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A New Theorem introduced by Piyush Goel with Four Proof

Piyush Goel

Mathematics for Piyush is a Passion from his childhood he was so passionate about Mathematics used to play with Numbersdraw figures and try to get sides distance one day I draw a AP SERIES Right Angle Triangle(thinking that the distance between the point of intersection of median & altitude at the base must be sum of rest sides that was in My Mind).

And at last Piyush Succeed. This new Theorem proved with Four Proof(Trigonometry/Co-ordinates Geometry/Acute Theorem/Obtuse Theorem).

Here are the Proofs:

This Theorem applies in Two Conditions:

1. The Triangle must be Right-Angled.
2. Its Sides are in A.P. Series.

1. Proof with Trigonometry
2. Proof with Obtuse Triangle Theorem
3. Proof with Acute Triangle Theorem
4. Proof with Co-ordinates Geometry

Four Proof (TRIGONOMETRY/CO-ORDINATES/OBTUSE TRIANGLE/ACUTE TRIANGLE)
(By PIYUSH GOEL)

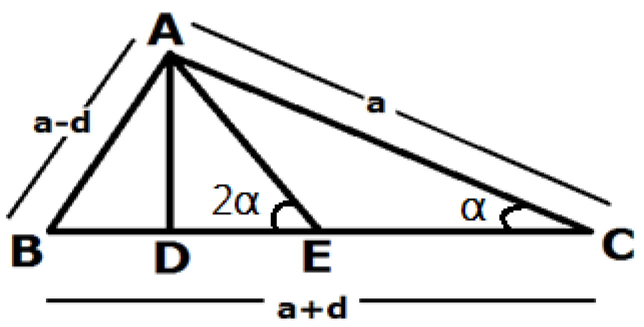
Theorem: In a Right-Angled Triangle with sides in A.P

Theorem: In a Right-Angled Triangle with sides in A.P. Series, the distance between the point of intersection of median & altitude at the base is 1/10 Th the sum of other two sides.

This Theorem applies in Two Conditions:

1. **The Triangle must be Right-Angled.**
2. **Its Sides are in A.P. Series.**

1. **Proof with Trigonometry**



tan α **=AD/DC**

AD= DC tan α **-----------------1**

tan α= **AD/DE**

AD= DE tan 2α **----------------2**

DC tan α**= DE** tan 2α

(DE+EC) tan α**= DE** tan 2α

DE tan α **+ EC** tan α**= DE** tan 2α

DE tan α **+ EC** tan α **= 2 DE** tan α **/ (1-** tan^2 α**)**

DE tan α **- DE** tan^3 α**+ EC** tan α**–EC** tan^3 α **= 2 DE** tan α

EC tan α**–EC** tan^3 α**- DE** tan^3 α**= 2 DE** tan α**– DE** tan α

tan α**(EC – EC** tan^2 α**– DE** t an^2 α**)= DE** tan α

DE tan^2 α **- DE = EC** tan^2 α **– EC**

-DE (tan^2 α**+ 1) = -EC (1 -** tan^2 α**)**

DE (sin^2α **/**cos^2α**+ 1) = EC (1-** sin^2α **/**cos^2α**)**

DE (sin^2α**+** cos^2α**/**cos^2α**) = EC (**cos^2α**-** sin^2α**/**cos^2α**)**

DE (sin^2α **+** cos^2α**) = EC(**cos^2α -sin^2α**)**

DE (sin^2α **+** cos^2α**) = EC (**cos^2α -sin^2α) **........where (**sin^2α **+** cos^2α**=1) & (**cos^2α -sin^2α=cos 2α **)**

DE= EC cos 2α

cosα **=a/a+d &**sinα**= (a-d)/ (a +d)**

cos^2α **= a ^{2}/ (a +b) ^{2}**

sin^2α **= (a-d) ^{2}/ (a+ d) ^{2}**

DE= EC (cos^2α **–** sin^2α**)**

= EC (a ^{2} / (a +b) ^{2} - (a-d) ^{2}/ (a +d) ^{2}

= EC (a ^{2} - (a-d) ^{2}/ (a +d) ^{2}

= EC (a –a +d) (a+ a-d)/ (a+ d) ^{2}

= EC (d) (2 a -d)/ (a+ d) ^{2}

= (a +d)/2(d) (2 a -d)/ (a +d) ^{2} ------------- where EC= (a +d)/2

= (d) (2 a -d)/2(a +d)

= (d) (8 d -d)/2(4 d+d) ------------------where a= 4 d (as per the Theorem)

= 7 d ^{2} /2 (5 d)

= 7 d /10

= (3 d+4 d)/10= (A B+AC)/10

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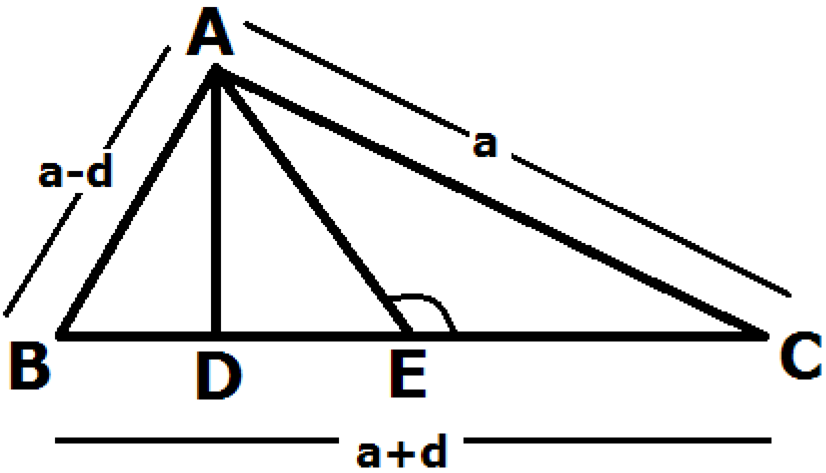
2. Proof with Obtuse Triangle Theorem



$AC^2 = EC^2 + AE^2 + 2\,CE.\,DE$ where $EC = (a+d)/2$, $AE = (a+d)/2$

$a^2 = (a+d/2)^2 + (a+d/2)^2 + 2(a+d)/2\ DE$

$= (a+d/2)(a+d+2\,DE)$

$= (a+d/2)(a+d+2\,DE)$ where $a = 4d$

$16d^2 = (5d/2)(5d+2\,DE)$

$32d/5 = 5d + 2\,DE$

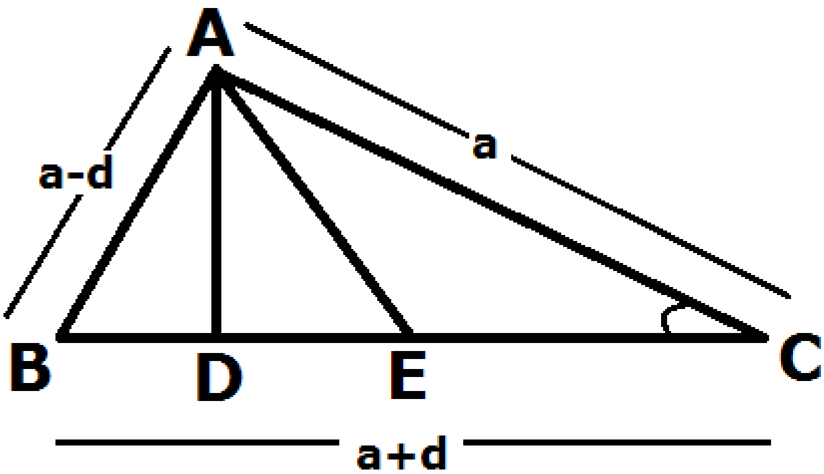
$32d/5 - 5d = 2\,DE$

$32d - 25d/5 = 2\,DE$

$DE = 7d/10$

$= (3d+4d)/10 = (AB+AC)/10$

3. Proof with Acute Triangle Theorem



$AB^2 = AC^2 + BC^2 - 2\,BC.DC$

$(a-d)\,2 = a^2 + (a+d)^2 - 2(a+d)(DE+EC)$ where $AB = (a-d)$, $AC = a$, $BC = (a+d)$ & $EC = (a+d)/2$

$(a-d)^2 - (a+d)^2 = a^2 -2(a+d)(DE+EC)$

$(a- d -a-d) (a -d +a +d) = a^2 -2(a+ d) (2 DE+a+d)/2$

$2(-2 d) (2 a) ^= 2 a^2 -2(a +d) (2 DE+a+d)$

$-8 ad ^- 2 a^2 = -2(a +d) (2 DE+a+d)$

$-2 a (4 d +a) = -2(a +d) (2 DE+a+d)$

$a (4 d + a) = (a +d)(2 DE+a+d)$

$4 d (4 d + 4 d) = (4 d+d) (2 DE+4 d+d)$

$4 d (8 d^) = (5 d) (2 DE+5 d)$

$32 d^2/5 d = (2 DE+5 d)$

$32 d/5 = (2 DE+5 d)$

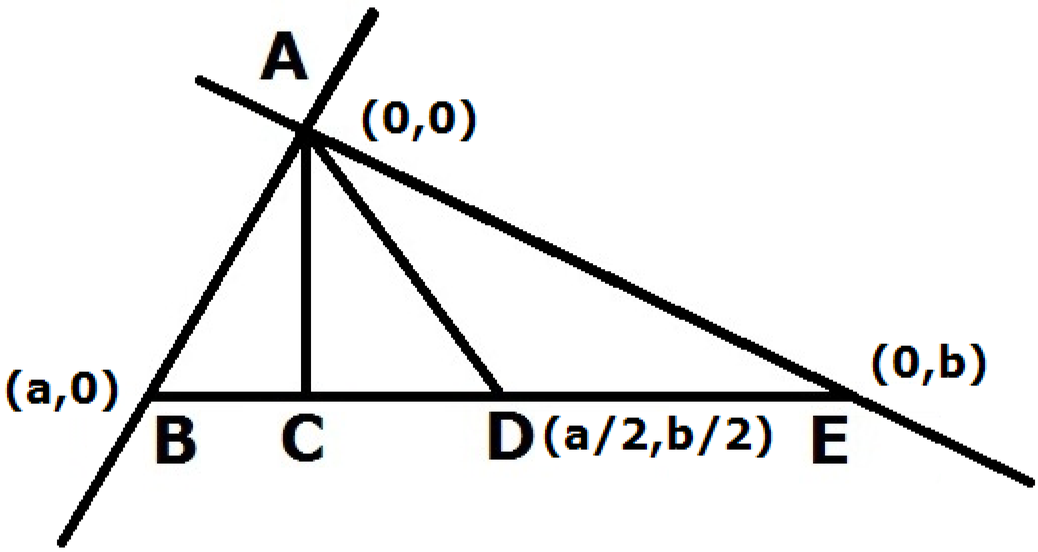
$32 d/5 - 5 d = 2 DE$

$(32 d - 25 d)/5 = 2 DE$

$DE = 7 d/10$

$= (3 d+4 d)/10 = (AB+AC)/10$

4. Proof with Co-ordinates Geometry



In Triangle A B C,point A,B&C,s co-ordinates are respectively (0,0) ,(a,0)&(0,b).
Point D is middle point ,co-ordinates of Point Dis (a/2,b/2)

Equation of BE is .... (Two Points equation)
$Y - Y1 = (X - X_1)(Y2 - Y1)/(X2 - X_1)$

$Y - 0 = b-0/0-a(X - a)$

$Y = -b/a(X) + b$------------------- (1)

$M_1 = -b/a$

For perpendicular

$M_1 M_2$= -1
M_2= -1/M_1
So M_2= a/b
Equation of AC
Y – 0 = a/b(X-0)
Y=a/b(X) ------------------ (2)
Put Y value in equation (1)
a/b(X) + b/a(X) =b
X (a^2+b^2/a b) = b
X = a b^2/ (a^2 + b^2)
To get Value of Y, put X value in equation (2)
Y = a/b (a b^2/ (a^2+b^2)
Y = a^2 b/ (a^2+b^2)
Here we got co-ordinates of Point C - a b^2/ (a^2 + b^2), a^2 b/ (a^2+b^2) and co-ordinates of point D is (a/2, b/2) because d is midpoint.
As per the A.P Series (z-d,z,z+ d)
Hers a= z-d, b= z, c = z+ d
$(z +d)^2$= $(z-d)^2$+ z^2
$(z +d)^2$- $(z-d)^2$ = z^2
(z +d +z-d)(z +d - z + d) = z^2
(2 z)(2 d) = z^2
4 z d = z^2
4 d = z
Put value of a & b
a b^2/ (a^2 + b^2), a^2 b/ (a^2+ b^2) & (a/2, b/2)
a b^2/ (a^2 + b^2) = 48 d/25
a^2 b/ (a^2+b^2) = 36 d /25
a/ 2=3 d/2
b/ 2 =4 d/2
CD^2= $(48 d/25 -3 d/2)^2$-$(3 6 d/25-4 d/2)^2$
= $(96 d-75 d/50)^2$ + $(72 d-100 d/50)^2$
= $(21 d/50)^2$ + $(-28 d/50)^2$
= (441 d^2/2500) + (784 d^2/2500)
= (1225 d^2/2500)
CD= 35 d/50 = 7 d/10
= 7 d/10 = (3 d+4 d)/10 = (A B+A E)/10