Stability of Hydro dynamical System Using Normal Mode Technique

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Abstract:

The subject of stability theory is of considerable importance because of its relevance to physical situations such as the convective instability in stars, heating of solar corona, stability of stellar interiors in magnetic stars, stability of highly ionized plasma surrounded by a slightly ionized cold gas, stability of the system where air is blown over mercury, thermal convective instability in stellar atmosphere, stability of tangential discontinuities in the solar wind, stability of streams of charged particles emanating from the sun and excitation of water waves by the wind. Thus the subject of fluid mechanics in general and the stability theory in particular have received great importance during the last few decades. Many physical phenomena are solved successfully with the help of these topics. In order to understand a given physical situation, an equivalent mathematical model is constructed. The more we try to make our mathematical model physically realistic, the more the problem becomes mathematically difficult to solve. To be specific, the compressibility of air, its thermally conducting nature and some other factors introduce complications in the study of mathematical model for any atmospheric study. One is bound to assume certain approximations and assumptions regarding the nature of fluid and the flow boundaries. These approximations and assumptions should be such that not only they simplify mathematical formulation of the problem, but should also agree considerably with the physical requirements of the problem and in this way they help us in providing the best representation to the physical phenomenon under investigation. In view of the above, the result, so obtained, is in accordance with the functioning of the physical system and confirms the experiments. But if these theoretical results do not agree with the experiments and observations, the departure of theoretical results from the experiments and observations needs a careful study. With the help of stability theory the question raised above can successfully be studied.

Keywords:

Stationary solution, unstable mode, parameters, marginal stability, linearized perturbation, wave number.

Fluid flows occur almost everywhere in nature and stability analysis of fluid flows is essential for understanding the nature of fluid flows and their applications in various fields. The subject of Fluid Mechanics, because of its vast applications has received great importance during the last few decades. Many physical phenomena are solved successfully with its help. In order to understand the behaviour of a fluid flow, we need to know how it would react to various forces acting on it. The basic equations of continuity, momentum and energy are all non-linear partial differential equations and as such their analytic solutions are generally not possible. Therefore to analyze any fluid flow theoretically we have to take some approximations depending upon the geometry of the flow and other considerations. In experimental investigation of any fluid flow, we must analyze the effect of these disturbances on the theoretically obtained solution.

In order to understand a given physical situation, an equivalent mathematical model is constructed. The more we try to take our mathematical model physically realistic, the more the problem becomes mathematically difficult to solve. To be specific, the compressibility of the air, its thermally conducting nature and other factors introduce much complications in the study of mathematical model of any situation in the atmospheric study. One is bound to assume certain approximations and assumptions regarding the nature of fluid and flow boundaries. These approximations and assumptions be such that not only they simplify mathematical formulation of the problem, but should agree considerably with the physical requirement of the problem and in this way they help us in providing the best representation to the physical phenomenon under investigation. In view of the above, the results, so obtained, are in accordance with the functioning of the physical system and confirm the experiments. But if these theoretical results do not agree with the experiments and observations, then the departure of the theoretical results from the experiments and observations needs a careful study.

In considering the stability of a hydrodynamic system, which, in accordance with the equations governing it, is in a stationary state, we essentially seek to determine the reaction of the system to small disturbances. Specifically, if the system is disturbed, will the disturbance gradually die down, or will the disturbance grow in amplitude in such a way that the system progressively depart from the initial state and never reverts to it? In former case, we say that the system is stable and in latter case we say that it is unstable. In the space of the governing parameters, the locus, which separates the stable and unstable state, is called the state of marginal stability. The determination of this locus is one of the prime objects in any investigation related to the stability of a system.

The mathematical formulation of the stability theory proceeds from the non-linear partial differential equations. The unknown quantities are the functions of three space co-ordinates and time and are subjected to some boundary conditions. Certain special solutions of such a general problem are usually of particular interest. The simplest among these permanent type solutions are the stationary solutions. If this perturbed solution goes on departing from basic solution, the system is said to be unstable and on the other hand if this perturbed solution approaches to theoretical solution, as the time passes, the system is said to be stable. Clearly, a system must be considered as unstable even if there is one special mode of disturbance with respect to which it is unstable and a system is not stable unless it is stable with respect to every possible disturbance to which it is subjected. In other words, stability must imply that there exists no unstable mode of disturbance.

For discussing the stability of a system, it is convenient to suppose that all parameters of the system are kept constant and a particular one is continuously varied. We shall then pass from stable to unstable state when this particular parameter takes a certain critical value. We say that the instability sets in at this value of the chosen parameter. Thus the marginal state is the locus which separates the stable state from the unstable state and occurs when there exist some perturbations whose amplitude remains constant with time, while the amplitude of the other tends to zero in course of time i.e. the marginal stability is the state of neutral stability. If at the onset of instability a stationary pattern of motion prevails, then one says that Principle of Exchange of Stabilities (P.E.S.) is valid if at the onset of instability, oscillatory motion prevails, and then we have a case of over stability.

Normal Mode Analysis:-

For determining the stability of a hydro dynamical system by normal mode analysis, the linearized perturbation equations are set up first in a single perturbation variable by eliminating the remaining variables from the linear equations derived from the equations of conservation of mass, momentum and energy, retaining only the linear terms in perturbed quantities. These equations are then solved either analytically or with the help of variational procedure or through an integral equation under a set of appropriate boundary conditions. This leads to the dispersion relation in the parameters determining the stability of a system. The dispersion relation thus obtained is quite complex and an analytical interpretation is not always possible. Therefore, in order to determine the effect of a particular physical parameter on the growth rates, we analyze the change by varying that parameter while keeping the other parameters fixed. An increase in growth rate implies the destabilizing influence of that particular parameter and a decrease in growth rate shows stabilizing influence of the parameter.

For the investigations in any stability analysis to be complete, it is assumed that the perturbations can be resolved into dynamically independent wave-like components, each component satisfying the linearized equations of motion and the boundary conditions separately. The essential point here is that the disturbances, in all cases, must be expanded in all possible forms of time function constituting the time behaviour of the quantities in the system, i.e. in terms of some suitable sets of normal modes which must be complete for such an expansion to be possible. Thus if A'(x, y, z, t) is a typical wave component describing a disturbance, we expand it in the manner,

$$A'(x, y, z, t) = A(z)\exp[i(\alpha x + \beta y + ct)],$$

Where $K = [\alpha^2 + \beta^2]^{1/2}$ the real wave number of the disturbance and c is is a constant to be determined and in general, is complex. It is to be remembered that the real parts are to be taken to get physical quantities, this being permissible for the linear problems. Further, since the perturbation equations are linear, the reaction of the system to a general disturbance can be determined if we know the reaction of the system to disturbances of all assigned wave numbers. In particular, the stability of the system will depend on its stability to disturbances of all wave numbers, and instability will follow from the instability with respect to even one wave number.

The assumption that a disturbance can be represented by wave components, according to the method of normal modes, serves to separate the variables and reduces the linearized equations of motion from partial to ordinary differential equations. The final process consists of solving the set of coupled, homogeneous, ordinary linearized differential equations governing the amplitude A (z), subject to appropriate boundary conditions of the problem under investigation. Indeed, the requirement that the equation allows a non-trivial solution satisfying the various boundary conditions leads directly to a characteristic value problem for c. In general, the characteristic value for c will be complex, whose real and imaginary parts will apart from various modes, depend on the physically significant parameters involved in the system.

Now, if the subscript k is attached to c in order to emphasize the fact that different values of c correspond to various modes appropriate to a particular problem (distinguished by k) we have,

(i) $C_k^{(r)} < 0$ for all k => stability

(ii) $C_k^{(r)} > 0$ for at least one k => instability

(iii) $C_k^{(r)}(R_1, R_2, R_3, \ldots\ldots\ldots\ldots, R_j) = 0 \Rightarrow$ The marginal state with respect to disturbance belonging to k.

Condition (iii) provides a locus in $(R_1, R_2 \ldots\ldots\ldots\ldots, R_j)$ space, which separates states which are stable from those, which are unstable with respect to the disturbance belonging to the particular mode k. Also if $C_k^{(r)} = 0$ implies $C_k^{(i)} = 0$ for every *k*, then the Principle of Exchange of Stabilities (PES) is valid at the marginal state and the instability sets in through stationary cellular convection. But if $C_k{}^{(r)} = 0$ implies $C_k{}^{(i)} \neq 0$ even for at least one k, then the PES is not valid at the marginal state and we have the case of over stability.

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