Stochastic Model for a Single Grade System with Same Renewal Process for Inter-Decision Times and Univariate Policy of Recruitment

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ABSTRACT

In this paper, an organization with single grade subjected to exodus of personnel due to policy decisions taken by the organization is considered. In order to avoid the crisis of the organization reaching a breakdown point, a suitable univariate recruitment policy based on shock model approach and cumulative damage process is suggested. A mathematical model is constructed and a performance measures namely the mean and variance of time to recruitment are obtained. The analytical results are graphically illustrated and the influence of nodal parameters on the performance measures are studied and relevant conclusions are presented.

Key Words : *Single grade system, A Univariate policy of recruitment, Time to recruitment.*

INTRODUCTION

Frequent wastage or exit of personnel is common in many administrative and production oriented organization. Whenever the organizations announces revised policies regarding sales target, revision of wages, incentives and perquisites, the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover in the organization. As the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitments.

Once the total amount of wastage crosses a certain threshold level, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is done at this point of time. The time to attain the breakdown point is an important characteristic for the management of the organization. Many models could be seen in ,Barthlomew[1] and Barthlomew and Forbes[2].Many researchers [3] [4] and[7] have considered the problem of time to recruitment in a marketing organization under different conditions. Performance measures for a single grade system with univariate policy of recruitment are obtained by Sathiyamoorthi and Parthasarathy [5] when the policy decision announced for the single grade are governed by same renewal process and the time between two consecutive decisions form a sequence of independent and identically distributed random variables. Also they have assumed that the threshold for the loss of manpower in the organization as the maximum of the thresholds of two grades. Suresh Kumar et al. [6] have analyzed the model by considering the threshold level of the organization as the sum of the thresholds of single grade.. In this paper a single grade organization with univariate of policy of recruitment is considered and performance measures are derived using different distributions.

MODEL DESCRIPTION

Consider a single grade organization with univariate policy of recruitment which takes decisions at random epoch. At every decision making epoch a random number of persons quit the organization. There is an associated loss of man-hours to the organization if a person quits. The loss of man-hours at any decision form a sequence of independent and identically distributed random variables. Threshold for the loss of manpower is considered as a continuous random variable following Erlang K=2 distribution with parameter μ. The inter-decision times are independent and identically distributed random variables. Recruitment takes place only at decision points and time of recruitment is negligible.

NOTATIONS

X_i : independent and identically distributed continuous random variable denoting the total depletion of manpower i^{th} decision, i = 1, 2, ... with probability density function g(.) and cumulative distribution function G(.)

Y : continuous random variable representing the threshold level following Erlang k=2 distribution with parameter μ.

H(.) : the probability distribution function of threshold.

T : continuous random variable representing the time to recruitment.

$V_k(t)$: probability that there are exactly k decisions in (0,t) with intensity represented as a Alpha poisson process with parameters "a" and α is

$$p_{a,\alpha}(n,t)=\sum_{k=0}^{\infty}(-1)^k\binom{k+n}{k}\frac{(at)^{\alpha(k+n)}}{\Gamma(\alpha(k+n)+1)},\quad a>0,\quad 0<\alpha\le 1,\quad n=0,1,2,\ldots$$

The inter decision times follow Mittag-Leffler (1990,2001)distribution which is given by

$$F_{a,\alpha}(t)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{\Gamma(\alpha k+1)}(at)^{\alpha k}\quad,\quad t\ge 0\;,\quad a>0,\; 0<\alpha\le 1$$

ANALYSIS

In this section the expected time to recruitment and variance of time to recruitment are derived.

P {T > t} =

P(there are exactly k decisions and threshold crossing is not taking place during (0,t)

$$=\sum_{k=1}^{\infty}V_k(t)\ \mathrm{P}\left\{\sum_{i=1}^{k}X_i<Y\right\}$$

$$\text{Now, } \mathrm{P}\{X<Y\}=\int_0^{\infty}G(x)f(x)dx$$

$$=\mu^2\int_0^{\infty}G(x)x\ e^{-\mu x}dx$$

$$=-\mu^2\frac{d}{d\mu}\left[\frac{g^*(\mu)}{\mu}\right]$$

Then $P\{X_1 + X_2 + ... + X_k < Y\} = -\mu^2 \frac{d}{d\mu}\left[\frac{g_k^*(\mu)}{\mu}\right]$

$$= -\mu^2 \frac{d}{d\mu}\left[\frac{g^*(\mu)^k}{\mu}\right]$$

$$= -\mu k\left[g^*(\mu)\right]^{k-1} g^{*/}(\mu) + \left[g^*(\mu)\right]^k$$

Now S (t) = P {T > t}

$$= \sum_{k=0}^{\infty} \frac{e^{-(at)^\alpha}\left[(at)^\alpha\right]^k}{(\alpha k)!}\left\{\left[g^*(\mu)\right]^k - \mu k\left[g^*(\mu)\right]^{k-1} g^{*/}(\mu)\right\}$$

$$= \exp\left\{-(at)^\alpha[1-g^*(\mu)]\right\}\left[1-\frac{(at)^\alpha \mu}{\alpha} g^{*/}(\mu)\right]$$

$$L(t) = P\{T \le t\} = 1 - e^{-(at)^\alpha[1-g^*(\mu)]}\left[1-\frac{(at)^\alpha}{\alpha}\mu\, g^{*/}(\mu)\right]$$

If g (.) has Mittag - Leffler distribution with parameter a and a, then

$$g^*(\mu) = \frac{a^\alpha}{a^\alpha + \mu^\alpha} \Rightarrow g^{*/}(\mu) = -\frac{\alpha a^\alpha \mu^{\alpha-1}}{(a^\alpha + \mu^\alpha)^2}$$

$$L(t) = P\{T \le t\} = 1 - \exp\left\{-(t)^\alpha\left[\frac{a^\alpha \mu^\alpha}{a^\alpha + \mu^\alpha}\right]\right\}\left[1 + \frac{a^{2\alpha} t^\alpha \mu^\alpha}{(a^\alpha + \mu^\alpha)^2}\right]$$

The probability density function of time to recruitment T is

$$\Psi(t) = \frac{\alpha a^{\alpha} \mu^{2\alpha}}{(a^{\alpha} + \mu^{\alpha})^{2}} \exp\left\{-(t)^{\alpha} \frac{a^{\alpha} \mu^{\alpha}}{\mu^{\alpha} + a^{\alpha}}\right\} t^{\alpha-1} \left[1 + \frac{a^{2\alpha} t^{\alpha}}{a^{\alpha} + \mu^{\alpha}}\right]$$

The expected time to recruitment T is given by

$$E(T) = \int_0^{\infty} t\psi(t)dt = \frac{1}{a^{\alpha} + \mu^{\alpha}} \left[\frac{a^{\alpha} + \mu^{\alpha}}{a^{\alpha} \mu^{\alpha}}\right]^{\frac{1}{\alpha}} \left[\mu^{\alpha} \Gamma(1/\alpha + 1) + a^{\alpha} \Gamma(1/\alpha + 2)\right]$$

$$\text{And } E(T^{2}) = \frac{1}{a^{\alpha} + \mu^{\alpha}} \left[\frac{a^{\alpha} + \mu^{\alpha}}{a^{\alpha} \mu^{\alpha}}\right]^{\frac{2}{\alpha}} \left[\mu^{\alpha} \Gamma(2/\alpha + 1) + a^{\alpha} \Gamma(2/\alpha + 2)\right)$$

On simplification, we get the variance

$$\mathrm{V(T)} = \frac{(a^{\alpha} + \mu^{\alpha})^{\frac{2}{\alpha}-1}}{a^{2} \mu^{2}} \left\{\left[\mu^{\alpha}\Gamma(2/\alpha+1) + a^{\alpha}\Gamma(2/\alpha+2)\right] - \left[\frac{1}{a^{\alpha} + \mu^{\alpha}}\right]\left[\mu^{\alpha}\Gamma(1/\alpha+1) + a^{\alpha}\Gamma(1/\alpha+2)\right]^{2}\right\}$$

Particular case

Case (i): When k = 1 the mean and variance of the time to recruitment becomes

$$\mathrm{E\ (T)} = \left[\frac{a^{\alpha} + \mu^{\alpha}}{a^{\alpha} \mu^{\alpha}}\right]^{\frac{1}{\alpha}} \Gamma(1/\alpha + 1)$$

$$V(T) = \left[\frac{a^{\alpha} + \mu^{\alpha}}{a^{\alpha}\mu^{\alpha}}\right]^{\frac{2}{\alpha}} \left[\Gamma(2/\alpha+1) - \Gamma(1/\alpha+1)^2\right]$$

This is the Mean and Variance of the time to recruitment when the threshold is exponential distribution.

Case (ii)

When α = 1, the mean and variance of the time to recruitment

E(T)= (μ+2a)/aμ and

$$V(T) = \frac{2a^2 + \mu^2 + 4a\mu}{a^2\mu^2}$$

This is the Mean and Variance of the time to recruitment when the threshold is Erlang k=2 distribution.

Case (iii): When a = 1 and k = 1, the Mean and Variance of the time to

$E(T) = (\mu + \alpha)/a\mu$

$$V(T) = \frac{(a+\mu)^2}{a^2\mu^2}$$

NUMERICAL ILLUSTRATION

The mean and variance of time to recruitment are obtained and represented in figures for the values $\mu = \mathbf{0.2, 0.40.6, 0.8.}\boldsymbol{and}\mathbf{1.0}$ and different values of α .

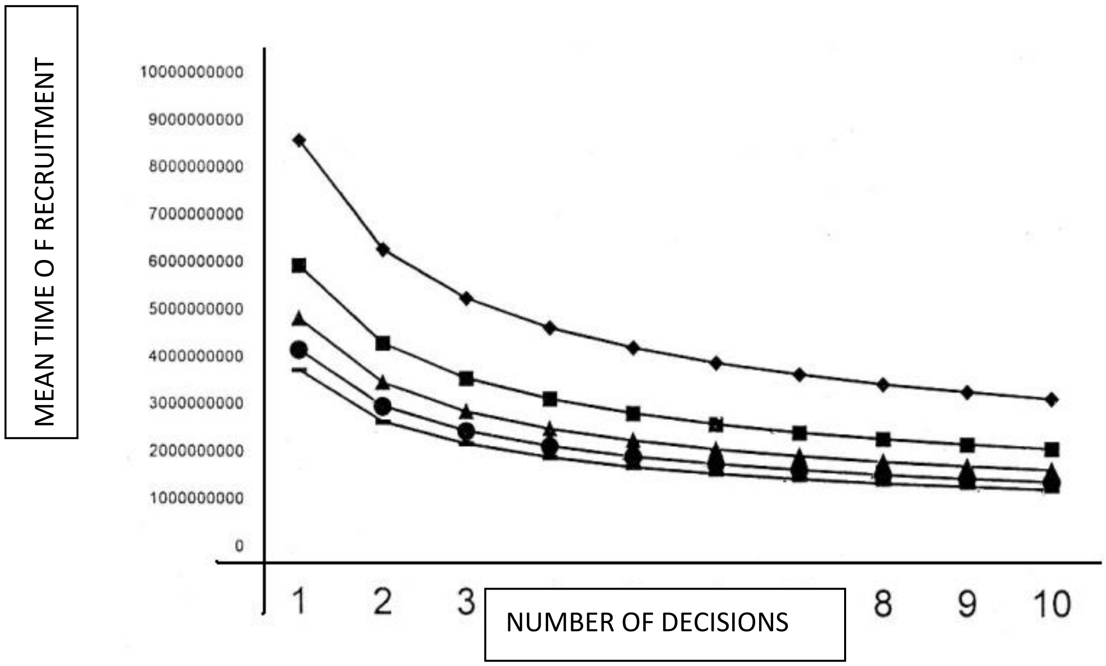

Fig.1 Mean of time recruitment with $\alpha = \mathbf{0.1}$

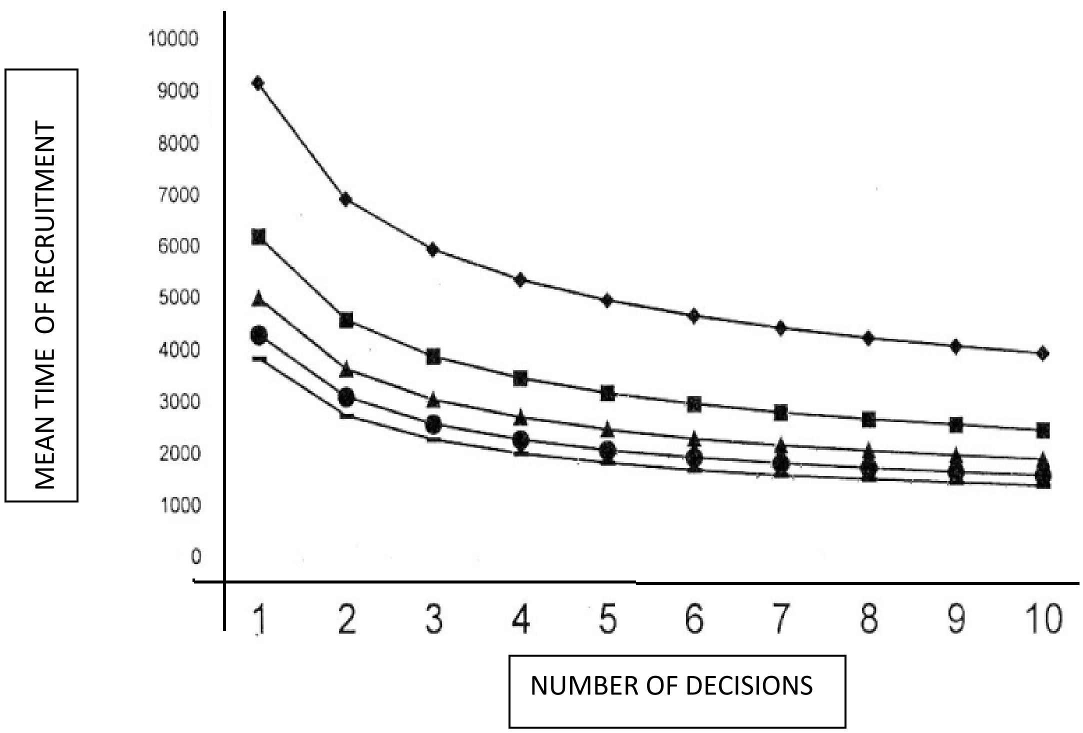

Fig.2 Mean of time recruitment with $\alpha = \mathbf{0.2}$

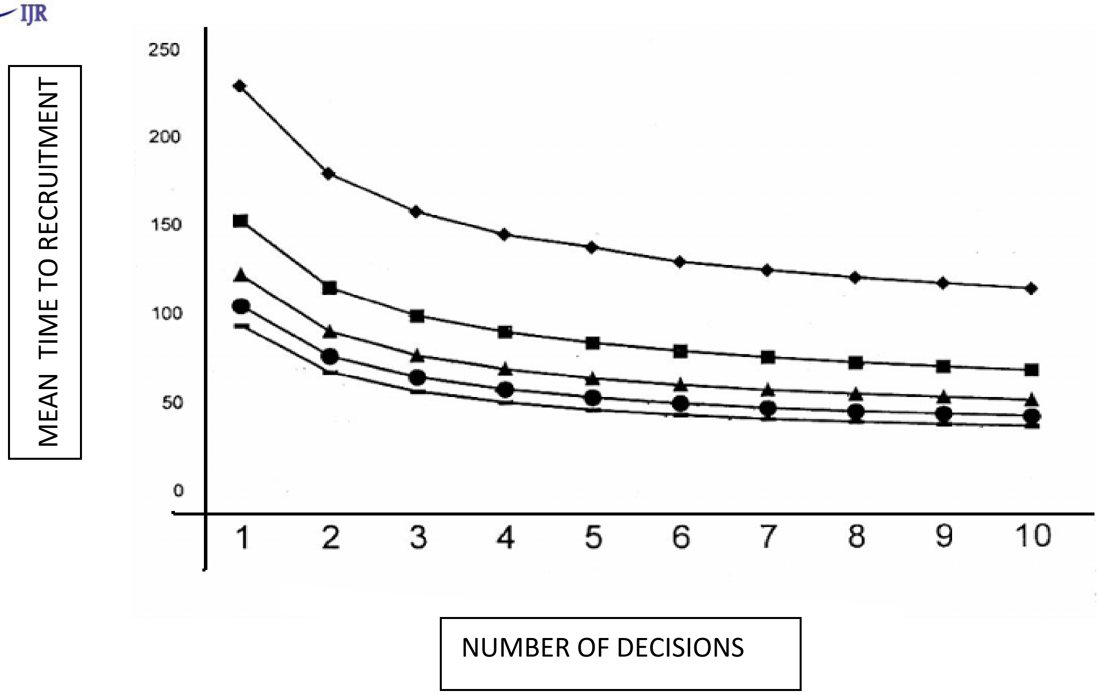

Fig.3 Mean of time recruitment with $\alpha = \mathbf{0.3}$

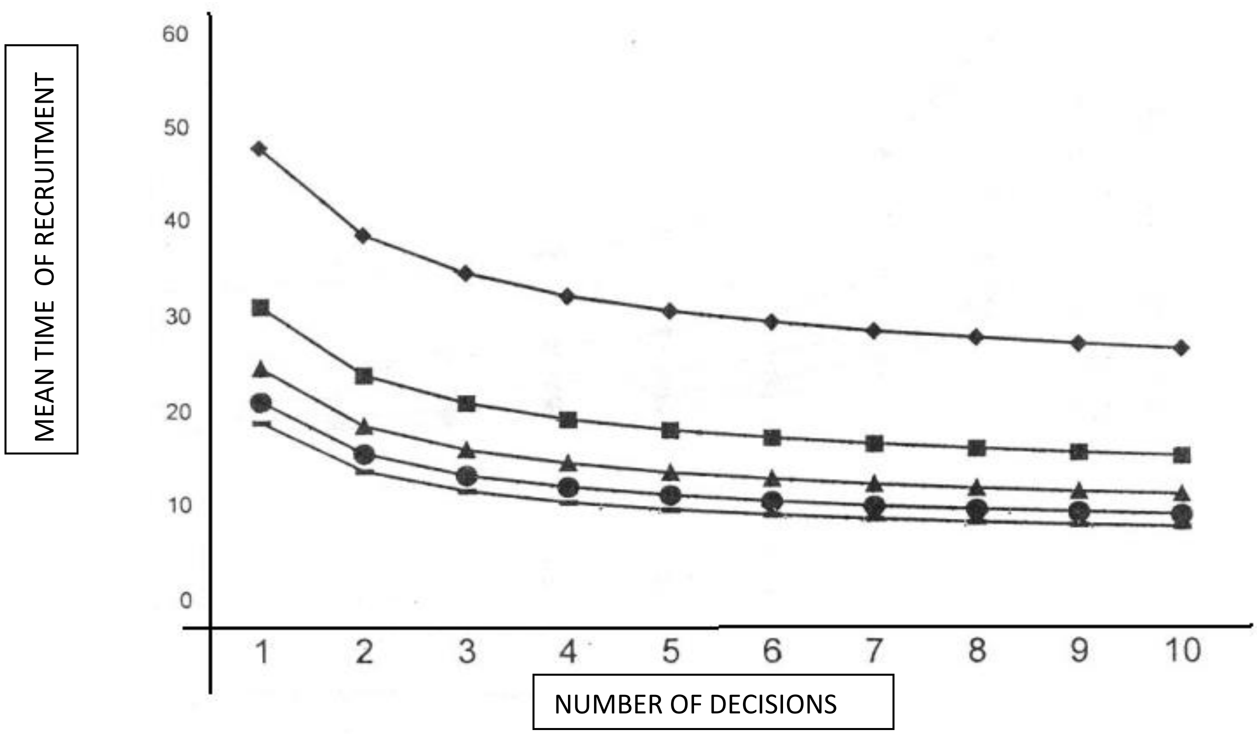

Fig.4 Mean of time recruitment with $\alpha = \mathbf{0.4}$

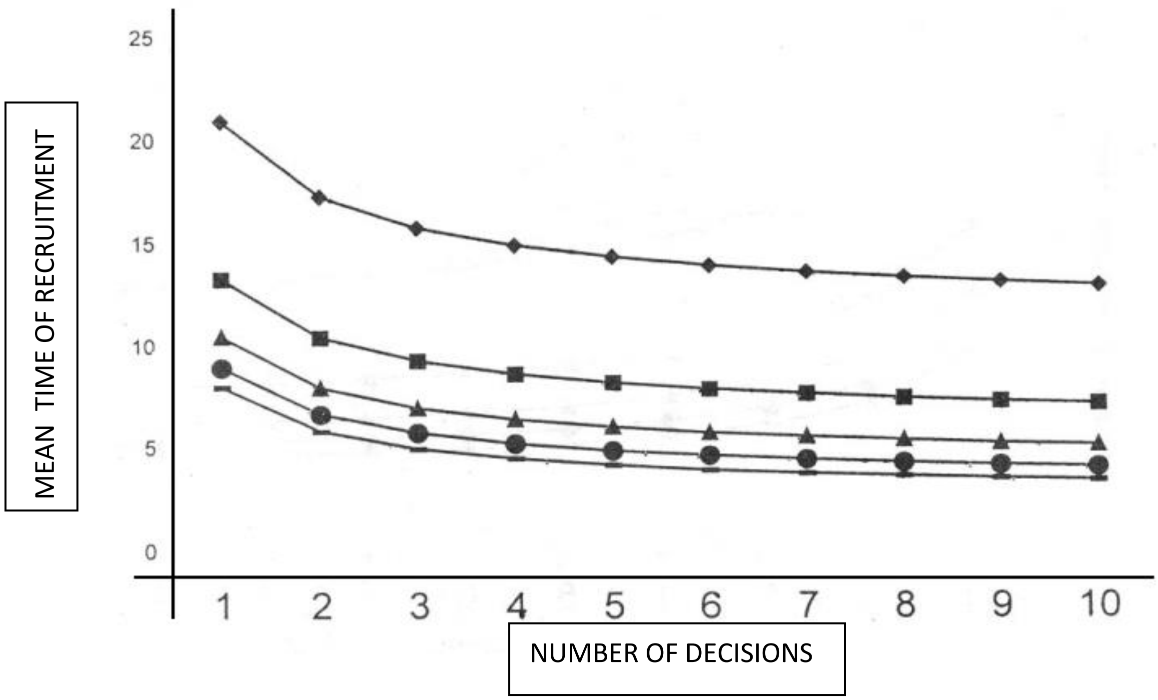

Fig.5 Mean of time recruitment with $\alpha = 0.5$

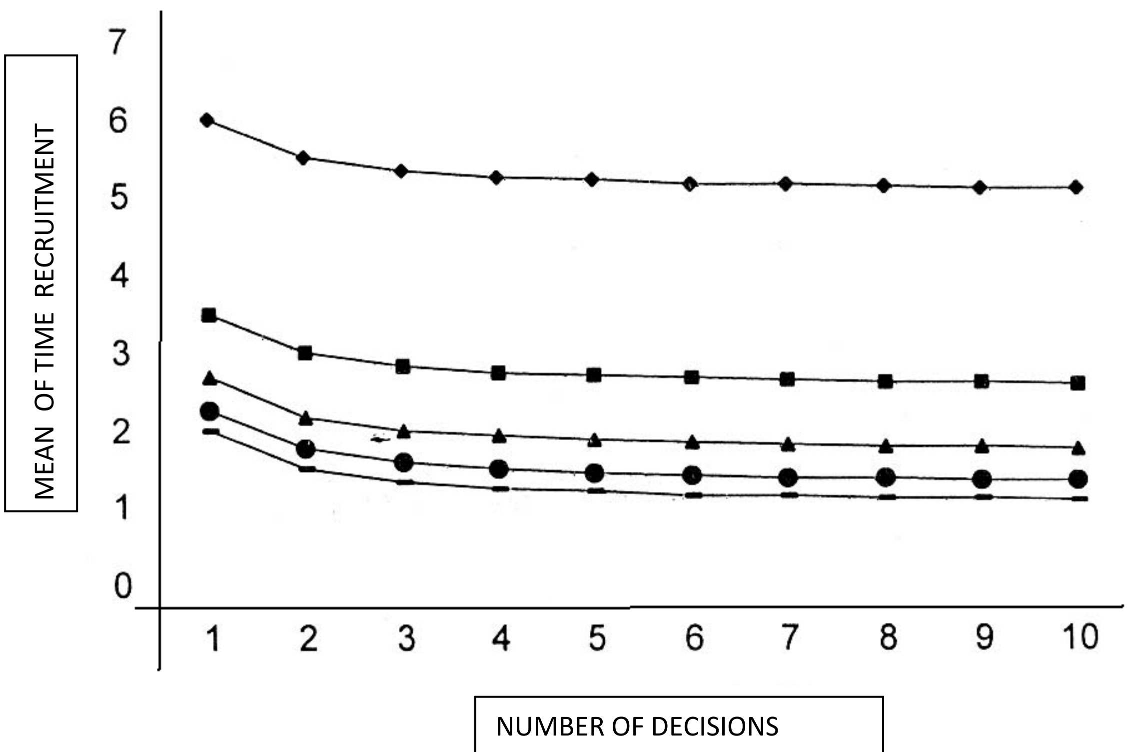

Fig.6 Mean of time recruitment with $\alpha = 0.6$

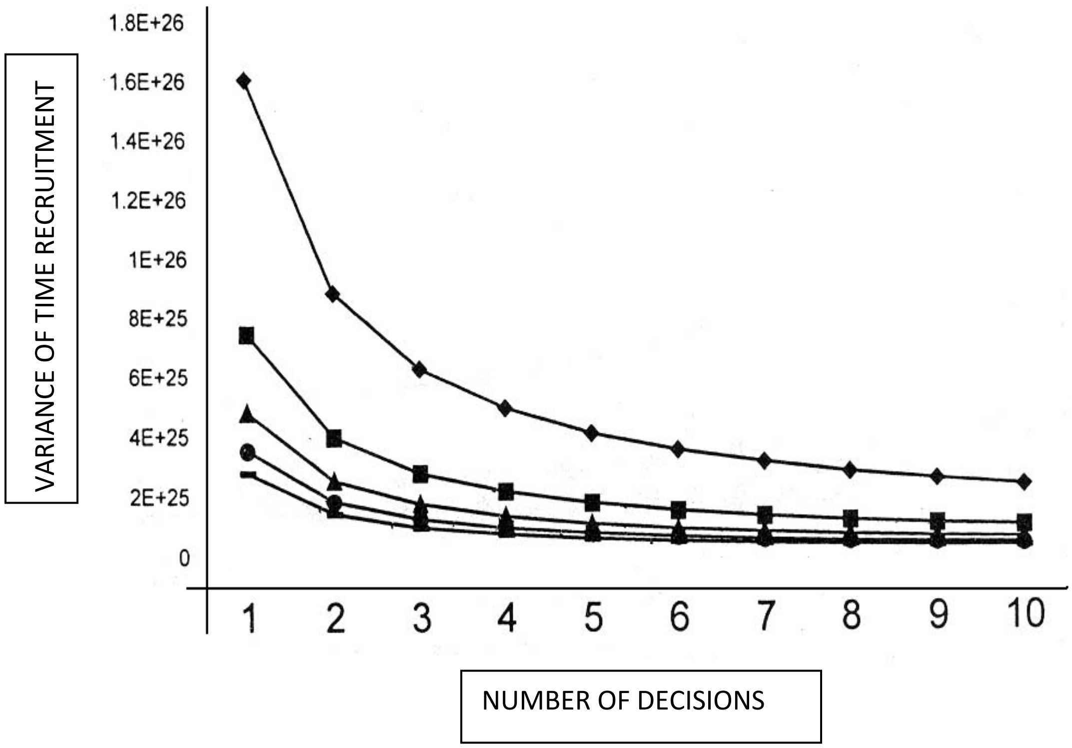

Fig.7 Variance of time recruitment with $\alpha = 0.1$

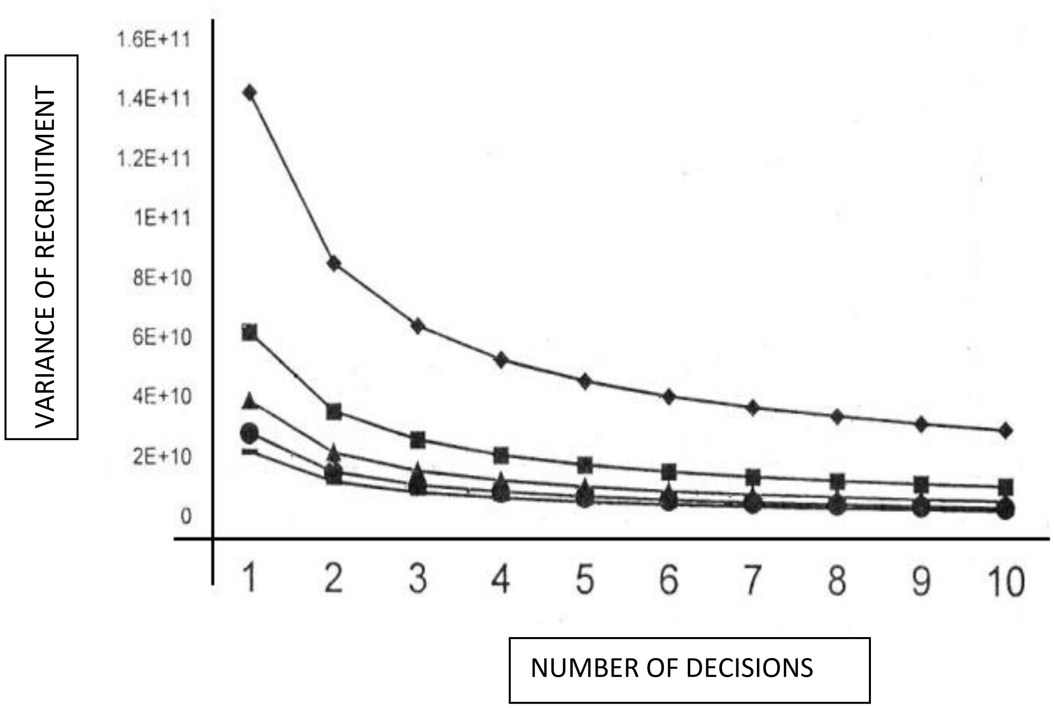

Fig.8 Variance of time recruitment with $\alpha = 0.2$

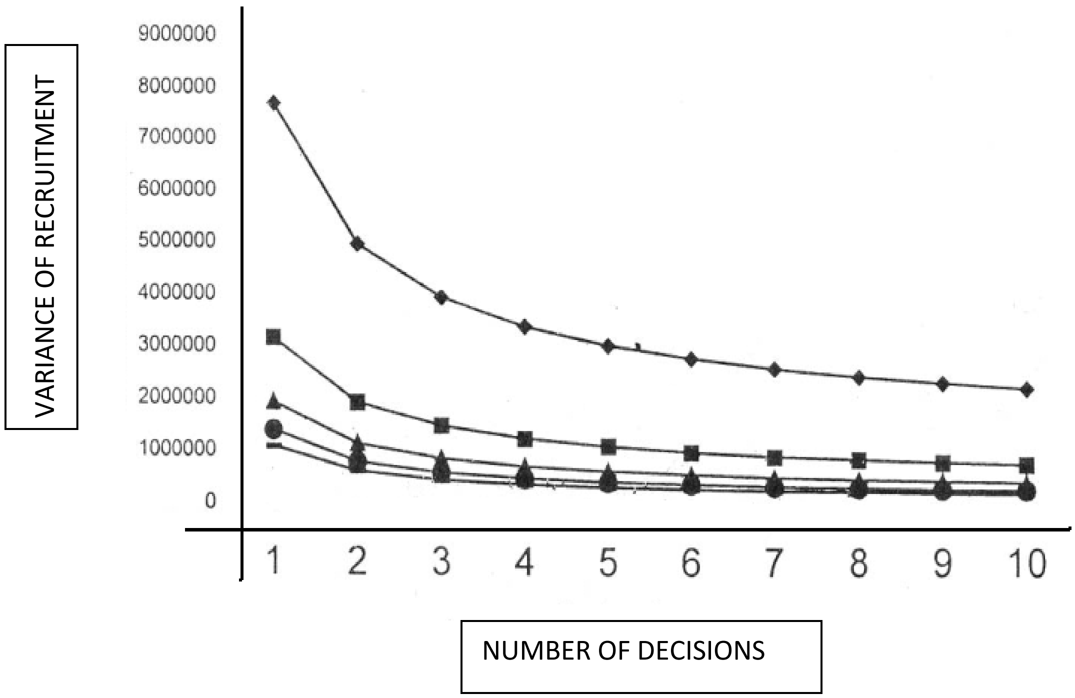

Fig.9 Variance of time recruitment with $\alpha = \mathbf{0.3}$

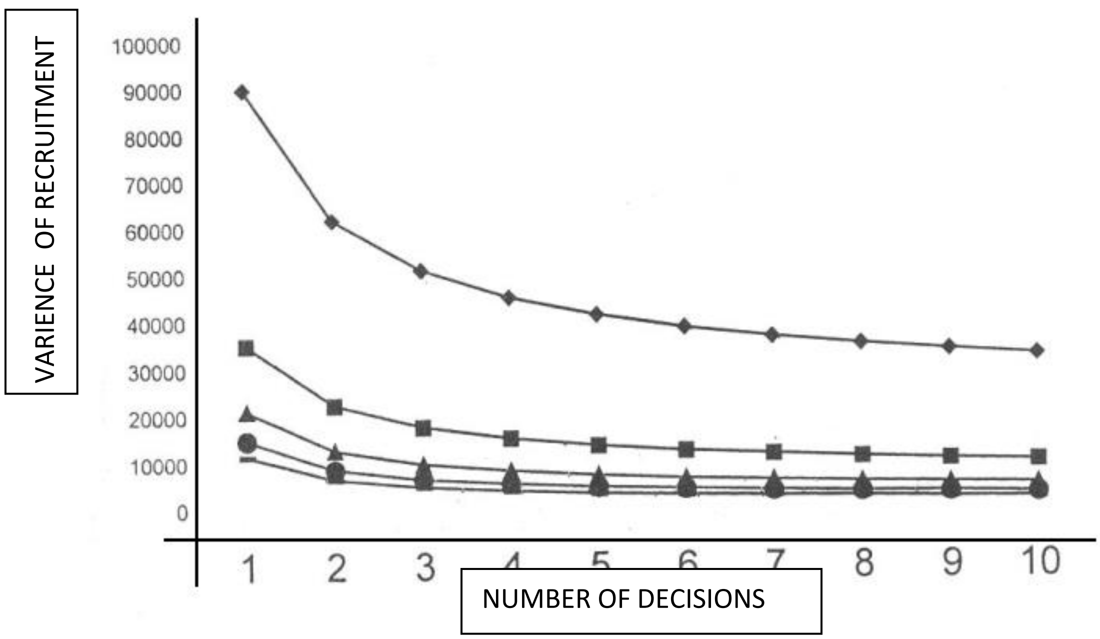

Fig.10 Variance of time recruitment with $\alpha = \mathbf{0.4}$

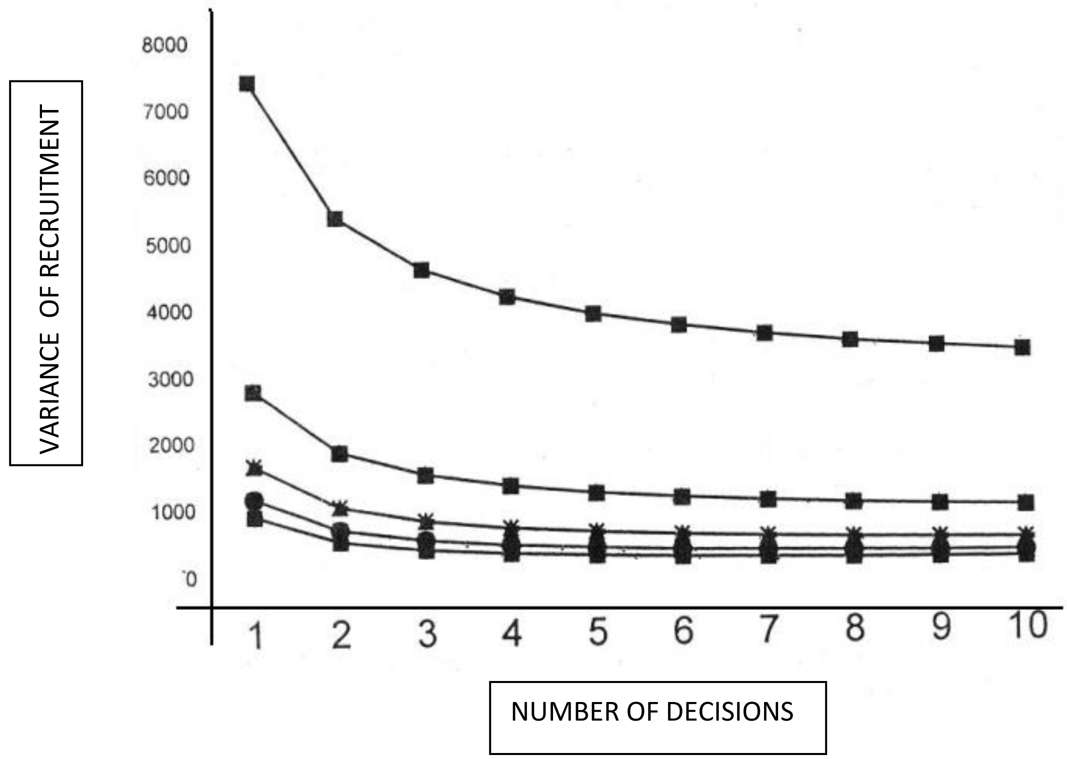

Fig.11 Variance of time recruitment with $\alpha = \mathbf{0.5}$

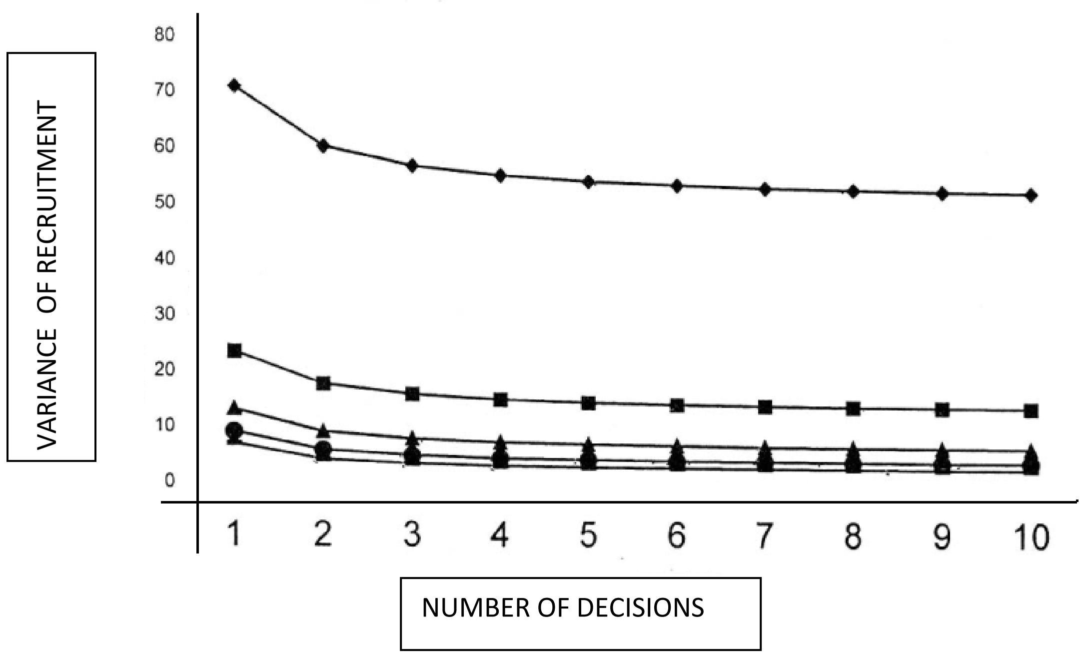

Fig.12 Variance of time recruitment with $\alpha = \mathbf{0.6}$

CONCLUSION

From the above figures, we observed that for fixed 'μ' when 'a' increases, the mean of time recruitment decreases. Also, if 'a' is fixed and 'μ' is allowed to increase then the mean of time recruitment decreases. The same tendency is also noticed on the variances of the time recruitment. Also the value of 'a' increases, the mean and variance of time recruitment decreases.

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