and analyze vector control technique of induction motors, the dynamic model is the major and necessary requirement. In this paper, the dynamic modeling of the induction motor with the chosen rotor frame of reference is explained in details.

Key Words:

Dynamic model; I_{mr} ; Rotor field orientation; I_{sq} ;
Frame of reference

Introduction

The induction motor especially the squirrel cage type, is the most widely used industrial electronic motor. Although the mathematical model of the induction motor has been known for decades, only after the introduction of vector control technique in the 1970's a powerful theoretical tool became available to control the IM for high performance applications. During the 1980's with the advance of semiconductor technology (power and digital electronics), the vector control

technique became technically and economically feasible [1].

The dynamic properties of the induction motor as a control plant can be described by a set of nonlinear differential equations. These equations relate the electrical quantities currents and voltages of the induction motor with the mechanical quantities torque and speed [2].

Induction motor model with rotor field

The analysis of the induction motor with rotor field orientation as a chosen frame of reference is the more normally used than the other methods of field orientation. In order to simplify the model and eliminate the time dependence of variables, transformations of phase variables are used [3]. Some usual approximations are made to simplifying the modeling analysis, These assumptions are;

- 1- The windings are sinusoidally distributed in the cylindrical stator and rotor cores (i.e. constant air gap). So, the mmf waves are sinusoidal that is, mmf space harmonics, stator harmonics and end effects are neglected.
- 2- The permeability of the fully laminated stator and rotor assumed infinite and disregarding magnetic circuit saturation and iron losses as well as eddy currents in the conductors.

The currents and voltages may be of arbitrary wave shape as long as their summations is zero which is dictated by the isolated neutrals and symmetry of the three-phase windings [4]. The three stator currents are given by:

$$i_{s1}(t) + i_{s2}(t) + i_{s3}(t) = 0 \quad (1)$$

The three rotor currents are given by:

$$i_{r1}(t) + i_{r2}(t) + i_{r3}(t) = 0 \quad (2)$$

The following vectorial equations of the induction motor with a short circuited rotor electrical circuit, in a rotor reference frame using space phase rotation are describe the non-stationary behavior of the motor:

$$R_s i_s(t) + L_s (di_s(t) / dt) + L_m [d(i_r e^{j\theta}) / dt] = U_s(t) \quad (3)$$

$$R_r i_r(t) + L_r (di_r(t) / dt) + L_m [d(i_s e^{-j\theta}) / dt] = 0 \quad (4)$$

$$J(d\omega / dt) = T_e - T_L = (2/3)L_m I_m (i_s(t)[i_r(t)e^{-j\theta}]^*) - T_L \quad (5)$$

$$(d\theta / dt) = \omega \quad (6)$$

Where:

i^* is the conjugate complex of the vector i .

L_s and L_r are the stator and rotor inductances per phase referred to stator and are given by:

$$L_s = (1 + \sigma_s)L_m \quad (7)$$

$$L_r = (1 + \sigma_r)L_m \quad (8)$$

Where:

$$\sigma_s = [(L_s / L_m) - 1] \quad (9)$$

And

$$\sigma_r = [(L_r / L_m) - 1] \quad (10)$$

The control of such a plant described by the Eq. 3 to Eq. 6 becomes complicated because of intricate coupling between all the control inputs. This problem can be overcome by the control of field oriented quantities which reduces the control of an ac induction motor to that of a separately excited compensated dc motor. The current space vectors are defined by vectorial combination of the phase currents of the stator and rotor respectively as:

$$i_s(t) = (2/3)(i_{s1}(t) + i_{s2}(t)e^{j\frac{2\pi}{3}} + i_{s3}(t)e^{j\frac{4\pi}{3}}) \quad (11)$$

$$i_r(t) = (2/3)(i_{r1}(t) + i_{r2}(t)e^{j\frac{2\pi}{3}} + i_{r3}(t)e^{j\frac{4\pi}{3}}) \quad (12)$$

Correspondingly, the terminal voltage space vector combines the phase to neutral voltages as:

$$u_s(t) = (2/3)(u_{s1} + u_{s2}(t)e^{j\frac{2\pi}{3}} + u_{s3}(t)e^{j\frac{4\pi}{3}}) \quad (13)$$

Applying the two axis theorem on the stator current $i_s(t)$ in order to convert the stator fed three phase system into a second stator fixed two phase rectangular (α - β) system. The α -axis is chosen to be in phase with the motor phase 1. Since, the stator current becomes

$$i_s(t) = (2/3)(i_{s\alpha}(t) + i_{s\beta}(t)e^{j\frac{\pi}{2}}) \quad (14)$$

Then the α - β stator currents are:

$$i_{s\alpha}(t) = i_{s1}(t) \quad (15)$$

$$i_{s\beta}(t) = (1/\sqrt{3})[i_{s1}(t) + 2i_{s2}(t)] \quad (16)$$

This model can be transformed into the rotating reference frame (defined by the rotating flux vector) to get the relation between field oriented quantities [5]. When the mmf wave of the stator currents described by $i_s(t)$ is referred to this frame of reference.

$$i_{mr}e^{j\phi} - i_s(t) = (1 + \sigma_r)i_r(t)e^{j\theta} \quad (26)$$

$$i_r(t)e^{j\theta} = [1/(1 + \sigma_r)][i_{mr}e^{j\phi} - i_s(t)] \quad (27)$$

Substituting from Eq. 19 in Eq. 24, then:

$$i_r(t)e^{j\theta} = [1/(1 + \sigma_r)][i_{mr}(t) - i_s(t)] \quad (28)$$

Or

$$i_r(t) = [1/(1 + \sigma_r)][i_{mr}(t) - i_s(t)]e^{-j\theta} \quad (29)$$

And

$$i_s(t) = i_{mr}(t) - (1 + \sigma_r)i_r(t)e^{j\theta} \quad (30)$$

At constant speed and torque the transformed variables are constant dc values except for ripple caused by inverter operation.

$$u_s(t) = R_s i_s(t) + L_s [di_s(t) / dt] + [L_m / (1 + \sigma_r)][d(i_{mr} - i_s(t)) / dt] \quad (31)$$

$$u_s(t) = R_s i_s(t) + [L_s - (L_m / (1 + \sigma_r))] [di_s(t) / dt] + [L_m / (1 + \sigma_r)][di_{mr}(t) / dt] \quad (32)$$

Eq. 7, Eq. 9, and Eq. 10 gives:

$$(1 + \sigma_r) = [L_m / L_s (1 - \sigma)] \quad (33)$$

Substituting from Eq. 33 in Eq. 32 to get:

$$u_s(t) = R_s i_s(t) + \sigma L_s [di_s(t) / dt] + (1 - \sigma)[di_{mr}(t) / dt] \quad (34)$$

By substituting from Eq. 19 and Eq. 20 in Eq. 34 and separating the real and imaginary parts, the direct and quadrature components of stator voltage can be defined as:

$$U_{sd} = R_{sd} I_{sd} + \sigma L_s [dI_{sd} / dt] - \sigma L_s \omega_1 I_{sq} + (1 - \sigma) L_s [di_{mr}(t) / dt] \quad (35)$$

$$U_{sq} = R_{sq} I_{sq} + \sigma L_s [dI_{sq} / dt] - \sigma L_s \omega_1 I_{sd} + (1 - \sigma) L_s di_{mr}(t) \quad (36)$$

Substituting from Eq. 29 in the rotor voltage Eq. 4, then:

$$[(R_r(i_{mr}(t) - i_s(t))e^{j\theta}) / (1 - \sigma_r)] + L_r / (1 - \sigma_r)[di_{mr}(t) - i_s(t) / dt]e^{-j\theta} \quad (37)$$

$$+ L_m [di_s(t)e^{-j\theta} / dt] = 0$$

Substituting from Eq. 19 in Eq. 37 and separating real and imaginary parts, the transformed variables can be defined as;

$$T_r [di_{mr} / dt] + i_{mr} = I_{sd} \quad (38)$$

$$d\phi / dt = \omega_1 = \omega_{s1} + \omega = [I_{sq} / T_r i_{mr}] + \omega \quad (39)$$

$$J(d\omega / dt) = T_e - T_L = K_t i_{mr} I_{sq} - T_L \quad (40)$$

$$d\theta / dt = \omega \quad (41)$$

Where:

K_t : is the torque constant and is equal to $(3/2)[L_m / (1 - \sigma_r)]$

Eq. 30 can be written as:

$$i_s(t)e^{-j\theta} = i_{mr}e^{-j\theta} - (1 + \sigma_r)i_r(t) \quad (42)$$

Substituting from Eq. 42 in the rotor voltage Eq. 4, then:

$$R_r i_r(t) + L_r (di_r(t) / dt) + L_m d[i_{mr}(t)e^{-j\theta} - (1 + \sigma_r)i_r(t) / dt] = 0 \quad (43)$$

By substituting from Eq. 19 in Eq. 43 and separating the real and imaginary parts, the direct and quadrature components of the rotor current can be defined as:

$$I_{rd} = (L_m / R_r)[di_{mr} / dt] = T_r (di_{mr} / dt) / (1 + \sigma_r) \quad (44)$$

$$I_{rq} = (L_m \omega_{s1} / R_r) i_{mr} = T_r \omega_{s1} i_{mr} / (1 + \sigma_r) = I_{sq} (1 + \sigma_r) \quad (45)$$

Discussion

The dynamic behavior and transient response of the induction motor are described by the Eq. 34 to Eq. 41. Equation 35 and Eq. 36 describe the transition from the field-oriented voltage components U_{sd} and U_{sq} to the current components I_{sd} and I_{sq} which involves small leakage time constants and some interactions.

The slow dynamics in the direct axis are governed by Eq. 38, with acting as control input. The angular position of the flux vector is determined by Eq. 39, with I_{sq} serving as a convenient input in order to control the torque from Eq. 40, as the product of i_{mr} , i_{sq} and Eq. 41, describe the rotor angular frequency.

Equations 44 and Eq. 45 describe the rotor field oriented currents of the induction motor. These equations show that, when the magnetizing current i_{mr} is fixed, the direct component of the rotor current i_{rd} , is eliminated, while the quadrature component results from the rotor induced emf (due to the main flux) which impressed on the rotor resistance.

Analogous to control of dc motors, it is the best policy to maintain i_{mr} at its maximum level, limited either by iron saturation below the base speed, or above the base speed (field weakening region) by ceiling voltage of the inverter. On the other hand, I_{sq} should be used to control the torque and the speed of the drive corresponds to the armature current in the separately excited dc motor.

Conclusion

In adjustable speed drives, the transient behavior of the induction motor has to be taken into consideration. Hence, to study the dynamic behavior of the induction motor under both transient and steady state conditions, the mathematical models of the induction motor have been developed in the stationary reference frame by using d-q axes. The induction motors characteristics can be described simply

using the dynamic model equations developed on a rotating reference frame. This paper describes in details, the modeling and analysis of the induction motor with rotor field orientation as the chosen frame of reference and it will be shown that the dynamic equations of the induction motor is simplified and analogous to a DC motor.

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