

Determination of Mean and Variance of the Time to Recruitment with Two Sources of Depletion

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ABSTRACT

In any organization the exit of personnel occurs, especially once the policy decisions relating to pay, perquisites and targets are proclaimed. In real time, once the exit of personnel happens, the recruitment cannot be introduced as a result of it's time overwhelming and costly. Frequent recruitments also are not fascinating. Thus, the recruitment is made solely once the additive loss of manpowers on sequent occasion cross the level referred to as threshold. Based on shock model approach, a mathematical model is constructed using a appropriate univariate policy of recruitment. The mean and variance of the time to recruitment is obtained when the inter-policy decision times and the inter-transfer decision times for the three grades form same renewal process. Numerical illustrations are additionally provided.

Key Words: Three grade system, two sources of depletion, univariate policy of recruitment, renewal process

INTRODUCTION

Frequent wastage or exit of personnel is common in many administrative and production oriented organization. Whenever the organizations announces revised policies regarding sales target, revision of wages, incentives and perquisites, the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover in the organization. As the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitments. Once the total amount of wastage crosses a certain threshold level, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is done at this point of time. The time to attain the breakdown point is an important characteristic for the management of the organization. Many models could be seen in, Barthlomew [1] and Barthlomew and Forbes [2] Researchers [3] and [4] have considered the problem of time to recruitment in a marketing organization under different conditions. In an organization, the depletion of manpower can occur due to two different cases:

1) whenever the policy decisions regarding pay, perquisites and work schedule are revised and

2) due to transfer of personnel to the other organization of the same management.

In the presence of these *two different sources* of depletion, Elangovan et.al [5] have studied the problem of time to recruitment for an organization consisting of one grade and obtained the variance of the time to recruitment using a univariate policy of recruitment when (i) the loss of man power in the organization due to the two sources of depletion and its threshold are independent and identically exponential random variables. (ii) Inter-policy decision times and inter-transfer decision times form the same renewal process. The Authors [6] have discussed a stochastic model for estimating the expected time to recruitment under the assumptions that i) the depletion of manpower is in terms of persons leaving at every epoch of decision making and at every epoch of transfer and hence they are in terms of discrete random variables. ii) the recruitment is carried out as and when the total depletion crosses a level called the threshold which is a discrete random variable. Usha et.al [7] proposed a model to determine the expected time to recruitment time under the assumption that the interarrival times between successive epochs of policy decisions are not independent but correlated, whereas the interarrival times between successive epochs

of transfers are i.i.d random variables. Vijaysankar et.al [8] have constructed a stochastic model by assuming the threshold with two components namely the level of wastage which can be allowed and the manpower which is available from what is known as backup resource. Recently, Uma and Roja Mary [9] have considered a single grade system which is subjected to exodus of personnel due to policy decisions given by the organization and the explicit expression for the long run average cost is derived by considering survival time process, which is a geometric process and the threshold has two components.

In this paper, it is proposed to determine time to recruitment for a three grade manpower system with two sources of depletion and the renewal processes governing the inter-policy decisions times and that of the inter-transfer decisions times are same. The performance measures namely mean time to recruitment and the variances of time to recruitment are derived.

MODEL DESCRIPTION

Consider a three grade organization with univariate policy of recruitment which takes decisions at random era. At every decision making an era a random number of persons quit the organization. There is an

associated loss of manhours to the organization if a person quits. The loss of manhours at any decision forms a sequence of independent and identically distributed random variables. For $i=1,2,3\dots$ let X_{i1}, X_{i2} and X_{i3} be the continuous random variables representing the amount of depletion of manpower in grades A,B and C respectively caused due to the i^{th} policy decision. $g_1(\cdot)$, $g_2(\cdot)$ and $g_3(\cdot)$ are the p.d.f of X_{i1}, X_{i2} and X_{i3} for each i and each form a sequence of i.i.d random variables. $\bar{X}_{m1} = \sum_{i=1}^m X_{i1}$, $\bar{X}_{m2} =$

$\sum_{i=1}^m X_{i2}$ and $\bar{X}_{m3} = \sum_{i=1}^m X_{i3}$. For $j=1,2,3\dots$ let Y_{j1}, Y_{j2} and Y_{j3} d Y_{j3} for each j and each form a sequence of i.i.d random variables. $\bar{Y}_{n1} = \sum_{j=1}^n Y_{j1}$, $\bar{Y}_{n2} = \sum_{j=1}^n Y_{j2}$ and $\bar{Y}_{n3} = \sum_{j=1}^n Y_{j3}$ be the

continuous random variables representing the amount of depletion of manpower in grades A,B and C respectively caused due to the j^{th} transfer decision. $h_1(\cdot)$, $h_2(\cdot)$ and $h_3(\cdot)$ are the p.d.f of Y_{j1}, Y_{j2} an. Let Z_1, Z_2 and Z_3 be independent exponentially distributed threshold levels for the depletion of manpower in grades A, B and C with mean $\frac{1}{\theta_1}, \frac{1}{\theta_2}$ and $\frac{1}{\theta_3}$ respectively and let Z be the

threshold level for the depletion of manpower in the organization with probability density

function $l(.)$. Let the inter-decision times and inter-transfer decision times for the three grades form the same renewal processes be i.i.d exponential random variables with distribution $F(.)$ and $V(.)$ and probability density function $f(.)$ and $v(.)$ respectively with mean $\frac{1}{\mu_1}, \frac{1}{\mu_2}$ ($\mu_1, \mu_2 > 0$). It is assumed that three sources of depletion are independent. Let T be the random variable denoting the time to recruitment with

$$P[T > t] = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} [F_m(t) - F_{m+1}(t)] [V_n(t) - V_{n+1}(t)] P[\bar{X}_{m1} + \bar{X}_{m2} + \bar{X}_{m3} + \bar{Y}_{n1} + \bar{Y}_{n2} + \bar{Y}_{n3} < Z] \tag{1}$$

By the total law of probability

$$P[\bar{X}_{m1} + \bar{X}_{m2} + \bar{X}_{m3} + \bar{Y}_{n1} + \bar{Y}_{n2} + \bar{Y}_{n3} < Z] = \int_0^{\infty} P[\bar{X}_{m1} + \bar{X}_{m2} + \bar{X}_{m3} + \bar{Y}_{n1} + \bar{Y}_{n2} + \bar{Y}_{n3} < Z] k(z) dz \tag{2}$$

The explicit expressions for $E(T)$ and $V(T)$ are obtained by using the equations (1) and (2)

Case(i): $Z = \min(Z_1, Z_2, Z_3)$

In this case,

$$k(z) = (\theta_1 + \theta_2 + \theta_3) e^{-(\theta_1 + \theta_2 + \theta_3)z} \tag{3}$$

From (1),(2) and (3)

$$\begin{aligned}
 P[T > t] &= \sum_{n=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_1 + \theta_2 + \theta_3) g_2(\theta_1 + \theta_2 + \theta_3) g_3(\theta_1 + \theta_2 + \theta_3)]^m \\
 &\quad \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_1 + \theta_2 + \theta_3) h_2(\theta_1 + \theta_2 + \theta_3) h_3(\theta_1 + \theta_2 + \theta_3)]^n \\
 &= \left\{ \begin{aligned} &1 - [1 - g_1(\theta_1 + \theta_2 + \theta_3) g_2(\theta_1 + \theta_2 + \theta_3) g_3(\theta_1 + \theta_2 + \theta_3)] \\ &\sum_{n=0}^{\infty} F_m(t) [g_1(\theta_1 + \theta_2 + \theta_3) g_2(\theta_1 + \theta_2 + \theta_3) g_3(\theta_1 + \theta_2 + \theta_3)]^{m-1} \end{aligned} \right\} \\
 &\quad \left\{ \begin{aligned} &1 - [1 - h_1(\theta_1 + \theta_2 + \theta_3) h_2(\theta_1 + \theta_2 + \theta_3) h_3(\theta_1 + \theta_2 + \theta_3)] \\ &\sum_{n=0}^{\infty} V_n(t) [h_1(\theta_1 + \theta_2 + \theta_3) h_2(\theta_1 + \theta_2 + \theta_3) h_3(\theta_1 + \theta_2 + \theta_3)]^{n-1} \end{aligned} \right\} \tag{4}
 \end{aligned}$$

Since

$$f_m(t) = \frac{\mu_1^m e^{-\mu_1 t} t^{m-1}}{(m-1)!} \text{ and } v_n(t) = \frac{\mu_2^n e^{-\mu_2 t} t^{n-1}}{(n-1)!}$$

distribution $L(.)$, mean $E(T)$ and variance $V(T)$.

Results

The probability distribution of T is given by

$P(T > t) = \{ \text{Probability that there are exactly } m \text{ decisions in all the three grades and } n \text{ transfer decisions and total loss of manpower does not cross the threshold } Z \}$

By hypothesis

$$\begin{aligned}
 & [1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] \\
 & \sum_{n=0}^{\infty} F_m(t) [g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)]^{n-1} \\
 & = 1 - e^{-\mu_1 [1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)]t} \\
 & [1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)] \\
 & \sum_{n=0}^{\infty} V_n(t) [h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]^{n-1} \\
 & = 1 - e^{-\mu_2 [1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]t}
 \end{aligned}
 \tag{5}$$

From (4) and (5)

$$L(t) = 1 - e^{-\{\mu_1 [1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] + \mu_2 [1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]\}t}$$

This is the exponential distribution with parameter

$$\mu_1 [1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] + \mu_2 [1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]$$

Hence

$$E(T) = \frac{1}{\mu_1 [1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] + \mu_2 [1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]}$$

$$V(T) = \frac{1}{(\mu_1 [1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] + \mu_2 [1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)])^2}$$

Case (ii): $Z = \max(Z_1, Z_2, Z_3)$

In this case

$$k(z) = \theta_1 e^{-\theta_1 z} + \theta_2 e^{-\theta_2 z} + \theta_3 e^{-\theta_3 z} - (\theta_1 + \theta_2) e^{-(\theta_1 + \theta_2)z} - (\theta_1 + \theta_3) e^{-(\theta_1 + \theta_3)z} - (\theta_2 + \theta_3) e^{-(\theta_2 + \theta_3)z} + (\theta_1 + \theta_2 + \theta_3) e^{-(\theta_1 + \theta_2 + \theta_3)z}
 \tag{6}$$

From (1),(2) and (6)

$$\begin{aligned}
 P(T > t) = & \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_1)g_2(\theta_1)g_3(\theta_1)]^m \\
 & \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_1)h_2(\theta_1)h_3(\theta_1)]^n \\
 & + \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_2)g_2(\theta_2)g_3(\theta_2)]^m \\
 & \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_2)h_2(\theta_2)h_3(\theta_2)]^n \\
 & + \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_3)g_2(\theta_3)g_3(\theta_3)]^m \\
 & \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_3)h_2(\theta_3)h_3(\theta_3)]^n \\
 & - \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_{1+\theta_2})g_2(\theta_{1+\theta_2})g_3(\theta_{1+\theta_2})]^m \\
 & \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_{1+\theta_2})h_2(\theta_{1+\theta_2})h_3(\theta_{1+\theta_2})]^n \\
 & - \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_{1+\theta_3})g_2(\theta_{1+\theta_3})g_3(\theta_{1+\theta_3})]^m \\
 & \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_{1+\theta_3})h_2(\theta_{1+\theta_3})h_3(\theta_{1+\theta_3})]^n \\
 & - \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_{2+\theta_3})g_2(\theta_{2+\theta_3})g_3(\theta_{2+\theta_3})]^m \\
 & \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_{2+\theta_3})h_2(\theta_{2+\theta_3})h_3(\theta_{2+\theta_3})]^n \\
 & + \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] [g_1(\theta_{1+\theta_2+\theta_3})g_2(\theta_{1+\theta_2+\theta_3})g_3(\theta_{1+\theta_2+\theta_3})]^m \\
 & \sum_{n=0}^{\infty} [V_n(t) - V_{n+1}(t)] [h_1(\theta_{1+\theta_2+\theta_3})h_2(\theta_{1+\theta_2+\theta_3})h_3(\theta_{1+\theta_2+\theta_3})]^n
 \end{aligned}$$

On simplification

$$\begin{aligned}
 P(T > t) = & \left\{ 1 - [1 - g_1(\theta_1)g_2(\theta_1)g_3(\theta_1)] \sum_{m=1}^{\infty} F_m(t) [g_1(\theta_1)g_2(\theta_1)g_3(\theta_1)]^{m-1} \right\} \\
 & \left\{ 1 - [1 - h_1(\theta_1)h_2(\theta_1)h_3(\theta_1)] \sum_{n=1}^{\infty} V_n(t) [h_1(\theta_1)h_2(\theta_1)h_3(\theta_1)]^{n-1} \right\} \\
 & + \left\{ 1 - [1 - g_1(\theta_2)g_2(\theta_2)g_3(\theta_2)] \sum_{m=1}^{\infty} F_m(t) [g_1(\theta_2)g_2(\theta_2)g_3(\theta_2)]^{m-1} \right\} \\
 & \left\{ 1 - [1 - h_1(\theta_2)h_2(\theta_2)h_3(\theta_2)] \sum_{n=1}^{\infty} V_n(t) [h_1(\theta_2)h_2(\theta_2)h_3(\theta_2)]^{n-1} \right\} \\
 & + \left\{ 1 - [1 - g_1(\theta_3)g_2(\theta_3)g_3(\theta_3)] \sum_{m=1}^{\infty} F_m(t) [g_1(\theta_3)g_2(\theta_3)g_3(\theta_3)]^{m-1} \right\} \\
 & \left\{ 1 - [1 - h_1(\theta_3)h_2(\theta_3)h_3(\theta_3)] \sum_{n=1}^{\infty} V_n(t) [h_1(\theta_3)h_2(\theta_3)h_3(\theta_3)]^{n-1} \right\} \\
 & - \left\{ 1 - [1 - g_1(\theta_1 + \theta_2)g_2(\theta_1 + \theta_2)g_3(\theta_1 + \theta_2)] \sum_{m=1}^{\infty} F_m(t) [g_1(\theta_1 + \theta_2)g_2(\theta_1 + \theta_2)g_3(\theta_1 + \theta_2)]^{m-1} \right\} \\
 & \left\{ 1 - [1 - h_1(\theta_1 + \theta_2)h_2(\theta_1 + \theta_2)h_3(\theta_1 + \theta_2)] \sum_{n=1}^{\infty} V_n(t) [h_1(\theta_1 + \theta_2)h_2(\theta_1 + \theta_2)h_3(\theta_1 + \theta_2)]^{n-1} \right\} \\
 & - \left\{ 1 - [1 - g_1(\theta_1 + \theta_3)g_2(\theta_1 + \theta_3)g_3(\theta_1 + \theta_3)] \sum_{m=1}^{\infty} F_m(t) [g_1(\theta_1 + \theta_3)g_2(\theta_1 + \theta_3)g_3(\theta_1 + \theta_3)]^{m-1} \right\} \\
 & \left\{ 1 - [1 - h_1(\theta_1 + \theta_3)h_2(\theta_1 + \theta_3)h_3(\theta_1 + \theta_3)] \sum_{n=1}^{\infty} V_n(t) [h_1(\theta_1 + \theta_3)h_2(\theta_1 + \theta_3)h_3(\theta_1 + \theta_3)]^{n-1} \right\} \\
 & - \left\{ 1 - [1 - g_1(\theta_3 + \theta_2)g_2(\theta_3 + \theta_2)g_3(\theta_3 + \theta_2)] \sum_{m=1}^{\infty} F_m(t) [g_1(\theta_3 + \theta_2)g_2(\theta_3 + \theta_2)g_3(\theta_3 + \theta_2)]^{m-1} \right\} \\
 & \left\{ 1 - [1 - h_1(\theta_3 + \theta_2)h_2(\theta_3 + \theta_2)h_3(\theta_3 + \theta_2)] \sum_{n=1}^{\infty} V_n(t) [h_1(\theta_3 + \theta_2)h_2(\theta_3 + \theta_2)h_3(\theta_3 + \theta_2)]^{n-1} \right\} \\
 & + \left\{ 1 - [1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] \sum_{m=1}^{\infty} F_m(t) [g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)]^{m-1} \right\} \\
 & \left\{ 1 - [1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)] \sum_{n=1}^{\infty} V_n(t) [h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]^{n-1} \right\}
 \end{aligned}$$

$$\begin{aligned}
 P(T > t) = & e^{-\{\mu_1[1-g_1(\theta_1)g_2(\theta_1)g_3(\theta_1)] + \mu_2[1-h_1(\theta_1)h_2(\theta_1)h_3(\theta_1)]\}t} \\
 & + e^{-\{\mu_1[1-g_1(\theta_2)g_2(\theta_2)g_3(\theta_2)] + \mu_2[1-h_1(\theta_2)h_2(\theta_2)h_3(\theta_2)]\}t} \\
 & + e^{-\{\mu_1[1-g_1(\theta_3)g_2(\theta_3)g_3(\theta_3)] + \mu_2[1-h_1(\theta_3)h_2(\theta_3)h_3(\theta_3)]\}t} \\
 & - e^{-\{\mu_1[1-g_1(\theta_1 + \theta_2)g_2(\theta_1 + \theta_2)g_3(\theta_1 + \theta_2)] + \mu_2[1-h_1(\theta_1 + \theta_2)h_2(\theta_1 + \theta_2)h_3(\theta_1 + \theta_2)]\}t} \\
 & - e^{-\{\mu_1[1-g_1(\theta_1 + \theta_3)g_2(\theta_1 + \theta_3)g_3(\theta_1 + \theta_3)] + \mu_2[1-h_1(\theta_1 + \theta_3)h_2(\theta_1 + \theta_3)h_3(\theta_1 + \theta_3)]\}t} \\
 & - e^{-\{\mu_1[1-g_1(\theta_3 + \theta_2)g_2(\theta_3 + \theta_2)g_3(\theta_3 + \theta_2)] + \mu_2[1-h_1(\theta_3 + \theta_2)h_2(\theta_3 + \theta_2)h_3(\theta_3 + \theta_2)]\}t} \\
 & + e^{-\{\mu_1[1-g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] + \mu_2[1-h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]\}t}
 \end{aligned}$$

Let

$$A = \{\mu_1[1 - g_1(\theta_1)g_2(\theta_1)g_3(\theta_1)] + \mu_2[1 - h_1(\theta_1)h_2(\theta_1)h_3(\theta_1)]\}$$

$$B = \{\mu_1[1 - g_1(\theta_2)g_2(\theta_2)g_3(\theta_2)] + \mu_2[1 - h_1(\theta_2)h_2(\theta_2)h_3(\theta_2)]\}$$

$$C = \{\mu_1[1 - g_1(\theta_3)g_2(\theta_3)g_3(\theta_3)] + \mu_2[1 - h_1(\theta_3)h_2(\theta_3)h_3(\theta_3)]\}$$

$$E = \{\mu_1[1 - g_1(\theta_1 + \theta_2)g_2(\theta_1 + \theta_2)g_3(\theta_1 + \theta_2)] + \mu_2[1 - h_1(\theta_1 + \theta_2)h_2(\theta_1 + \theta_2)h_3(\theta_1 + \theta_2)]\}$$

$$F = \{\mu_1[1 - g_1(\theta_1 + \theta_3)g_2(\theta_1 + \theta_3)g_3(\theta_1 + \theta_3)] + \mu_2[1 - h_1(\theta_1 + \theta_3)h_2(\theta_1 + \theta_3)h_3(\theta_1 + \theta_3)]\}$$

$$G = \{\mu_1[1 - g_1(\theta_3 + \theta_2)g_2(\theta_3 + \theta_2)g_3(\theta_3 + \theta_2)] + \mu_2[1 - h_1(\theta_3 + \theta_2)h_2(\theta_3 + \theta_2)h_3(\theta_3 + \theta_2)]\}$$

$$D = \{\mu_1[1 - g_1(\theta_1 + \theta_2 + \theta_3)g_2(\theta_1 + \theta_2 + \theta_3)g_3(\theta_1 + \theta_2 + \theta_3)] + \mu_2[1 - h_1(\theta_1 + \theta_2 + \theta_3)h_2(\theta_1 + \theta_2 + \theta_3)h_3(\theta_1 + \theta_2 + \theta_3)]\}$$

$$P(T > t) = e^{-At} + e^{-Bt} + e^{-Ct} - e^{-Et} - e^{-Ft} - e^{-Gt} + e^{-Dt} \tag{8}$$

It is known that

$$E(T^r) = r \int_0^{\infty} t^{r-1} P(T > t) dt, \quad r \geq 1 \tag{9}$$

From (8) and (9)

$$E(T) = \frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \frac{1}{D} - \frac{1}{E} - \frac{1}{F} - \frac{1}{G}$$

$$V(T) = \frac{1}{A^2} + \frac{1}{B^2} + \frac{1}{C^2} + \frac{1}{D^2} - \frac{3}{E^2} - \frac{3}{F^2} - \frac{3}{G^2} - \frac{2}{A} \left(\frac{1}{B} + \frac{1}{C} + \frac{1}{D} - \frac{1}{E} - \frac{1}{F} - \frac{1}{G} \right) - \frac{2}{B} \left(\frac{1}{C} + \frac{1}{D} - \frac{1}{E} - \frac{1}{F} - \frac{1}{G} \right) - \frac{2}{C} \left(\frac{1}{D} - \frac{1}{E} - \frac{1}{F} - \frac{1}{G} \right) + \frac{2}{D} \left(\frac{1}{E} + \frac{1}{F} + \frac{1}{G} \right) - \frac{2}{E} \left(\frac{1}{F} + \frac{1}{G} \right) - \frac{2}{FG}$$

NUMERICAL ILLUSTRATION:

The values of the mean and variance of the time to recruitment can be determined numerically using the above expressions when the values of the various parameters are given. The impact of the nodal

parameters α_{11} and μ_1 on these measures is given as findings.

Table 1

$$\mu_1 = 0.1, \mu_2 = 0.2, \alpha_{12} = 0.2, \alpha_{13} = 0.3,$$

$$\beta_{11} = 0.1, \beta_{12} = 0.2, \beta_{13} = 0.3, \theta_1 = 0.1, \theta_2 = 0.2, \theta_3 = 0.3$$

α_{11}	Case(i)		Case(ii)	
	E(T)	V(T)	E(T)	V(T)
0.1	3.3735	11.3805	4.6963	21.0601
0.2	3.3837	11.4493	4.9100	22.7760
0.3	3.3917	11.5033	5.0242	23.7142
0.4	3.3981	11.5468	5.0950	24.3052
0.5	3.4033	11.5826	5.1429	24.7114
0.6	3.4077	11.6125	5.1774	25.0076

Table 2

$$\alpha_{11} = 0.1, \mu_2 = 0.2, \alpha_{12} = 0.2, \alpha_{13} = 0.3,$$

$$\beta_{11} = 0.1, \beta_{12} = 0.2, \beta_{13} = 0.3, \theta_1 = 0.1, \theta_2 = 0.2, \theta_3 = 0.3$$

μ_1	Case(i)		Case(ii)	
	E(T)	V(T)	E(T)	V(T)
0.1	3.3735	11.3805	4.6963	21.0601
0.2	2.5301	6.4015	3.5222	11.8463
0.3	2.0241	4.0970	2.8178	7.5816
0.4	1.6867	2.8451	2.3482	5.2650
0.5	1.4458	2.0903	2.0127	3.8682
0.6	1.2651	1.6004	1.7611	2.9616

Conclusions:

From the above tables, when α_{11} increase and keeping all the parameters fixed, the mean and variance of time to recruitment increases and μ_1 increases and keeping all the parameters fixed, the mean and variance of the time to recruitment decreases.

For real applications, the results of any research work should be viable. In the case of stochastic models this is very much essential since the results derived are based on real factors. In any industry or organization, the applications of stochastic model are of great need and it is also useful in every areas of human activity. It is important to identify those areas of human activity where the disequilibrium arises on the demand for manpower and the supply. For the development of human resource management the transformation of real life situations into mathematical model and identification of those areas are to be analyzed.

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