

# Methods to Solve Linear Programming Problems

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**1. Abstract**

*In this paper we are focusing on the different methods of Solving the LPP(Linear Programming Problems).*

*The graphical method is applied to find out the feasible region of particular example.*

*Simplex method can be applied on several problems related to accounting and it is mostly used in capital budgeting.*

*Simplex method is used over graphical method as*

*1. It usually solves the problems in which more than two variables are used.*

**2. Introduction**

(A) History

The linear programming was invented during the tenure of second world war (1939-1945), when the American and British military called up a group of scientists to study and plan the maximum damage that could be imposed on the enemy at a minimum rate and loss of the resources, since this method was a great successful at that time therefore, later on this technique was applied on different functional areas.

(B) Linear programming refers to a technique for the optimization

(i.e. maximize or minimize) of a linear function

A general linear programming problem having n variable and m constraints can be given as

$$\text{Optimize } Z = C_1X_1 + C_2X_2 + C_3X_3 + \dots + C_nX_n$$

Subject to the constraints

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \dots + a_{1n}X_n (\leq \text{or } \geq) b_1$$

$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \dots + a_{2n}X_n (\leq \text{or } \geq) b_2$$

$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + \dots + a_{3n}X_n (\leq \text{or } \geq) b_3$$

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$$a_mX_1 + a_m2X_2 + \dots + a_mnX_n (\leq \text{or } \geq) b_m$$

where  $X_1, X_2, \dots, X_n \geq 0$

Here the  $X_1, X_2, \dots, X_n$  are the structural variables

Here the function Z which is to be optimized is known as the Objective Function of the linear programming problem.

$b_i$  ( $i=1,2,3,\dots,m$ ) is the constants which refers the availability

of the  $i$ th constraints.

Below are some methods of solving the problems related to linear programming problems

1. 1.Graphical method

2. Simplex method

**3.Graphical method(Corner point method)**

Graphical method is a basic method of solving the problem related to linear programming having two variables  $X_1$  and  $X_2$ .

The main motive of using the graphical method is to find the common feasible region.

The corner point method is based upon the theorem of EXTREME POINT THEOREM.

**Steps for solving the graphical method**

(A) Form the function to be optimize (i.e. maximize or minimize) from the given question.

(i) The equation is to be formed in the form  $ax_1 + bx_2 = S$  or  $ax + by = S$  from the given question.

(B) Plot all the points on a graph and shade the common region formed by all the lines satisfying the constraints.

This shaded region is called as the common feasible region.

(a) The points can be determined by selecting one point on x-axis by putting y as 0; and one point on y-axis by putting x as 0, thus a straight line is formed which can be shaded towards or away from the origin depending upon the type of inequality that is used in the constraint.

If there is a less than inequality then shading is towards the origin and vice versa.

(C) Determine the coordinates of the corner points of the shaded feasible region.

(D) Now place the coordinates of the corner points of the feasible region into the main function or optimization function.

(a) The coordinate giving the optimizing value is the required solution.

#### 4. Simplex Method

The simplex method is one of the best methods for solving the LPP and also a step-by-step process. This method was invented in year 1947 by George B. Dantzig.

The reason for using the simplex method over graphical method is that all the problems having more than two variables cannot be solved graphically thus simplex method instantly solves the problems which usually occur in case of graphical method.

To use simplex method it is necessary to convert the inequality of the constraints into equality by adding slack variables on the LEFT HAND side of each inequality in case of maximizing problems and can be converted into equality by subtracting a surplus variable from LEFT HAND side of the inequality in case of minimization problem.

Below are the following steps used in simplex method to solve the maximization problem.

(A) Express the problem in the standard form

(a) The objective function is always to be of maximizing nature.

but if the function is of minimizing nature then convert it into maximizing as

$$\text{Min. } Z = \text{Max. } (-Z)$$

The RHS of all the constraints should be constant and positive.

(a) By adding slack variables on the left hand sides of the inequality (if less than in equality is given).

(b) By subtracting a surplus variable from left hand side of the inequality (if greater than inequality is given).

(C) Set up the simplex table

(a) Insert the  $c_j$  which is the co-efficient of the variables of the optimization function in the top most row.

(b) In the first column write BV (i.e. the slack or surplus variable) the basic variable and CB which is the coefficient of basic variable from the optimization function.

(c)  $C_{B_i}$  is the coefficient of  $i$ th basic variable

(d) Write  $X_b (= b_i)$  in the first column which is the solution i.e. the values of the corresponding basic variable.

(e) In second column write all the variables having their respective values below them.

An identity matrix will form under  $S_1, S_2, S_3, \dots, S_n$  representing the coefficients of the slack or surplus variables present in the constraints set.

(D) Calculate Z by multiplying  $C_{B_i}$  and  $X_{B_i}$

$$\text{As } Z = \sum C_{B_i} \cdot X_{B_i}$$

(E) Calculate value of  $Z_j$  which represents amount by which the value of the optimization function Z would be increase or decrease if one unit of the variable not included in the solution is brought into the solution.

$$\text{i.e. } Z_j = \sum C_{ij} \cdot a_{ij}$$

(F) Determine the incoming variable by choosing the highest value of  $C_j - Z_j$ .

(G) Determine the outgoing variable by dividing each number of XB (solution column) by their corresponding positive entry in the key column thus a new ratio column is to be formed, now the row having/corresponds to the smallest positive quotient is called the pivot row, and the intersection value of both the incoming and outgoing variable is known as key or pivot element.



(H) Setup a new simplex table and calculate the values of the row element with the help of key or pivot element.

(a) If there is any positive value in  $C_j - Z_j$  then repeat the above steps (from step (D)).

(b) If there is no positive value in  $C_j - Z_j$  then the final solution is obtained.

### 3. References

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[2]

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