

Structural Framework of Riemann Hypothesis with Insight into the Spiral Structure for Elementary Particles

Suraj Kumar

Abstract

In this paper we try to review the relation between the zeroes of zeta function with the symmetry observed in the elementary particles. We try to establish the framework for Riemann hypothesis with the physical correspondence to the spiral structure for elementary particles which explains the hypothesis i.e. all non-trivial zeroes of the zeta functions have real part of one-half and provides a probabilistic field to analyze through the spin orientation of stable elementary particles. The only pole at $s=1$ due to discontinuity can be explained through the structural dynamics of force carriers with unit spin. We also derive the pattern for prime numbers based on its geometric distribution analogous to the spiral structure for the elementary particles.

MSC: 11M06, 51P05, 78A35

Introduction

What is Riemann Hypothesis?

It starts with the definition for non-trivial zeroes of the zeta function or can also be called as the root or eigenvalues of zeta function i.e. the value of s for $\zeta(s)$ which requires the real part of s to be greater than 1 or the sum for the series in zeta function as provided below won't converge.

$$\zeta(s) = \sum_n \frac{1}{n^s} = \prod_p \frac{1}{1 - \frac{1}{p^s}} \quad (\text{Euler's zeta function}) \quad \dots [1]$$

A trivial zero could be derived simply by observation. Let's find the trivial zeroes of zeta function by observations. For example, when $s = \text{negative even integer}$, we have $s = -2n \rightarrow \zeta(-2) = \sum_n n^2 = \infty$ (**positive sum**). Thus [1] only holds for s with real part greater than 1. For others we have different form of zeta function $\zeta(s)$ expressed in terms of $\zeta(1-s)$ as:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s) \quad (\text{Functional equation of zeta function}) \dots [2]$$

The equation holds for all the values of s except for $s = 1$ or $-2n$ where $s = 0$. The plot of the zeta function yields the following graph in complex plane given in [Figure 1].

The roots lying between the critical strip **0 and 1** provides the probable region to search for zeros. Thus $\zeta(s)$ must lie within the critical strip. Hence the zeros of zeta function must be symmetrical

about the real axis and the non-trivial zeros of $\zeta(s)$ are symmetrical about the critical line **Real part of $s = \frac{1}{2}$** . Thus a single zero off the line has three more companions zeros that do not lie on line. If zero lie on the real axis then there is only one companion. But there are no such zeros.

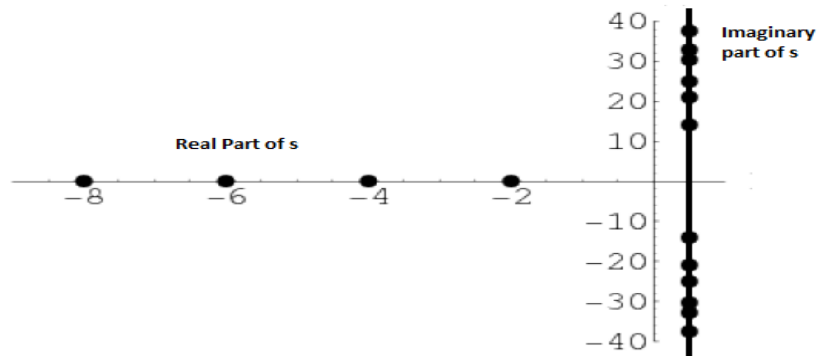


Figure 1: Plot of zeta function on complex plane.

The earlier version of Riemann Hypothesis derives the s to be $\frac{1}{2} + it$. Thus, $\zeta(s)$ would not converge to zero and t must be real to give s to have one-half.

Describing the Spiral Structure for Elementary Particles using the Riemann Zeta Function

Plotting the $\zeta(s)$ on four dimensions entangling the dimension to be projected in three dimensional spaces provides us the following graph:

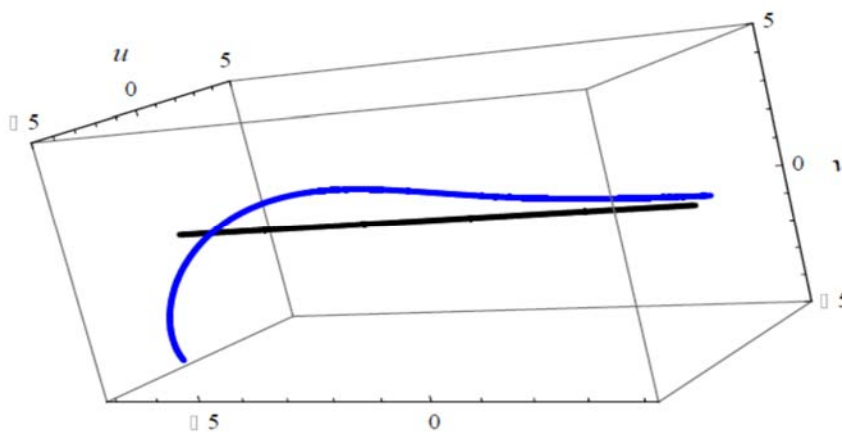


Figure 2: Snapshot of three dimensional plots for $\zeta(s)$ where u represents the real part and v represents the imaginary part of zeta function.

The straight line shows the trivial dimension i.e. $u=v=0$. This line is called line

of self intersection also called as **line of infinity** which can be compared to the zero spin gluon at the core of composite particles resulting from the charge orientation of the individual one-half spin quarks entangled together. **The Spiral represents the zeta function curls around but never touches axis of trivial dimension (i.e. line of infinity).** This provides us with the framework equivalence of the Riemann Zeta Function and Spiral Structure for Elementary Particles. The

Riemann Zeta Function can be considered to provide a glimpse of the structure for the accelerated elementary particles where the parameters can be described as:

$$\frac{\zeta(s)}{\zeta(1-s)} = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) = \frac{(2\pi)^s}{\pi} \sin\left(\frac{\pi s}{2}\right) (-s)! \quad \dots [3]$$

The left side of the equation above describes the staircase analogy which can be described as imagining several staircases with unit structure which are added to each other to create a staircase with uneven steps. These steps represent the energy levels added to the system of accelerated elementary particles where s is the variation in the unit structure of the staircase corresponding to the variation of acceleration to the elementary particles.

The right hand side of the above equation provides the quantum chaos arising due to slightly induced shift in the unit structure staircase mentioned above which provides increase in the length of spiral structure for elementary particles during acceleration which consecutively increases the mass of the particle. Thus equation [3] defines the spiral structure of accelerated particle shown below in the diagram.

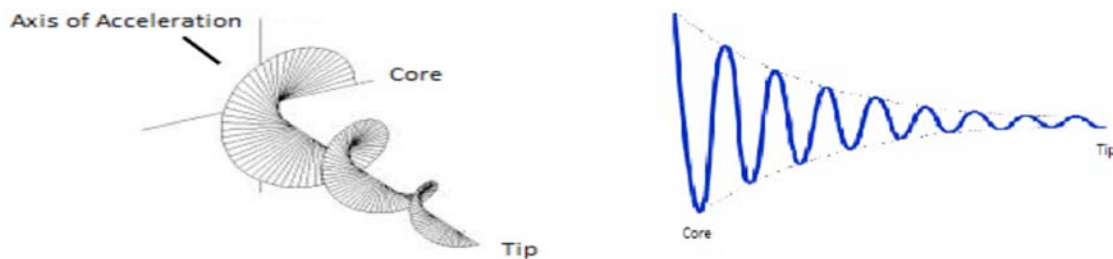


Figure 3: Representation of structure of antispiral when accelerated with velocity closer to the speed of light.

The detailed description of equation [3] includes the individual treatment of terms on the spiral structure for the elementary particle. Lets partition equation [3] into three terms with separate effects: $\frac{(2\pi)^s}{\pi}$, $\sin\left(\frac{\pi s}{2}\right)$ and $(-s)!$. The first term $\frac{(2\pi)^s}{\pi}$ shows that with each quantum chaos induced by s , there is an addition of spiral arm represented by $(2\pi)^s$ in the numerator. The π in the denominator is a normalization term which defines the unit of induced quantum chaos. The Second term $\sin\left(\frac{\pi s}{2}\right)$ describes the curl and defines the phase required for different properties deduced from spiral structure for elementary particles. The third term $(-s)!$ explains the change in the overall quantum chaos due to change in the energy levels at each step. This provides us an equivalence relation between Riemann Zeta Function and the Spiral Structure for Elementary Particle.

To find the roots of Riemann Zeta Function and its equivalence with the Spiral Structure for Elementary Particles

The roots of the Riemann Zeta Function are the values of s for which $\zeta(s) = 0$. This can also be described as the value of s (the variation of acceleration to the elementary particles) for which the acceleration tends towards zero resulting in ground state of the elementary particle.

What is structure of s ?

Given $s = \frac{1}{2} + it$ from the earlier calculated roots for Riemann zeta function. With respect to the Spiral Structure for Elementary Particles, s can be assumed to describe the spin property. The real part of s represent the spin state of stable elementary particles that can exist independently without decaying off and thus can have increase in mass when accelerated close to speed of light. Hence, the Riemann zeta function can be applied to describe the Spiral Structure for such Elementary Particles when accelerated. The second term is an important quantity as it is required to remove the effect of acceleration by cancelling out the effect of spin taken in real term to bring the elementary particle considered in ground state as required for the roots of Riemann zeta function. This imaginary quantity is denoted by t . As mentioned earlier the value of t must be real and should not be equal to zero to bring $\zeta(s) = 0$. The origin of t lies in the fact that the elementary particle considered is in acceleration and t is an induced property of accelerated spiral structure for elementary particles.. There are two ways to cancel out the effect of one-half spin required for the accelerated spiral structure for elementary particles; one is when t induces the value of $\frac{1}{2}$ which creates an imaginary boson with unit spin which decays out. As we know that from equation [1], with $s = 1$, the Riemann Zeta Function $\zeta(s)$ is **discontinuous** i.e. with spin 1 the bosons cannot sustain the accelerated Spiral Structure and disintegrates into lighter masses fermions or decays off. Similarly the other is when t induces the value of $-\frac{1}{2}$ which also creates an imaginary boson with zero spin which decays out as well since the second term of equation [3] i.e. $\sin\left(\frac{\pi s}{2}\right)$ vanishes off. Thus the critical strip of the Riemann zeta function requires t to be between $[-\frac{1}{2}, \frac{1}{2}]$.

Relating the zeros of $\zeta(s)$ to the eigenvalues of Hermitian Matrices which has all the eigenvalues as real. These Hermitian matrices can be compared to the Pauli Matrices which are a set of three 2×2 complex matrices which are Hermitian and unitary. These Pauli Matrices describes the interaction of spin of an elementary particle with external electromagnetic fields spanning the space of observables of the two dimensional complex Hilbert space. An expression has been discovered that represent the statistical distribution of distances between consecutive, non-trivial zeros that lie on the critical line. Dyson studied the behavior of the differences between the eigenvalues of certain random Hermitian matrices which represents the energy levels of heavy atomic nuclei. Thus, it was deduced that the imaginary parts of some of the Riemann zeta function's zeros probably represent the energy levels of some physical object.

How these energy levels represented by t do provide the diminishing effect to accelerated spiral structure for elementary particle?

The energy levels represented by t can be considered to have complementary effect to the existing energy levels in quantum chaos originating due to the real part of s induced due to accelerated spiral structure for the elementary particles.

Symmetry and the zeros of Riemann Zeta Function

As suggested by Wigner, the resonance lines of heavy nucleus might be modeled by the spectrum of a large random matrix. This is named as Random Matrix Model. He considered different probability distribution (i.e. ensembles) on spaces of matrices. The most probable ensembles are termed as Gaussian Orthogonal Ensemble and Gaussian Unitary Ensemble which lives on linear space of Hermitian $N \times N$ matrices and are unitary invariant ensembles. Dyson derived the circular Gaussian Symplectic Ensemble which may be realized as the compact Riemannian symmetric spaces with their volume form as probability measures. Considering the Spiral Structure for Elementary Particles we may define an ensemble to have $SU(1)$ kernel symmetry which provides the trivial existence of the **discontinuity** of $\zeta(s)$ for s having real part of 1 .

Deducing pattern from Prime numbers on the basis of Riemann zeta function description for Spiral Structure for Elementary Particles

The value of genus for an orientable surface is equal to the number of “holes” in it. For a non-orientable closed surface, the value of genus is the positive integer representing the number of self intersections attached to the sphere. The sphere here is the combined cavity for the composite particles which provides self intersection for the spiral structure for the quark to create a stable structure with **line of intersection** at the centre to provide a zero spin gluon representing the eigenvalues to the Riemann zeta function to provide stable structure to the composite particle. We can apply the Riemann Zeta Function to the unaccelerated spiral structure for composite particle as its constituent elementary particles are always in coupled acceleration through self intersection which provides space for Riemann zeta function at constituent quark level. The Gauss estimates for the logarithmic integral to find the patterns of prime which provides the potential variation for the interaction of the spiral structure for elementary particles can be plotted as:

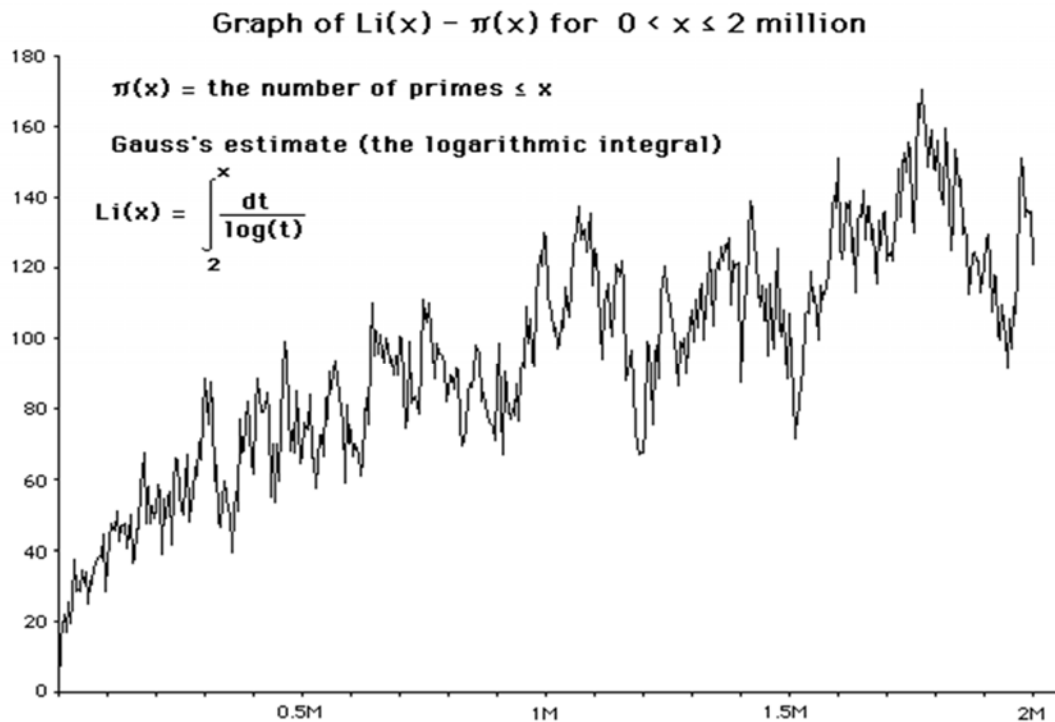
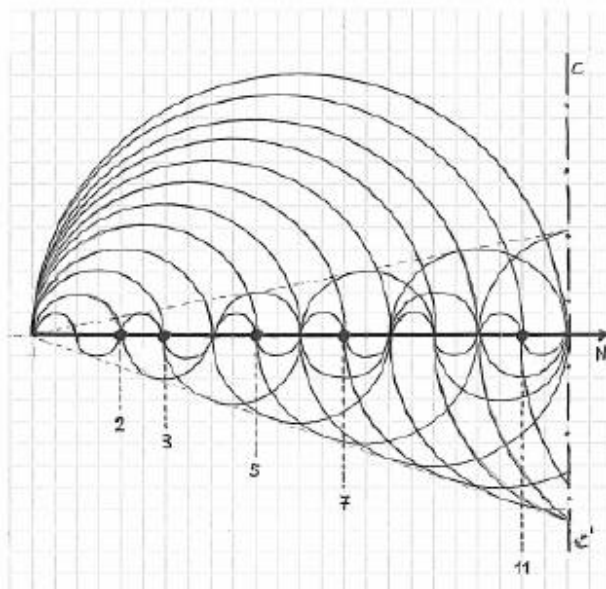


Figure 4: Plot for Gauss estimates for the logarithmic integral for prime function.

It's believed that if a zero occurs off the critical line, then the prime numbers can be considered to be biased and will form a pattern. This biasness increases with further the zero is from critical line. This can be well related with the non existence of particles with fractional spin except $\frac{1}{2}$. The existence of such particle will behave to be biased and annihilate off.

Based on the work of Omar E. Pol, the figure below by Jason Davies reveals the prime number's unique pattern, as a periodic curve, superposed with the unique pattern of every other number.



“For each natural number n , we draw a periodic curve starting from the origin, intersecting the x -axis at n and its multiples. The prime numbers are those that have been intersected by only two curves: the prime number itself and one.”

Figure 5: Illustration of unique pattern for prime numbers.

“This pattern cannot merely be a coincidence. A mathematician who finds a pattern of this sort with instinctively ask, ‘Why? What is the reason behind this order?’ Not only will all mathematicians

wonder what the reason is, but even more importantly, they will all implicitly believe that whether or not anyone ever finds the reason, there must be a reason for it. Nothing happens 'by accident' in the world of mathematics. The existence of a perfect pattern, a regularity that goes on forever, reveals — just as smoke reveals a fire — that something is going on behind the scenes. Mathematicians consider it a sacred goal to seek that thing, uncover it, and bring it out into the open." — Douglas Hofstadter (*I Am A Strange Loop*, p. 117)

Comparing the pattern of Prime Number to the Spiral Structure for the Elementary Particles, we can find the prime numbers to be representing the structural description for the accelerated spiral structure of elementary particles through the Riemann Zeta Function. The curls in the [Figure 4] provide an analogous framework to the spiral structure for the elementary particles. Its sum of divisors $\sigma(n)$ (i.e. number of allowed disintegration), and its aliquot sum $s(n) = \sigma(n) - n$, indicates whether the number is prime, deficient, perfect or abundant provides the stability condition for the particles by defining the **allowed degree of freedom**. For example, if $n=12$, $\sigma(n) = 1+2+3+4+6+12 = 28$. The aliquot sum is calculated to be $16 > n$, which is condition of abundance and the corresponding particle, would disintegrate into daughter nuclei. The behavior for spiral structure for elementary particles analogous to the unique pattern of prime numbers can be described as:

Aliquot Sum $s(n)$	Behavior of spiral structure for elementary particles	Example
Prime	Stable (i.e. will not disintegrate)	Fermions
Deficient	Absorbs other lighter nuclei.	Composite Particles
Perfect	Decays off or Annihilates	Bosons
Abundant	Disintegrates into lighter nuclei.	Radioactive Nuclei

In simple physical term we can say that the prime numbers are analogous to accelerated spiral structure for elementary particles which cannot be **disintegrated** (i.e. is not divisible into lighter particles) or **decayed** (i.e. prime number > 1) further.

Conclusion

Thus, the physical structure represented by Riemann zeta function has singularity at $s = \frac{1}{2}$ (real part) as in black holes with an event horizon of $\frac{1}{2}$. The absolute spin of elementary particles $|\frac{1}{2}|$ is the singularity point of Riemann Zeta Function and provides the stable spiral structure for elementary particles as in the galactic structure. It also provides a relation between these stable spiral structures with the existence of prime numbers which provides us with a unique pattern for these prime numbers.

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